The free electron model

The Drude Model: the free electrons gas in the relaxation time approximation. Calculation of DC and AC conductivity, dielectric constant and Hall coefficient; plasma frequency. Optical properties of a free electron metal: hints. Quantum approach to the free electron case: the Sommerfeld model. Boundary conditions for the free electrons gas, Fermi energy and Fermi level, density of states. Failures of the free electron model. (Chapt. 1, 2 [1])

Electrons in a periodic potential

The ions periodic potential and Von Karman boundary conditions; the Bloch's theorem, Schrodinger equation in momentum space. Fermi energy and Fermi surface. Electrons in a weak periodic potential: gap opening near a Bragg plane, energy bands and their representation (extended, reduced, and repeated zone scheme). The Fermi surface and its deformation close to Bragg planes. (Chapt. 8, 9 [1]). Different methods for the calculation of electron energy levels in a solid: tight-binding (TB) method (application of the TB method to a band arising from s-levels), (Chapt. 10 [1]); orthogonalized plane wave method and pseudopotentials (Chapt. 11 [1]).

The semiclassical model

The main hypotheses of the semiclassical model; interplay between band structure and electronic transport. Effective mass, effective mass tensor, electrons and holes. Electronic motion in presence of a constant electric field, of a constant magnetic field and in presence of perpendicular magnetic and electric fields; closed and open orbits. Hall effect and magnetoresistance. (Chapt. 12 [1]) Calculation of DC metal conductivity in the relaxation time approximation. (Chapt. 13 [1])

Fermi Surface and band structure

The shape of the Fermi surface. Band structure for alkali and noble metals; representation of the Fermi surface. Methods used to probe the Fermi surface: the de Haas – Van Alphen effect. (Chapt. 14 [1]) Optical properties of metals, direct transitions. (Chapt. 15 [1]; [2])

The contribution of electron-electron interactions

Hartree and Hartree-Fock approximation; Slater determinant as N-electrons wavefunction. Representation of the Slater determinant as a combination of single electron wavefunction permutations ([3]). Representation of the effect of one-body and two-body operators on the Slater determinant. Interaction effects on electron levels energy; exchange contribution. (Chapt. 17 [1]). Second quantization approach: outline. Construction and destruction operators, (Chapt. 2 [4]) field operators. Representation of one-body and two-body operators in second quantization. Electron-electron interaction: direct and exchange processes. Electrons gas susceptibility and contribution to the metal dielectric constant (screening) (Chapt. 17 [1]).

The contribution of lattice vibrations

Failure of the static lattice approach; adiabatic and harmonic approximations. Normal modes of a linear chain of atoms; normal modes of oscillation of the three-dimensional lattice. Dynamical matrix approach; acoustical and optical branches. **Oscillations representation in terms of space co-ordinates and in terms of normal modes: phonons.** Phonons: energy dispersion curve, contribution to specific heat, Debye model; comparison with the electrons contribution. Probes used to access phonons energy dispersion curves: neutrons, photons (Chapt. 22 - 24 [1]).

Electron-phonon interaction: temperature dependence of electronic conductivity, umklapp processes and crystal momentum conservation (Chapt. 25 [1]).

Phonons contribution to the dielectric constant of a metal; phonons contribution to electronelectron interaction: effective electron-electron interaction (Chapt. 26 [1]). Binding energy of a Cooper pair (Section 3.3.1 [4]); features that stabilize the Cooper pair: gap opening and Fermi sphere. Properties of superconductors: outline. (Chapt. 3[4])

Dielectric constant of a ionic crystal: atomic and displacement polarizability. Clausius-Mossotti equation; dielectric constant dependence on frequency in the electrostatic approximation. (Chapt. 27 [1])

[1] Solid State Physics, Neil W. Ashcroft and N. David Mermin

- [2] Representations of the shape of the Fermi Surface for different materials may be found here or here.
- [3] The Slater determinant.
- [4] Appunti di Meccanica Quantistica, prof. Fabio Ortolani, Università degli Studi di Bologna
- [5] Many Body Quantum Theory in Condensed Matter Physics, H. Bruus and K. Flensberg.

Note: the examination will not focus on the parts of the program that are written in **blue**; however, one of those parts may be the topic of the first question, chosen by the student.