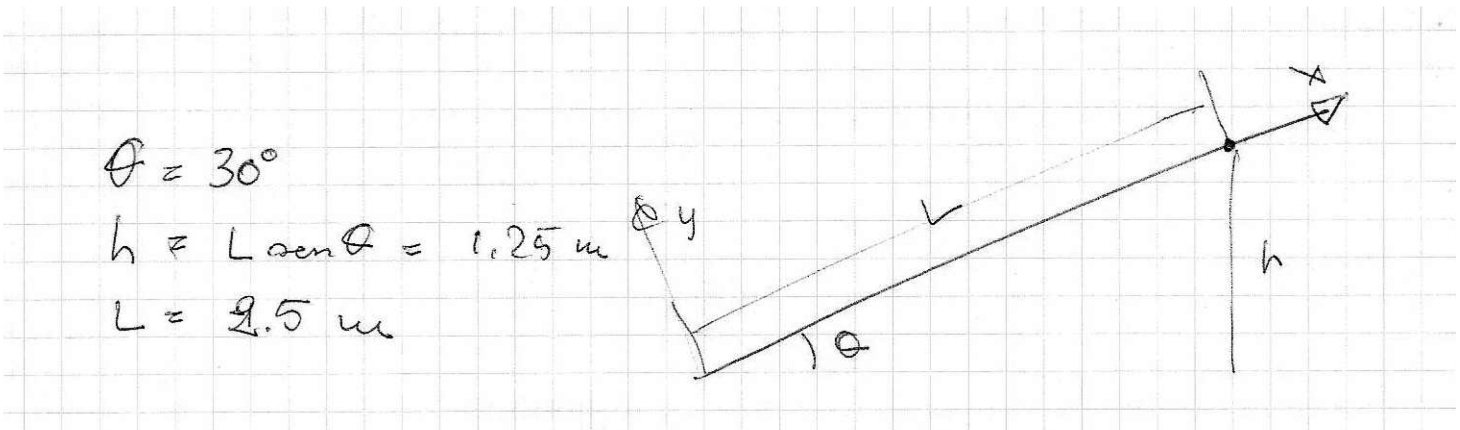
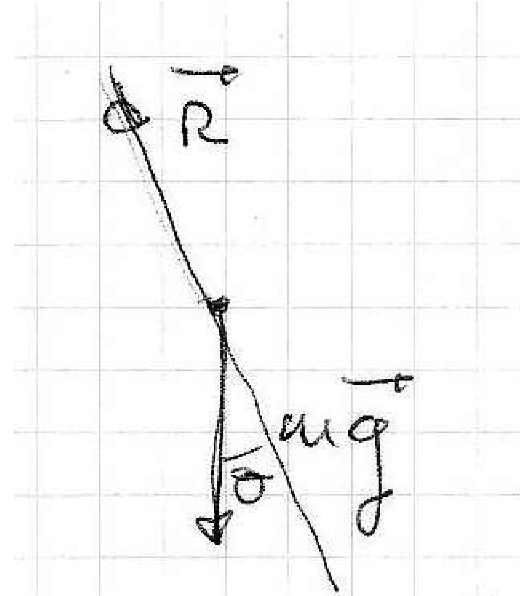
Esercizio risolto: piano inclinato



Un giocatore di minigolf cerca di mandare la pallina in una buca che si trova a 2.5 m su un piano inclinato di 30° rispetto all'orizzontale. Ignorando il rotolamento della pallina e trascurando l'attrito, riuscirà nell'intento se, alla base del piano inclinato, imprime alla pallina una velocità v_0 = 5 m/s ?

E se ci fosse un attrito dinamico con coefficiente μ_K = 0.3 ?

$$
\begin{cases}
y: \quad R - mg\cos\theta = 0 \\
x: \quad -mg\,\text{sen}\theta = m\,a_x
\end{cases}
$$



$$a_x = -\,g\ \text{sen}\theta$$

$$\boldsymbol{a_x = \frac{dv_x}{dt}}$$

$$\boldsymbol{\int_{v_0}^{v_x} dv_x = \int_0^t a_x\, dt}$$

$$v_x - v_0 = \int_0^t -g \sin \theta \, dt$$

$$v_x = -\, g\, sen\theta\, t + v_0$$

$$v_x = -\,g\ sen\theta\ t\ +\ v_0$$

$$\boldsymbol{v_x = \frac{dx}{dt}}$$

$$\boldsymbol{\int_{x_0}^{x} dx = \int_0^t v_x\, dt}$$

$$x(t) - x_0 = \int_0^t (-\,g\ sen\theta\ t\ +\ v_0)\, dt =$$

$$x(t) = -\,\frac{1}{2}\,\mathrm{g}\ \sin\theta\ \ t^2\ +\ v_0 t + x_0$$

$$x(t) = -\frac{1}{2} g \operatorname{sen}\theta \, t^2 + V_0 t + x_0$$

$$V_x(t) = -g \operatorname{sen}\theta \, t + V_0$$

$$x_0 = 0$$

$$\begin{cases} x_{MAX} = -\frac{1}{2} 0.5 \cdot 9.8 \, t_M^2 + V_0 t_M \\ V_x(t_M) = 0 = -4.9 \, t_M + V_0 \end{cases}$$

$$t_M = \frac{V_0}{4.9}$$

$$x_M = -2.45 \frac{V_0^2}{4.9^2} + V_0 \frac{V_0}{4.9}$$

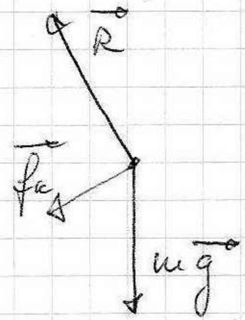
$$= -0.102 \, V_0^2 + 0.204 \, V_0^2$$

$$= 0.102 \, V_0^2$$

$V_0 = 5 \, m/s$

$x_M = 2.55 \, m$

Con attrito μ_D



$$y: \quad R - mg\cos\theta = 0$$

$$x: \quad -mg\,\mathrm{sen}\,\theta - f_k = ma$$

$$f_k = \mu_k R$$
$$= 0.3\, mg\cos\theta$$

$$-mg\,\mathrm{sen}\,\theta - \mu_k\, mg\cos\theta = ma$$

$$a = -g\left(\mathrm{sen}\,\theta + \mu_k \cos\theta\right)$$
$$= -9.8\left(0.5 + 0.3 \cdot 0.866\right)$$
$$= -9.8\left(0.5 + 0.260\right)$$
$$= -7.45 \quad m/s^2$$

$$\begin{cases} x_{MAX} = -\frac{1}{2} \cdot a\, t_m^2 + V_0\, t_m \\ 0 = -a\, t_m + V_0 \end{cases}$$

$$t_m = \frac{V_0}{a}$$

$$x_m = -\frac{1}{2} a \frac{V_0^2}{a^2} + V_0 \frac{V_0}{a} =$$

$$= \frac{1}{2} \frac{V_0^2}{a}$$

$$= \frac{1}{2} \frac{25}{7.45} = 1.68 \quad m$$