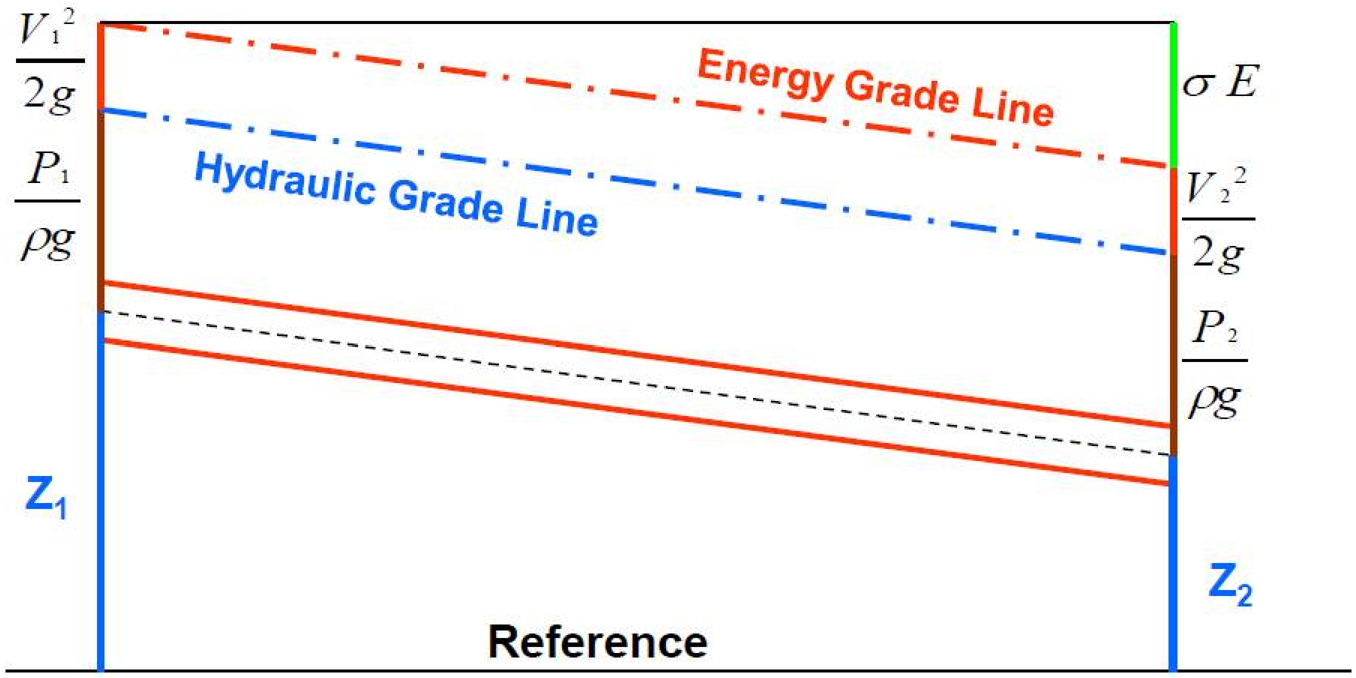
Energy, Piezometric and pressure Head

Bernoulli equation states that for constant flow, an energy balance between two pipes cross section can be written as: $E_1=E_2+\sigma E$ or expressed in developed form:

$$Z_1+\frac{P_1}{\rho g}+\frac{V_1^2}{2g}=Z_2+\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+\sigma E$$



Energy, Piezometric and pressure Head

- **Elevation Head:** this is an amount of flow potential energy in one cross section defined by the elevation. This corresponds to **Z** in cross section
- **Pressure Head**: $P/\rho g$ this is an amount of the flow potential energy in one cross section defined by the water pressure.
- **Piezometric Head**: $Z + P/\rho g$ this is the sum of elevation and pressure head in one cross section.
- **Velocity Head**: $V^2/2g$ this is an amount of flow kinetic energy in one cross section defined by the water velocity

Hydraulic Head

In physics every movement is caused by a potential gradient (ex: heat, electricity...). A system tends to decrease its potential energy and to reach an equilibrium status

- the displacement is proportional to the potential gradient
- the displacement is from the highest to the lowest potential

Which is the potential that drives flow in aquifers and how can we represent it?

- hydraulic head (piezometric head) h at a certain point = water energy at a certain point

- Water moves from higher to lower hydraulic heads

Il carico idraulico è composto da tre parti:

- ☐ Velocity head (kinetic energy).
- ☐ Elevation head determined by the height of the point relative to some reference plane (gravitational head)
- ☐ Pressure head determined by the height of the water column resting on that point (hydrostatic head)

$$H = \frac{V^2}{2 \cdot g} + \frac{P}{\rho_w g} + z$$

$$H \approx \frac{P}{\rho_\omega \cdot g} + z = \frac{u}{\rho_\omega \cdot g} + z$$

➢ Carico idraulico h in un certo punto= energia dell’acqua in quel punto

➢ L’acqua si muove dai punti a carico idraulico più elevato a quelli a carico idraulico più basso

The hydraulic head is composed of two parts:

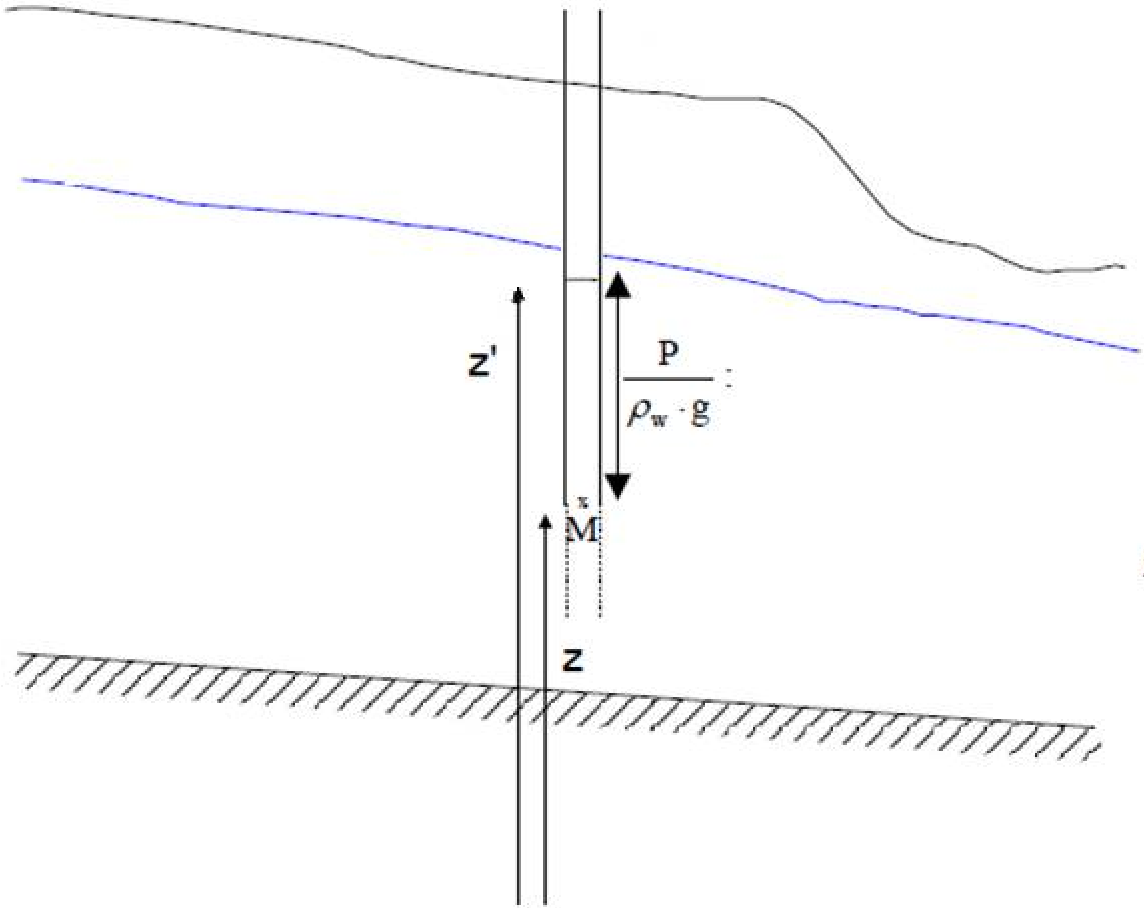
- ❑ Carico di velocità (energia cinetica).
- ❑ Carico di elevazione determinato dall’elevazione di quel punto relativamente ad un piano di riferimento
- ❑ Carico di pressione determinato dall’altezza della colonna d’acqua in quel punto

$$H = \frac{V^2}{2 \cdot g} + \frac{P}{\rho_w g} + z$$

$$H \approx \frac{P}{\rho_\omega \cdot g} + z = \frac{u}{\rho_\omega \cdot g} + z$$

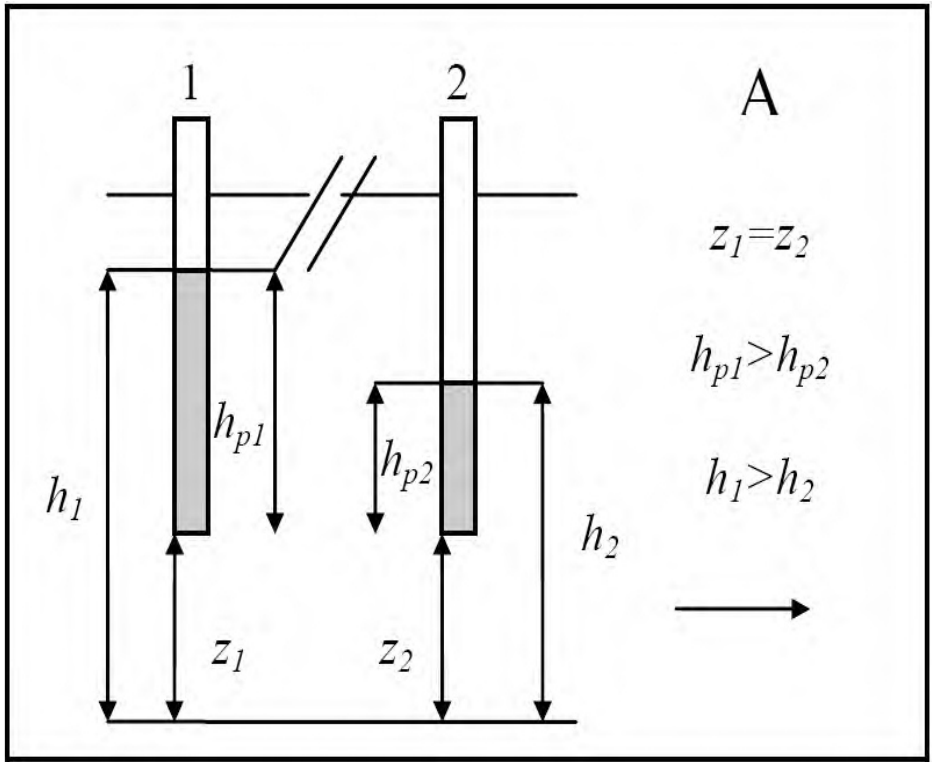
The energy gradient has 3 components:

Elevation:	**potential energy**	z
Pressure:	**“virtual” elevation**	$p/\rho g$
Velocity:	**kinetic energy**	~~$v^2/2g$~~

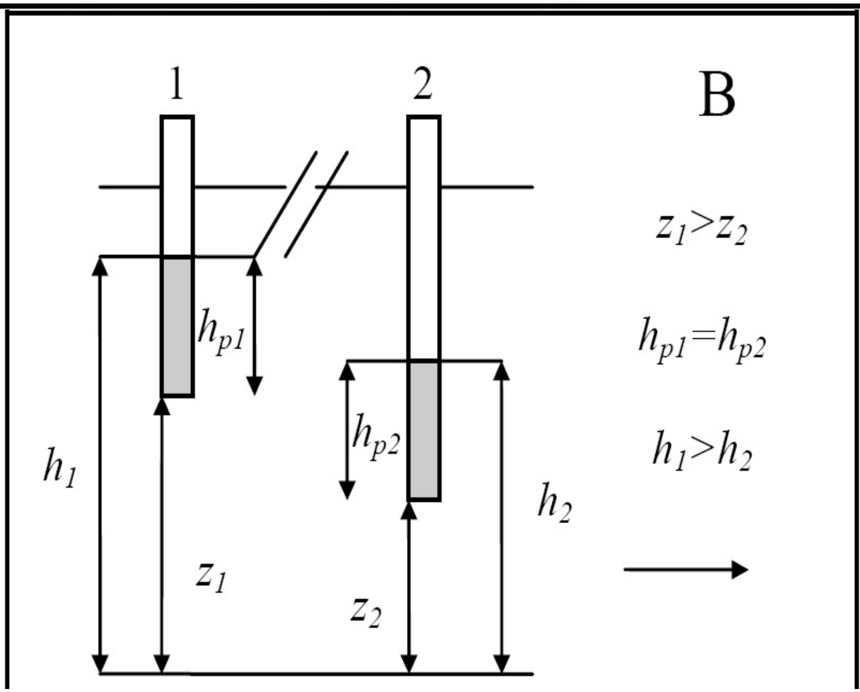


$$H = \frac{P}{\gamma_w} + z = \frac{\gamma_w\left(z' - z\right)}{\gamma_w} + z = z'$$

Piezometric surface: the imaginary surface to which groundwater rises under hydrostatic pressure in wells or spring



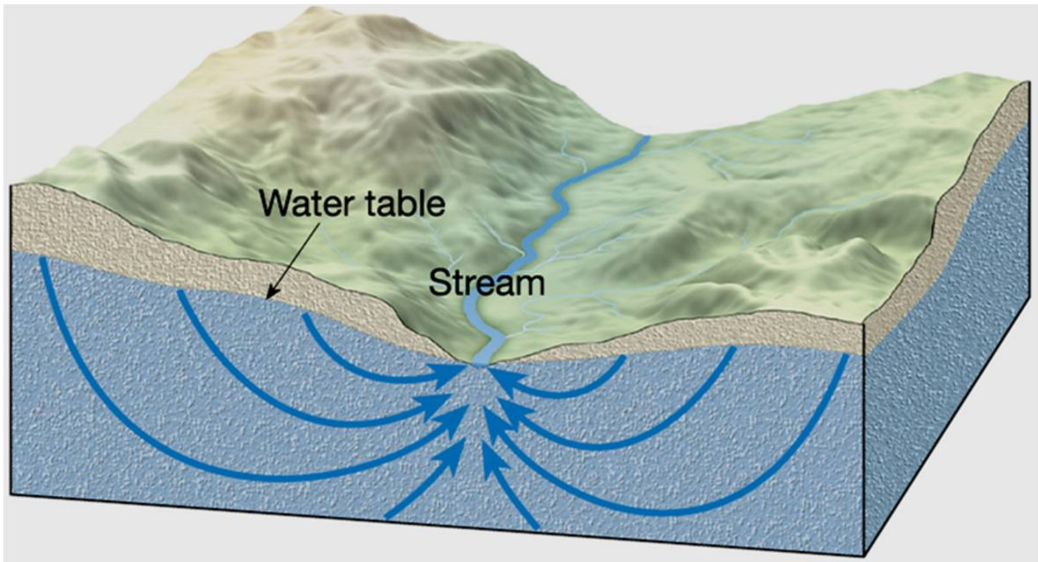
It's fluid pressure that controls the direction of flow between two points of the same elevation
Flow from hp_1 to hp_2



Same water pressure but different elevation
Flow from hp_1 to hp_2

Sistema di flusso sotterraneo

- Falda si muove da carico h più elevato al carico più basso.
- In un acquifero libero h è legato alla morfologia della tavola d'acqua che spesso "sposa" la topografia.



Hydraulic head (piezometric head):

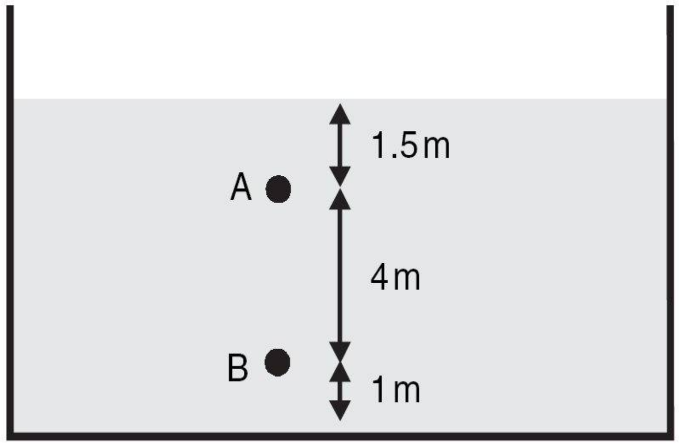
$$h(m) = \frac{P}{\rho g} + z$$

P : pressure in A and B (Pa)

z : height of the water column resting on that point (L)

ρ : water density (ML^{-3}) : ($kg\ m^{-3}$)

g : gravitational acceleration (LT^{-2}): 9.81 ms^{-2}



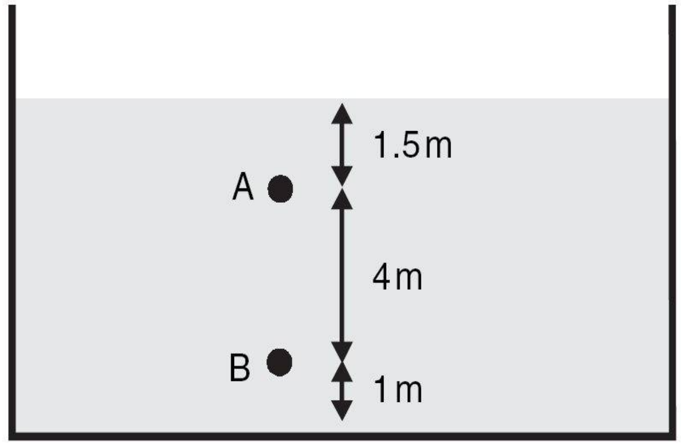
Water pressure in point A : 14 715 Nm^{-2}

Water pressure in point B : 53 955 Nm^{-2}

Does water move from B to A?

The movement is from the higher to the lower potential

$$h(m) = \frac{P}{\rho g} + z$$



The hydraulic head (m)

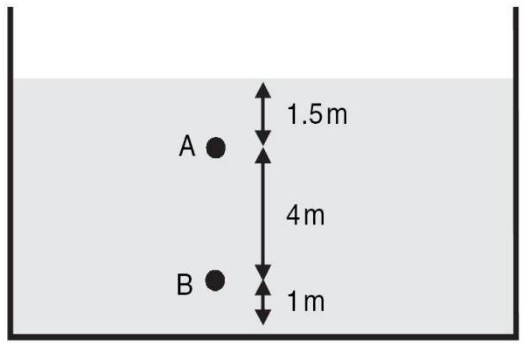
We divide per g :

$$\mathbf{h(m) = \frac{P}{\rho g} + z}$$

z : height of the point relative to some reference plane [L] : (m)

$$h_A = \frac{14715}{1000 \times 9.81} + 5 = 6.5m$$

$$h_B = \frac{53995}{1000 \times 9.81} + 1 = 6.5m$$



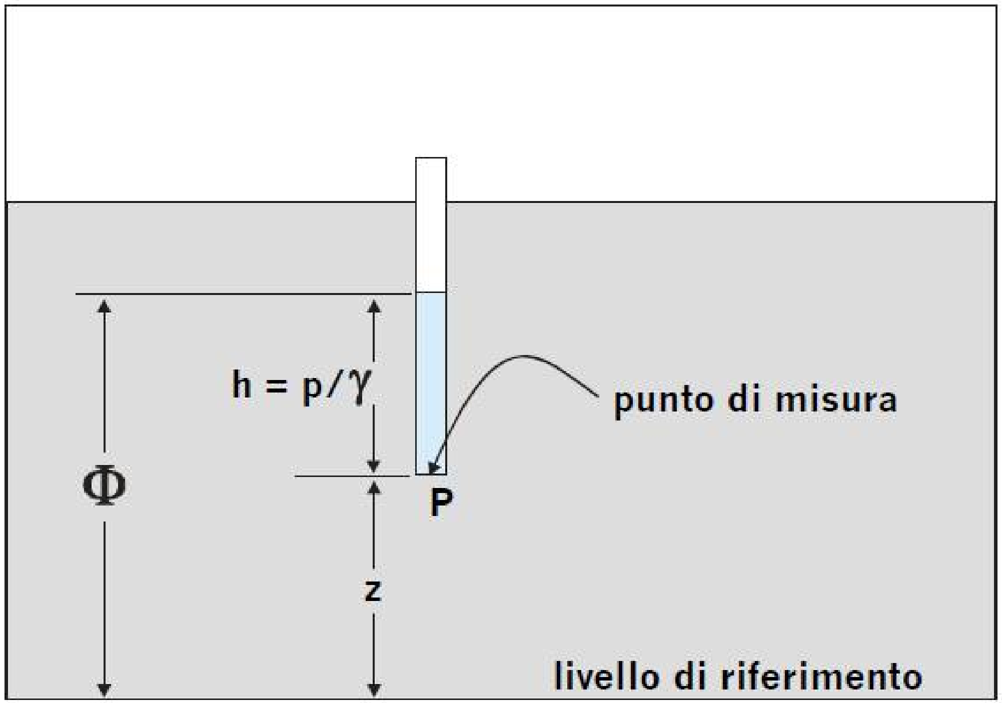
Nell'esempio in basso

$z = 2{,}2$ m

$p/\gamma = 9{,}8$ m

$\Phi = 9{,}8 + 2{,}2 = 12$ m (carico piezometrico al punto di misura P)

$P = \gamma h$



h = altezza di pressione; z = altezza di carico; Φ = carico idraulico; p = pressione; γ = densità

La pressione nel liquido al punto P di misura è regolata dal principio di Pascal:

$P = \gamma h$ γ = *peso di volume*

Come misuriamo h?

- Il carico piezometrico si può determinare misurando la profondità del livello di falda in un piezometro.
- Misurare l’ elevazione a cui l’ acqua risale in un pozzo o piezometro (osservare il “livello statico”) sopra qualche livello di riferimento.
- Tale elevazione è il carico relativo al mezzo che sta a contatto con il tratto filtrato del pozzo-piezometro.

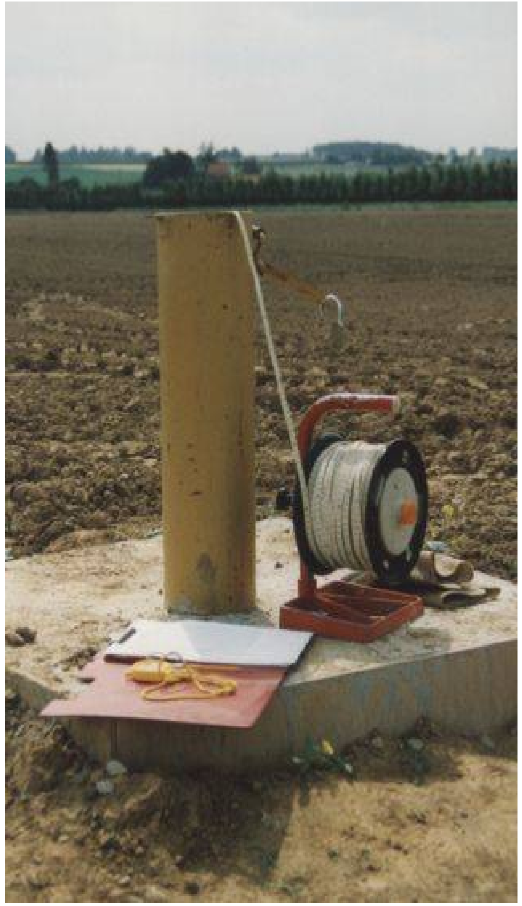
Un piezometro è un pozzo non in pompaggio, di solito di piccolo diametro e provvisto di un filtro attraverso cui entra l’acqua di falda. In alcuni casi è necessario usare dei piezometri a grappolo, molto vicini tra loro ma con filtri a diverse profondità, in questo modo si possono evidenziare movimenti verticali negli acquiferi.

Quando si effettua una misura nel piezometro si deve conoscere la quota topografica della bocca, la sua lunghezza totale, la lunghezza del tratto filtrante e la profondità alla quale si trova.

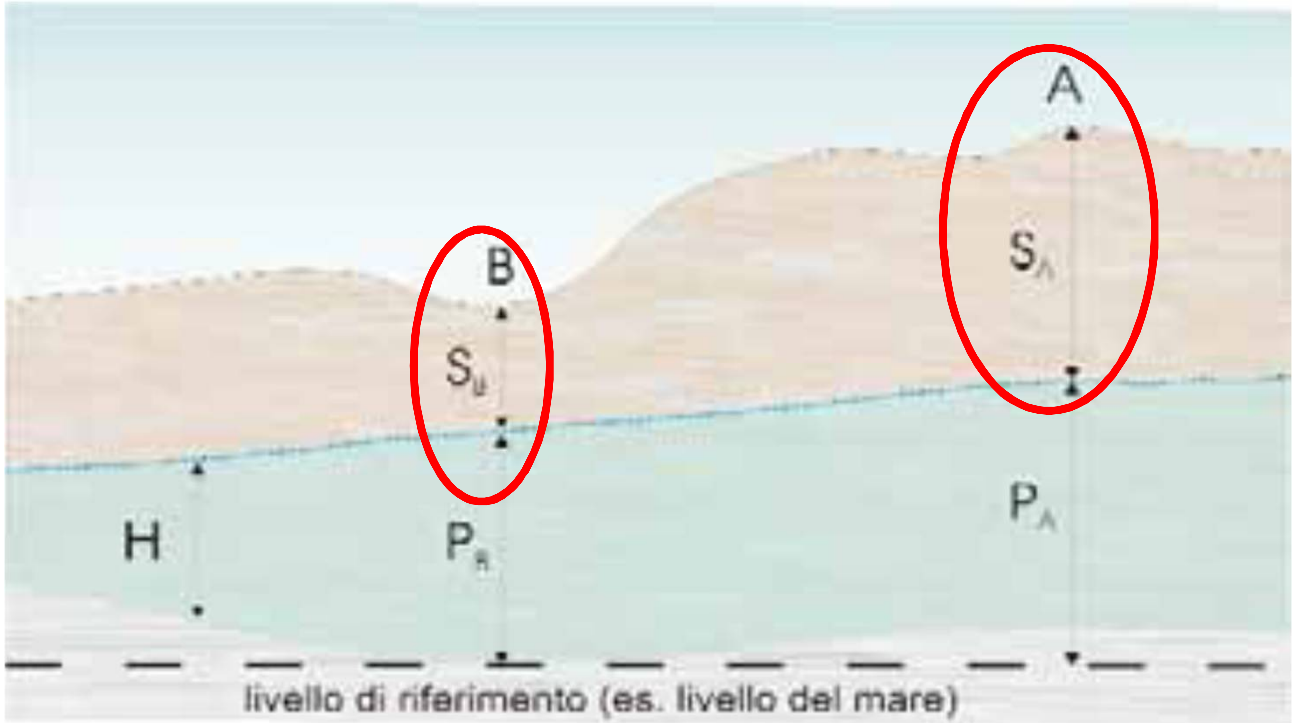
How to measure h?

Piezometers in durable casings can be buried or pushed into the ground to measure the groundwater pressure at the point of installation.

The pressure gauges (transducer) convert pressure into an electrical signal.



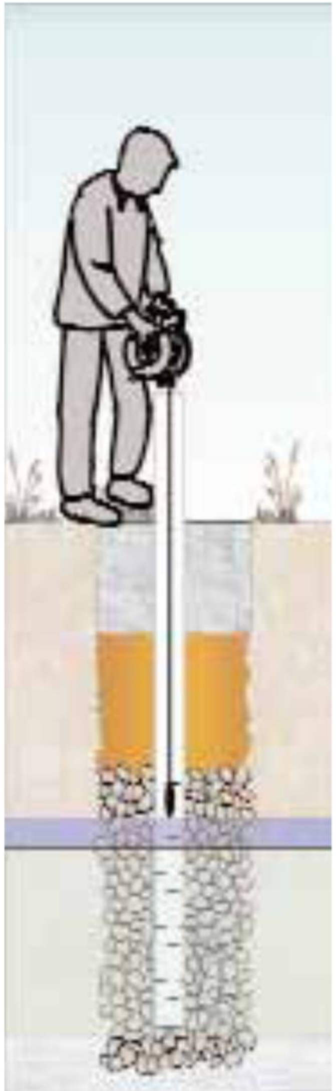
Misura della profondità della superficie di falda



E' detta soggiacenza della falda la profondità della sua superficie rispetto alla superficie topografica

L’idrogeologo usa un sondino (freatimetro) per misurare la profondità dell’acqua (soggiacenza) entro il pozzo, a partire dal boccaforo. La quota assoluta del boccaforo va conosciuta.

Misura della soggiacenza

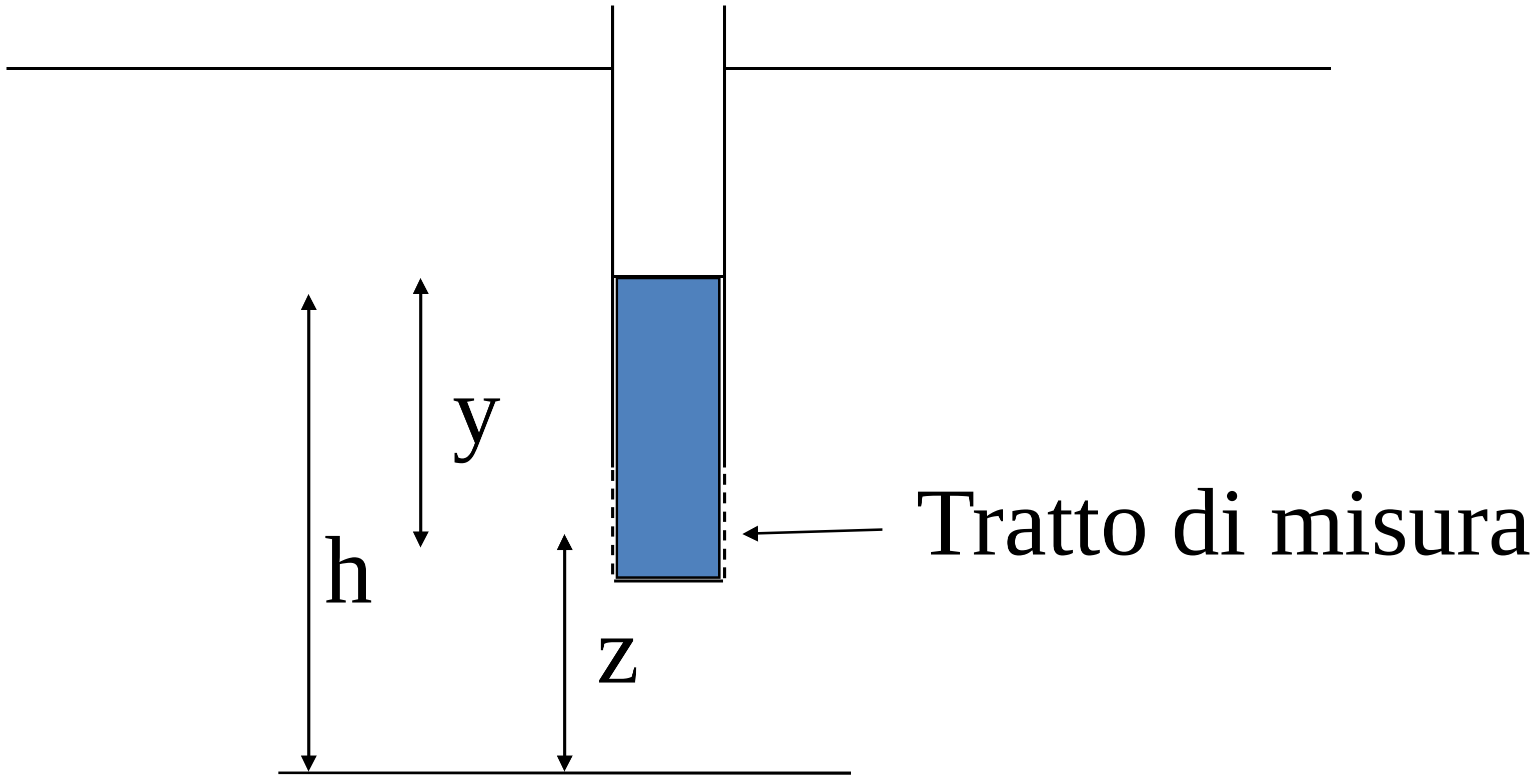


La misura della **soggiacenza** si effettua per mezzo del **freatimetro**.

Il **freatimetro** è uno strumento costituito da una sonda di lettura alimentata a batteria che viene calata all'interno del piezometro per mezzo di un cavo millimetrato o centimetrato.
Il circuito elettrico si chiude nel momento in cui la sonda viene a contatto con la superficie dell'acqua

Piezometri

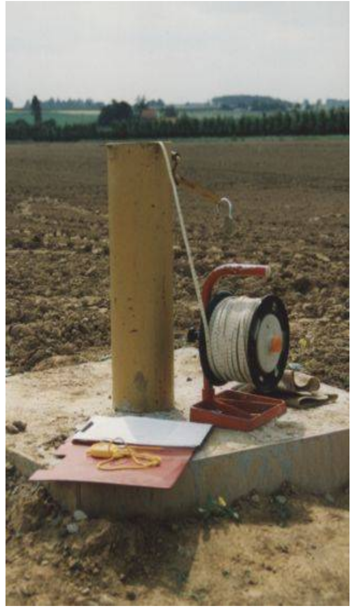
Tubi di piccolo diametro, aperti (*filtrati*) al fondo; misuratori di carico idraulico nella zona satura



Livello di riferimento
per z (es. livello mare)

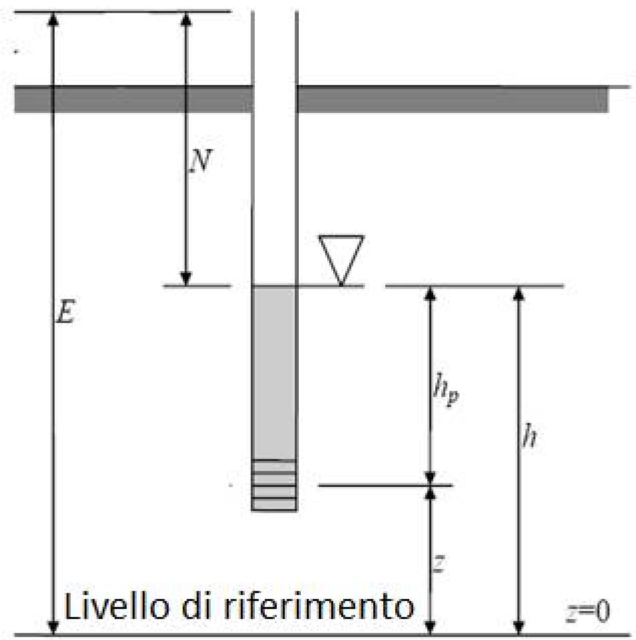
$y = P/\gamma$

- N is the measure of the depth of groundwater



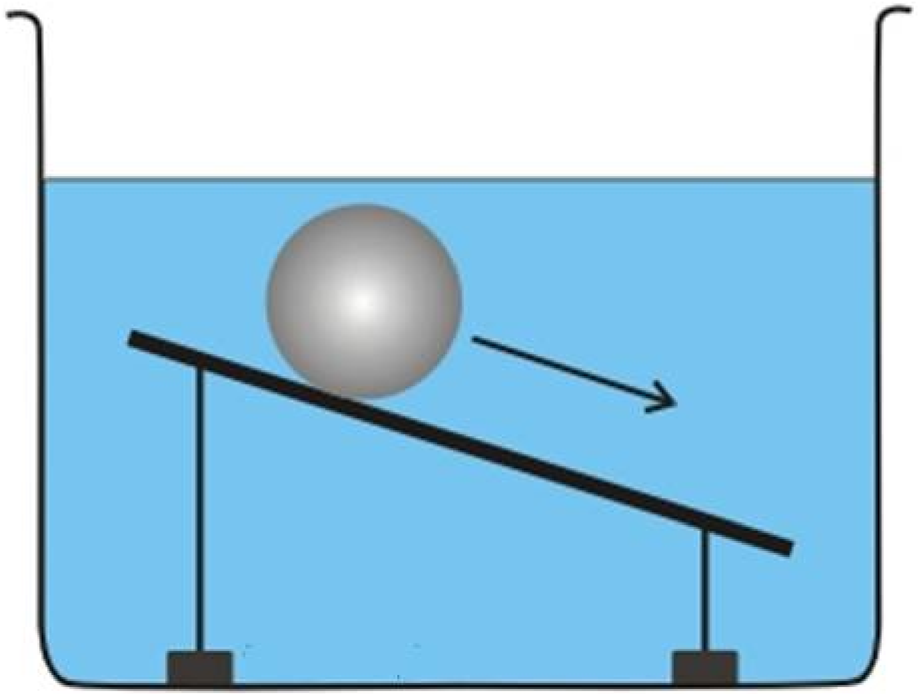
- E is the height of the highest point of the casing

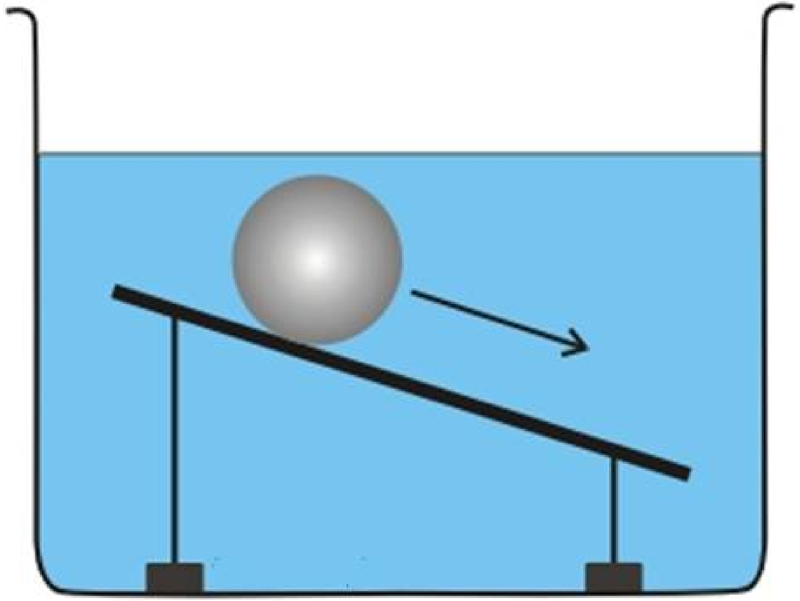
To obtain the hydraulic head (also called piezometric level)
H = E-N



Gradiente idraulico

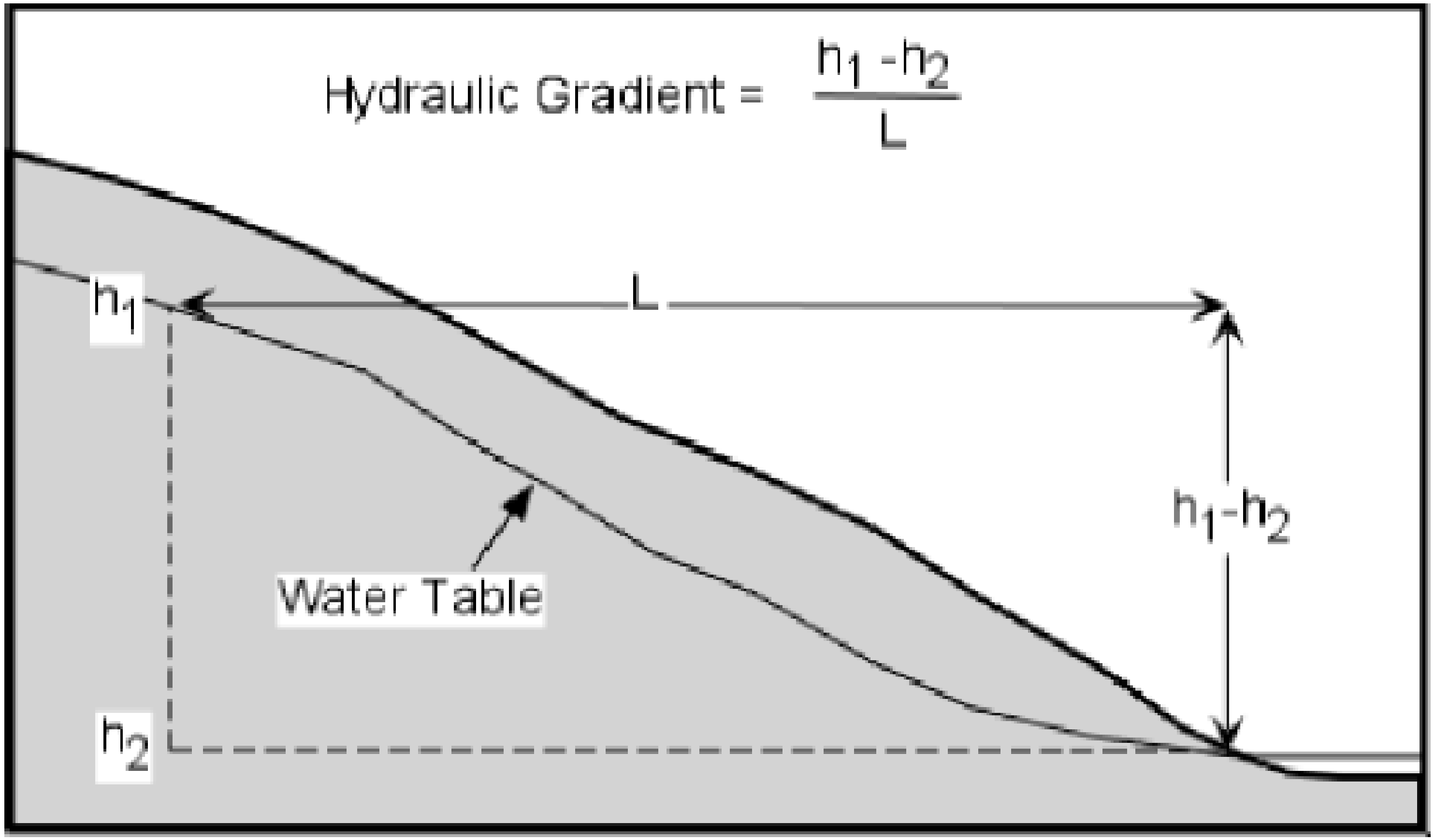
Il gradiente idraulico è il parametro che quantifica la forza motrice che permette all'acqua di spostarsi da un punto all'altro dell'acquifero vincendo la resistenza opposta dal terreno.
Per capire in modo intuitivo il rapporto fra gradiente idraulico, coefficiente di permeabilità (K) e filtrazione dell'acqua può essere utile immaginare un piano inclinato immerso in un acquario sul quale si muove una pallina d'acciaio.





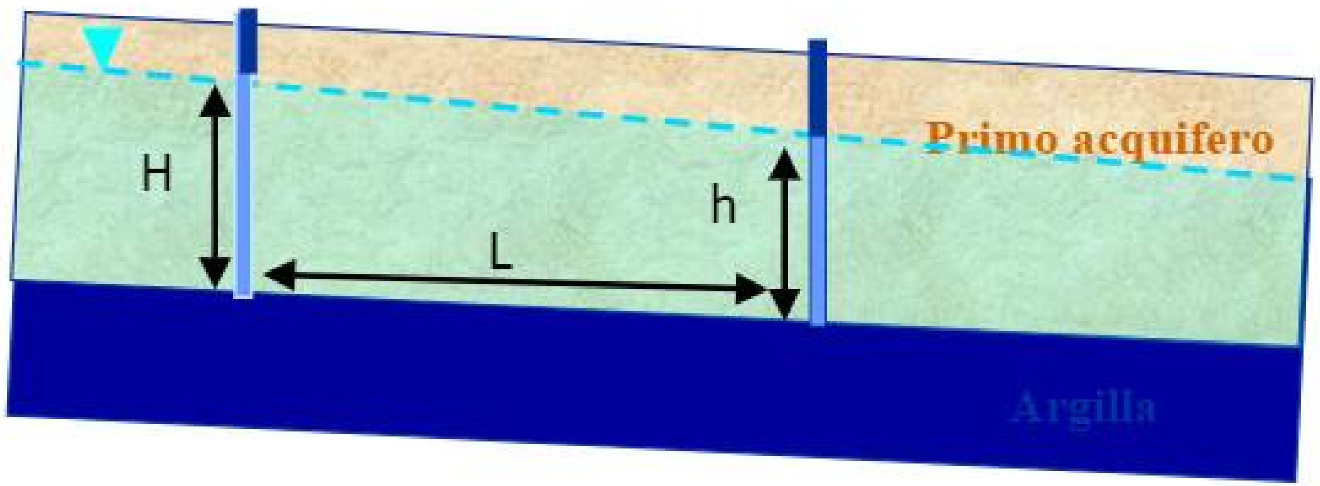
L'inclinazione del piano inclinato corrisponde al gradiente idraulico, la viscosità del fluido che riempie l'acquario corrisponde all’inverso coefficiente di permeabilità e la velocità della pallina corrisponde alla velocità di filtrazione dell'acqua nel terreno. La velocità della pallina aumenta all'aumentare dell'inclinazione del piano e al diminuire della viscosità del fluido.
A parità di inclinazione del piano la pallina scende più velocemente se l'acquario è riempito con acqua e scende più lentamente se l'acquario è riempito con olio.
Analogamente la velocità di filtrazione dell'acqua nel terreno, e quindi la portata di filtrazione, aumenta all'aumentare del gradiente idraulico e all'aumentare del coefficiente di permeabilità. Se il gradiente idraulico è nullo, situazione che nell'esempio equivarrebbe ad un piano orizzontale, manca la forza motrice e quindi non c'è movimento d'acqua a prescindere dal valore del coefficiente di permeabilità.

<u>Gradiente idraulico di un acquifero</u>

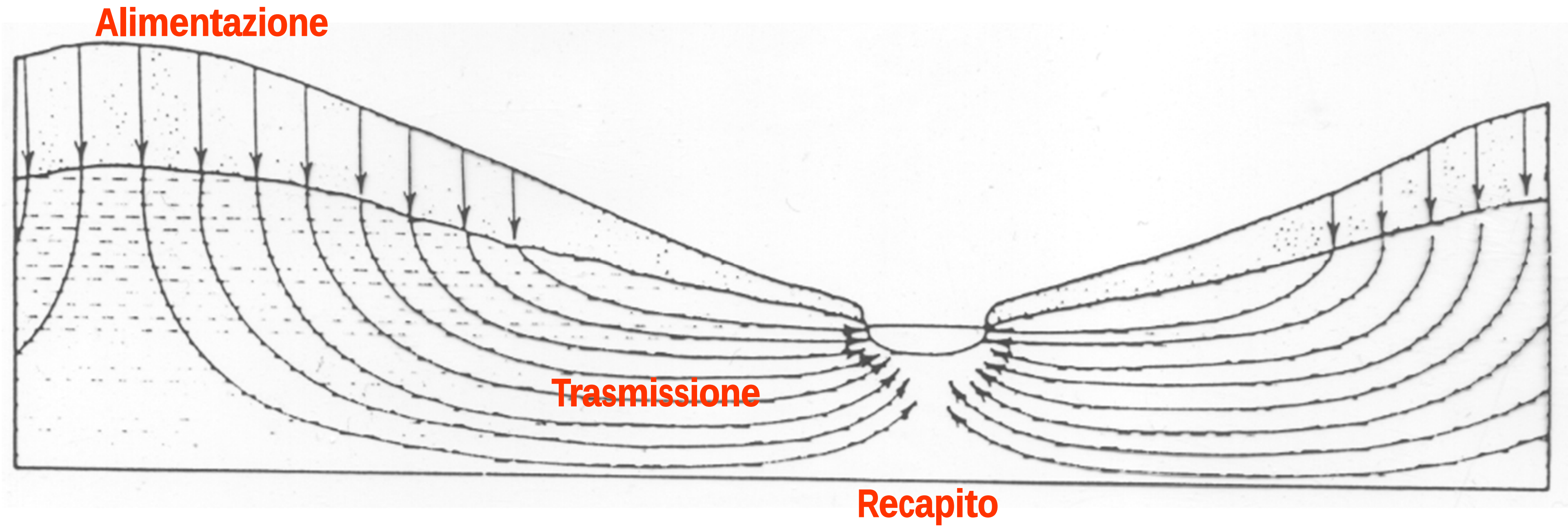


Gradiente idraulico: è la caduta di pressione (perdita di carico idraulico) per unità di lunghezza.
'Pendenza' della falda

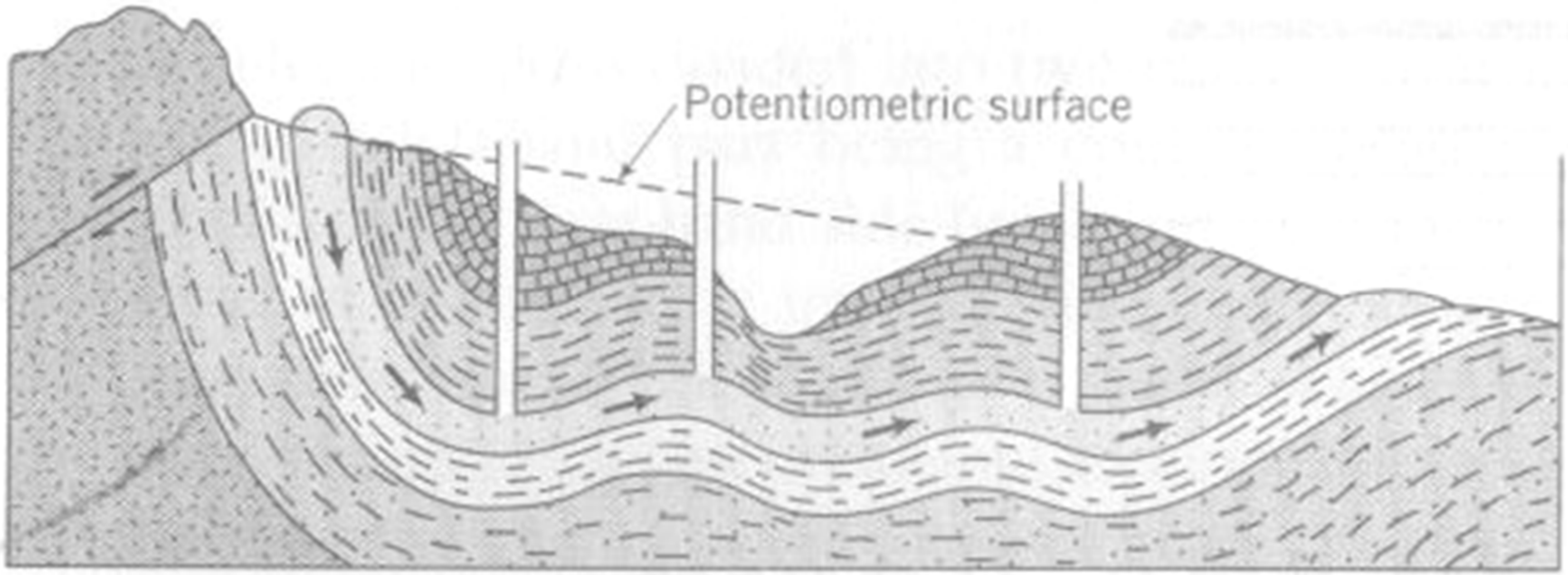
$$i = (H-h)/L \qquad [\%]$$



Flusso controllato da gravità in falda libera

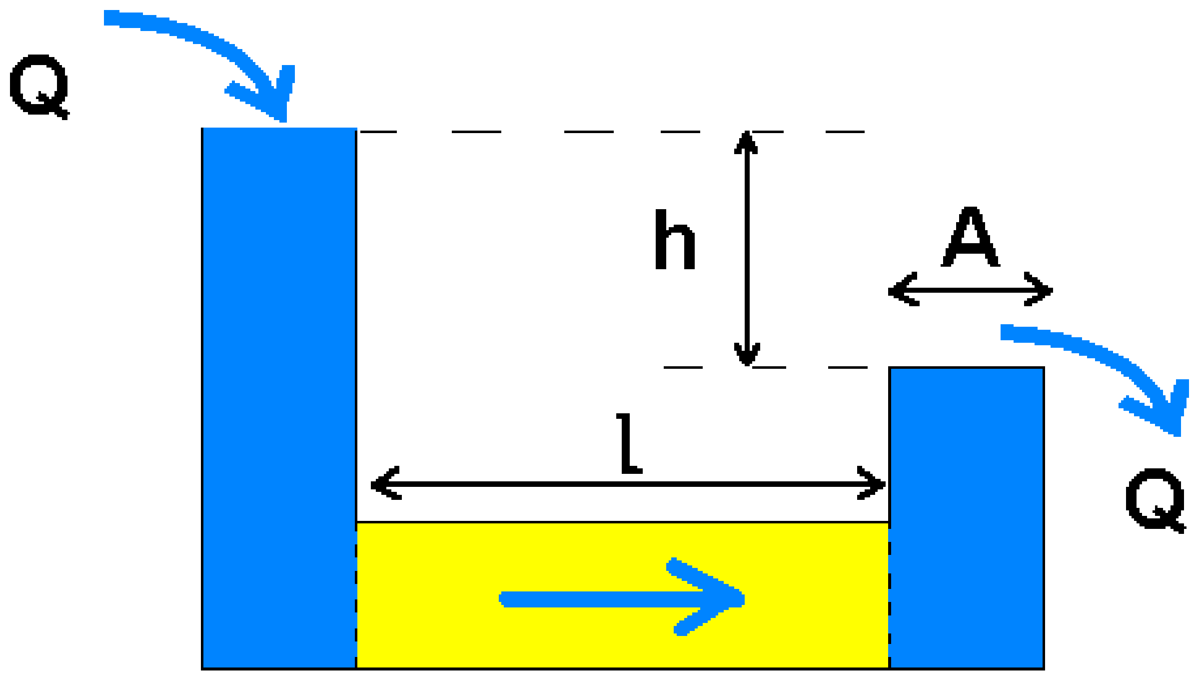


Flusso controllato da gravità in falda confinata



Darcy's law

The flow through porous media was studied by Darcy in 1856.



$$Q = K.A.\,h/L$$

Henry Darcy

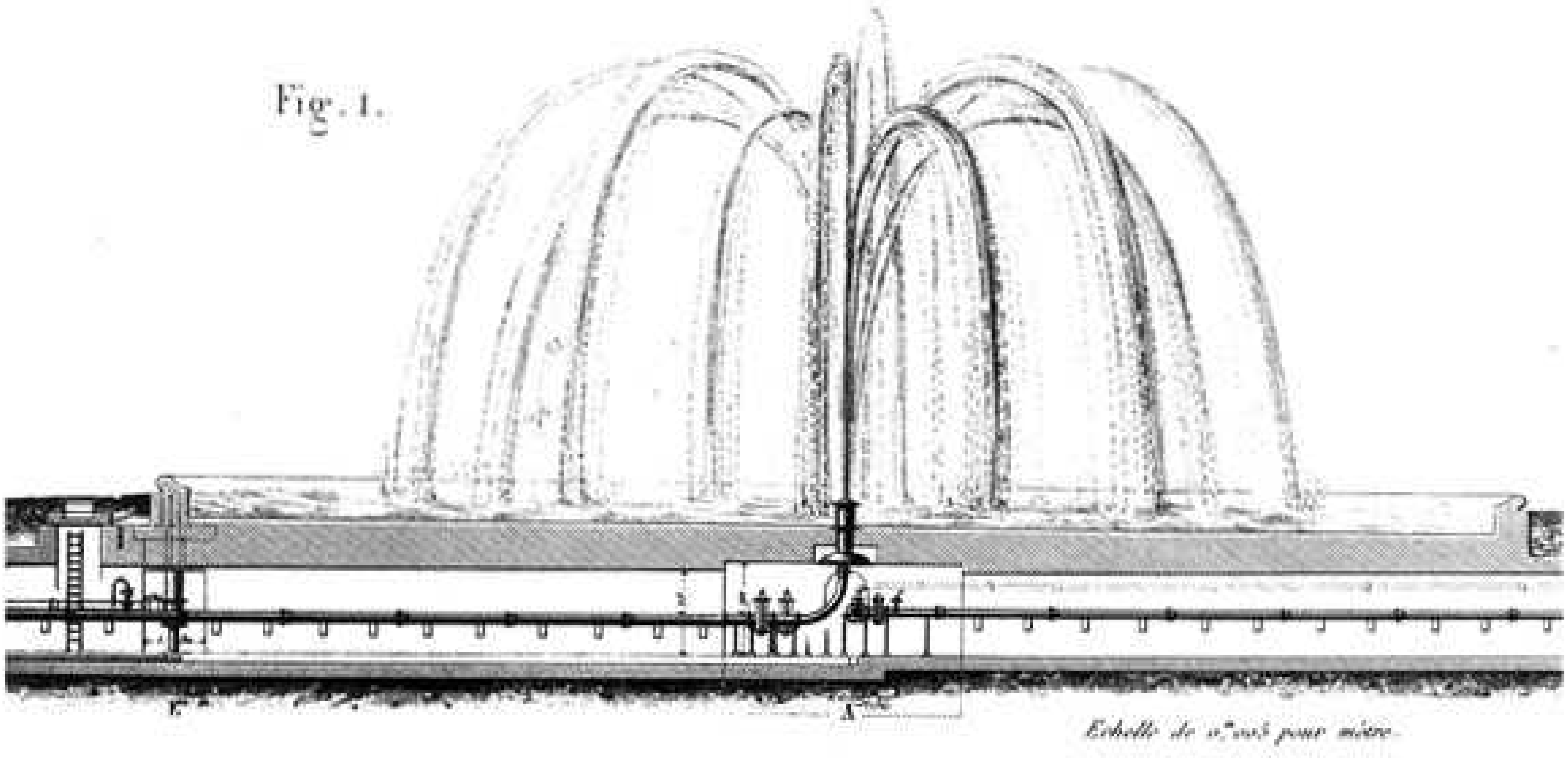
Fellow students at
***<u>L'École des Ponts et Chaussées</u>*<u>*:</u>**
Cauchy
Chézy
Coriolis
Dupuit
Fresnel
Navier
Pitot
St. Venant

*** <u>The school of Bridges and Roads</u>**

Henry graduated 12th in his class. He <u>had</u> been 1st, but he got violently upset with a chemistry professor over a question about cooking.

Henry Darcy

"As much as possible, one should favor the free drawing of water because it is necessary for public health. A city that cares for the interest of the poor class should not limit their water, just as daytime and light are not limited."



Most famous publication

LES

FONTAINES PUBLIQUES

DE LA VILLE DE DIJON

EXPOSITION ET APPLICATION

DES PRINCIPES A SUIVRE ET DES FORMULES A EMPLOYER

DANS LES QUESTIONS

DE

DISTRIBUTION D'EAU

OUVRAGE TERMINÉ

PAR UN APPENDICE RELATIF AUX FOURNITURES D'EAU DE PLUSIEURS VILLES

AU FILTRAGE DES EAUX

ET

A LA FABRICATION DES TUYAUX DE FONTE, DE PLOMB, DE TOLE ET DE BITUME

PAR

HENRY DARCY

INSPECTEUR GÉNÉRAL DES PONTS ET CHAUSSÉES.

La bonne qualité des eaux étant une des choses qui contribuent le plus à la santé des citoyens d'une ville, il n'y a rien à quoi les magistrats aient plus d'intérêt qu'à entretenir la salubrité de celles qui servent à la boisson commune des hommes et des animaux, et à remédier aux accidents par lesquels ces eaux pourraient être altérées, soit dans le lit des fontaines, des rivières, des ruisseaux où elles coulent, soit dans les lieux où sont conservées celles qu'on en dérive, soit enfin dans les puits d'où naissent des sources.

(De Jussieu, *Hist. de l'Académie royale des sciences*, 1733, p. 351.)

PARIS

VICTOR DALMONT, ÉDITEUR,

Successeur de Carilian-Gœury et Vve Dalmont,

LIBRAIRE DES CORPS IMPÉRIAUX DES PONTS ET CHAUSSÉES ET DES MINES,

Quai des Augustins, 49.

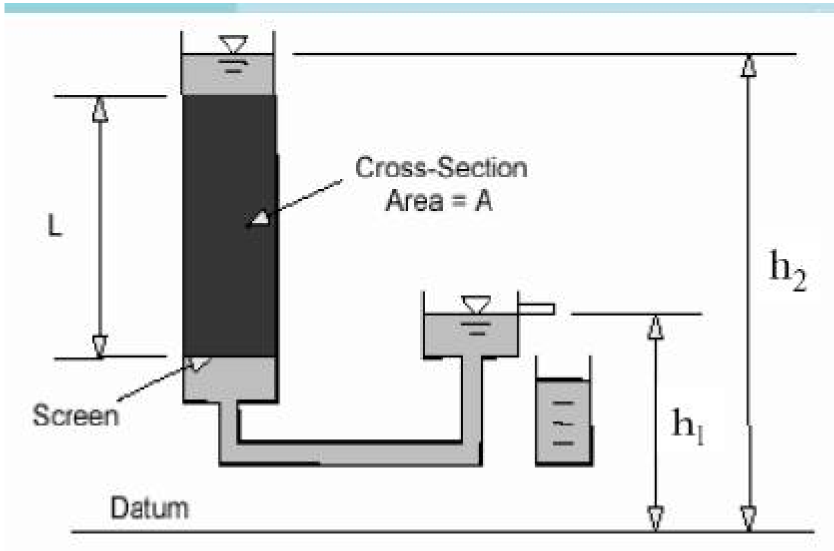
1856

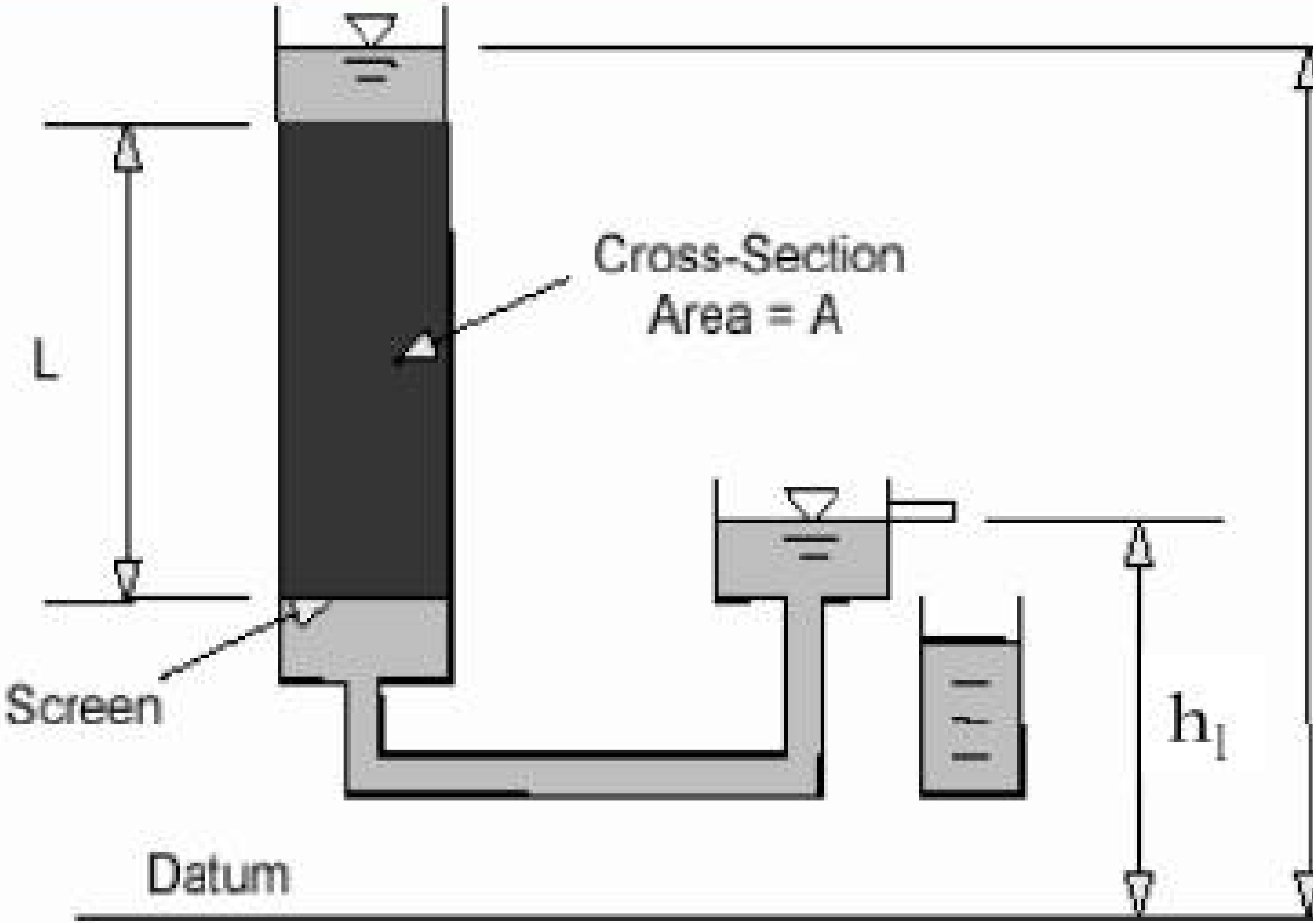
Henry Darcy has published a hydrogeological study in 1856. He established an empirical relationship for the flow of water through porous media

This study represented the birth of hydrogeology : Darcy has measured the flow rate for different values of hydraulic head h;

He has done tests with different grain size distributions and different values of L, h_1 and h_2

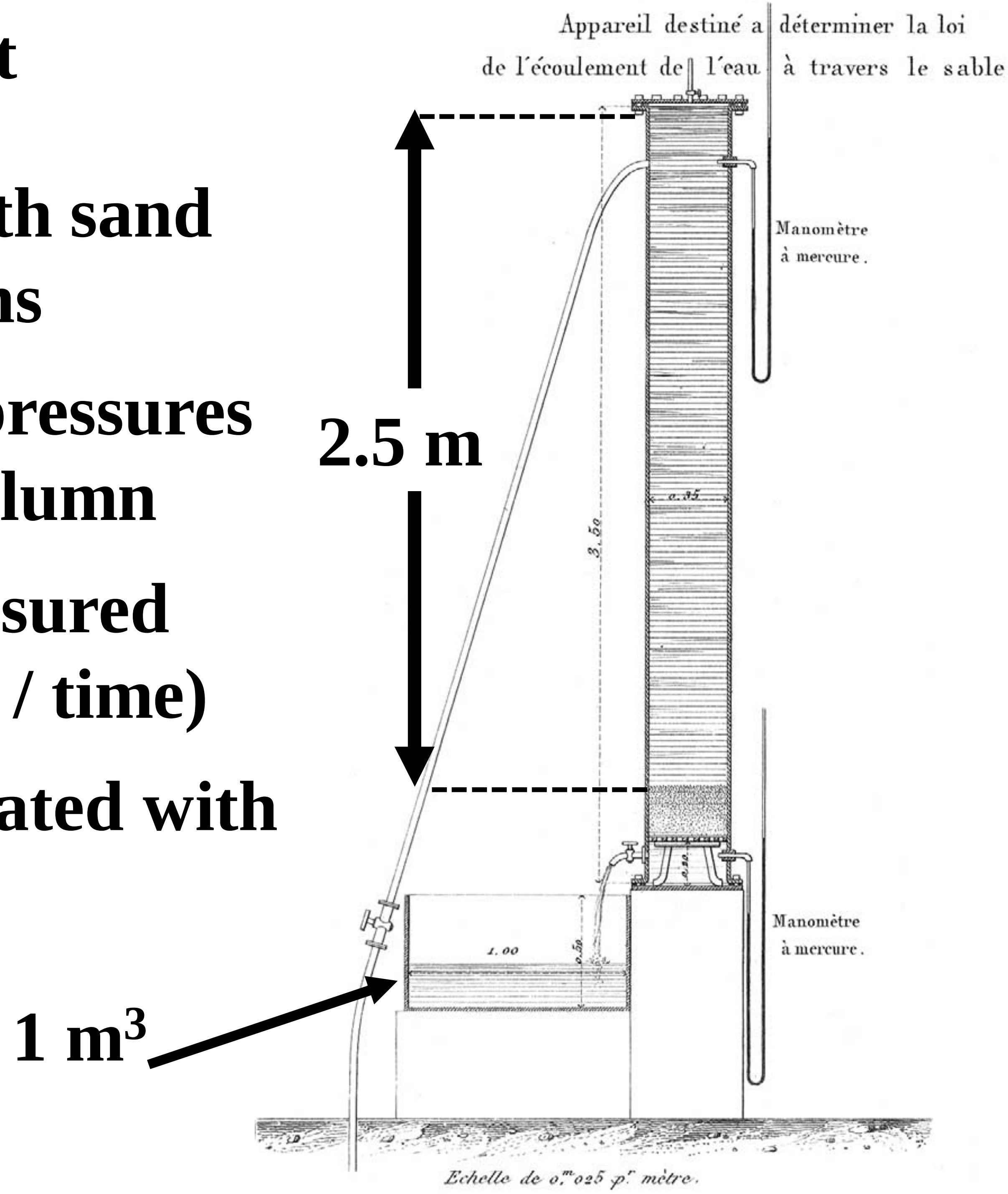
Henry DARCY (1802-1858) in 1856 was playing around with movement of water through sand filtration columns for the City of Dijon, France.




Cross-Section
Area = A
L
Screen
Datum
h_l

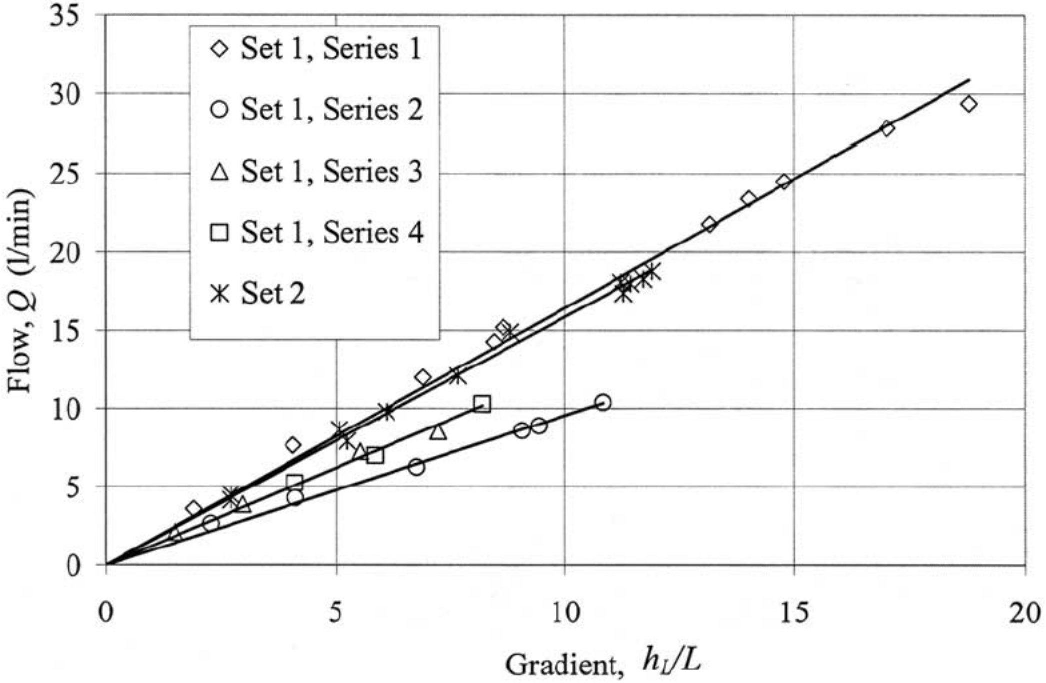
Darcy's experiment

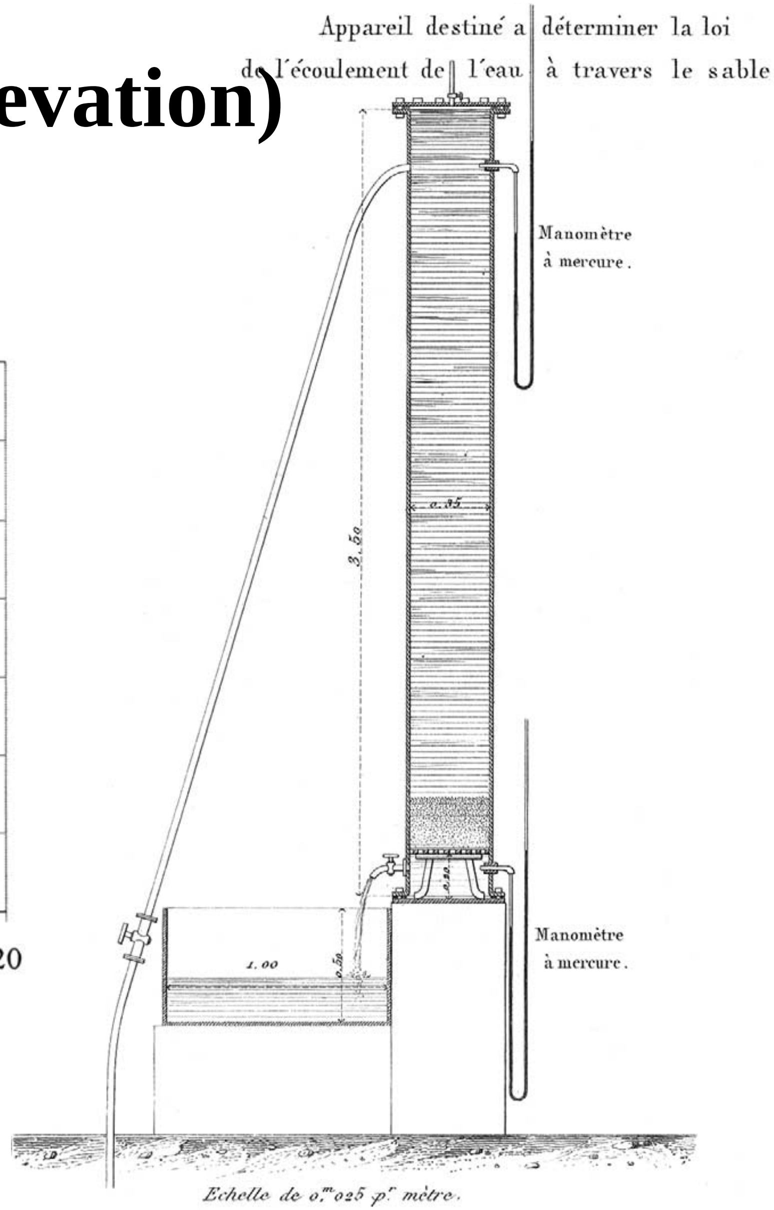
- Column filled with sand to different depths
- Different water pressures applied across column
- Discharge *Q* measured (volume of water / time)
- Experiment repeated with different sands



Darcy's 3 observations:

1: $Q \propto$ cross-sectional Area
2: $Q \propto$ drop in (pressure + elevation)
3: $Q \propto 1$ / flow distance

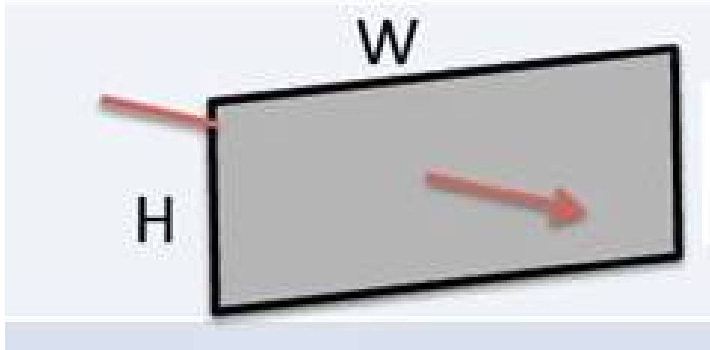




Darcy's 1st observation

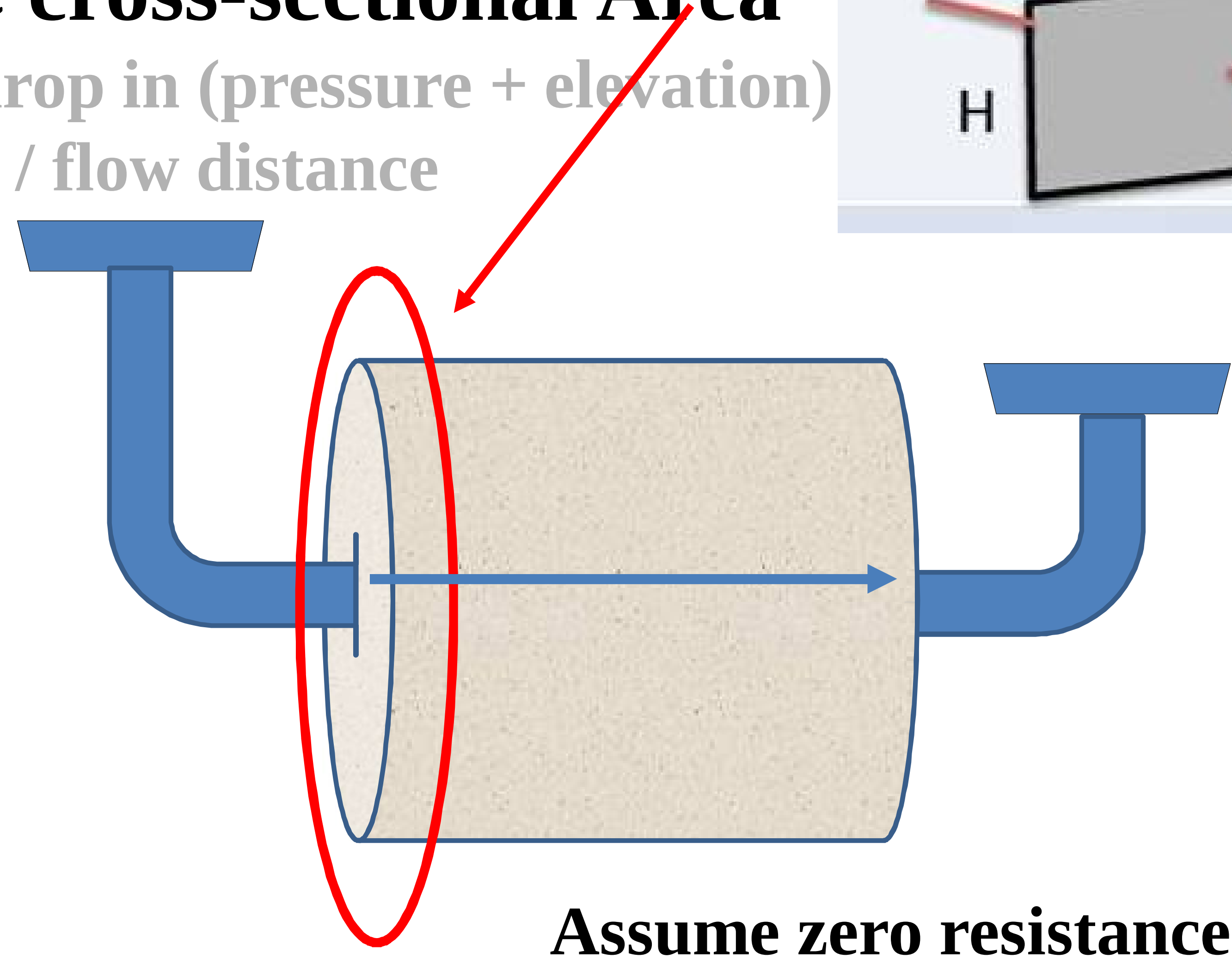
Darcy's experiments used a vertical column, but a horizontal column is simpler.

1: $Q \propto$ cross-sectional Area



2: $Q \propto$ drop in (pressure + elevation)

3: $Q \propto 1$ / flow distance



Assume zero resistance in pipes

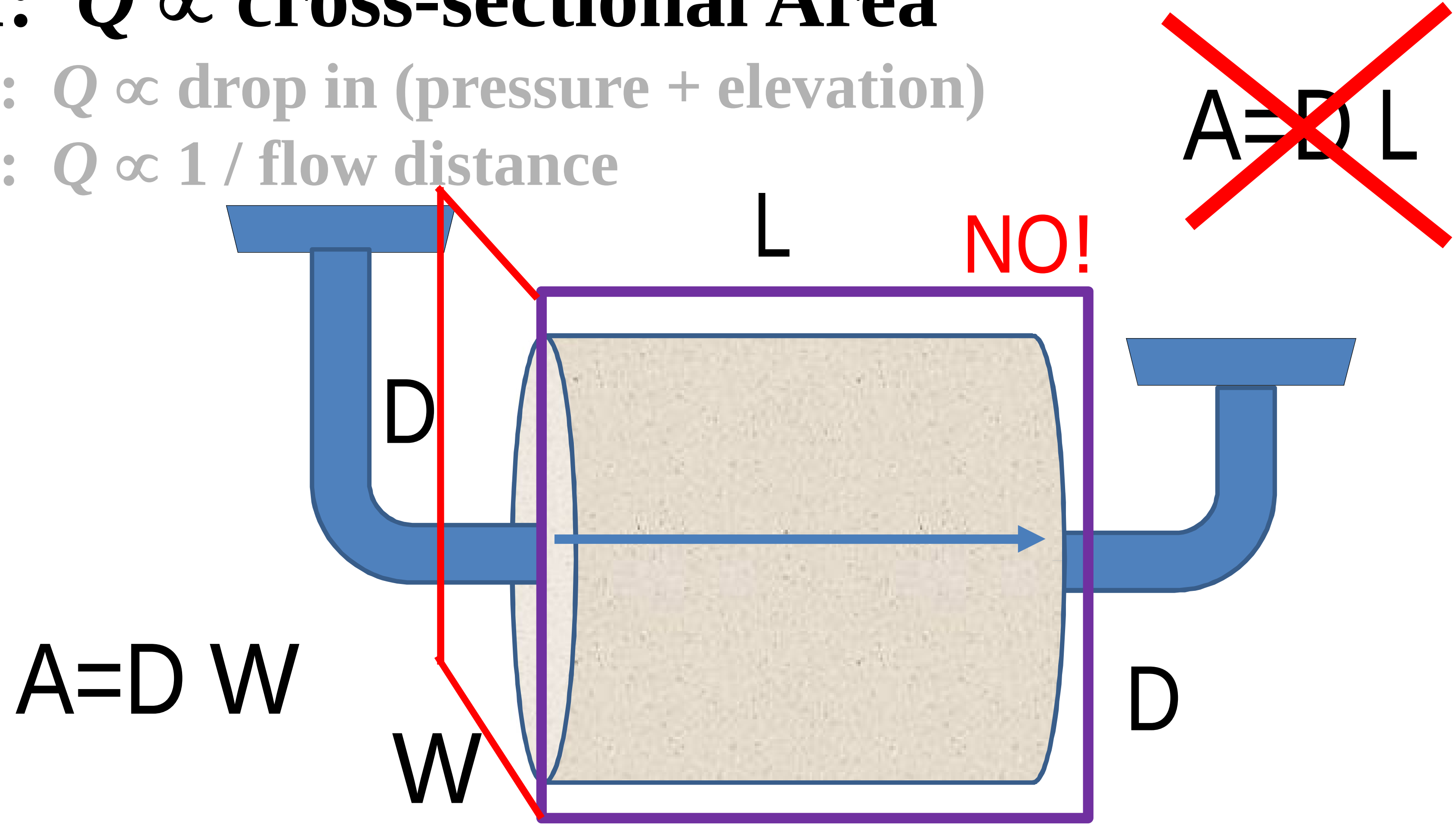
Darcy's 1st observation

Darcy's experiments used a vertical column, but a horizontal column is simpler.

1: $Q \propto$ cross-sectional Area

2: $Q \propto$ drop in (pressure + elevation)

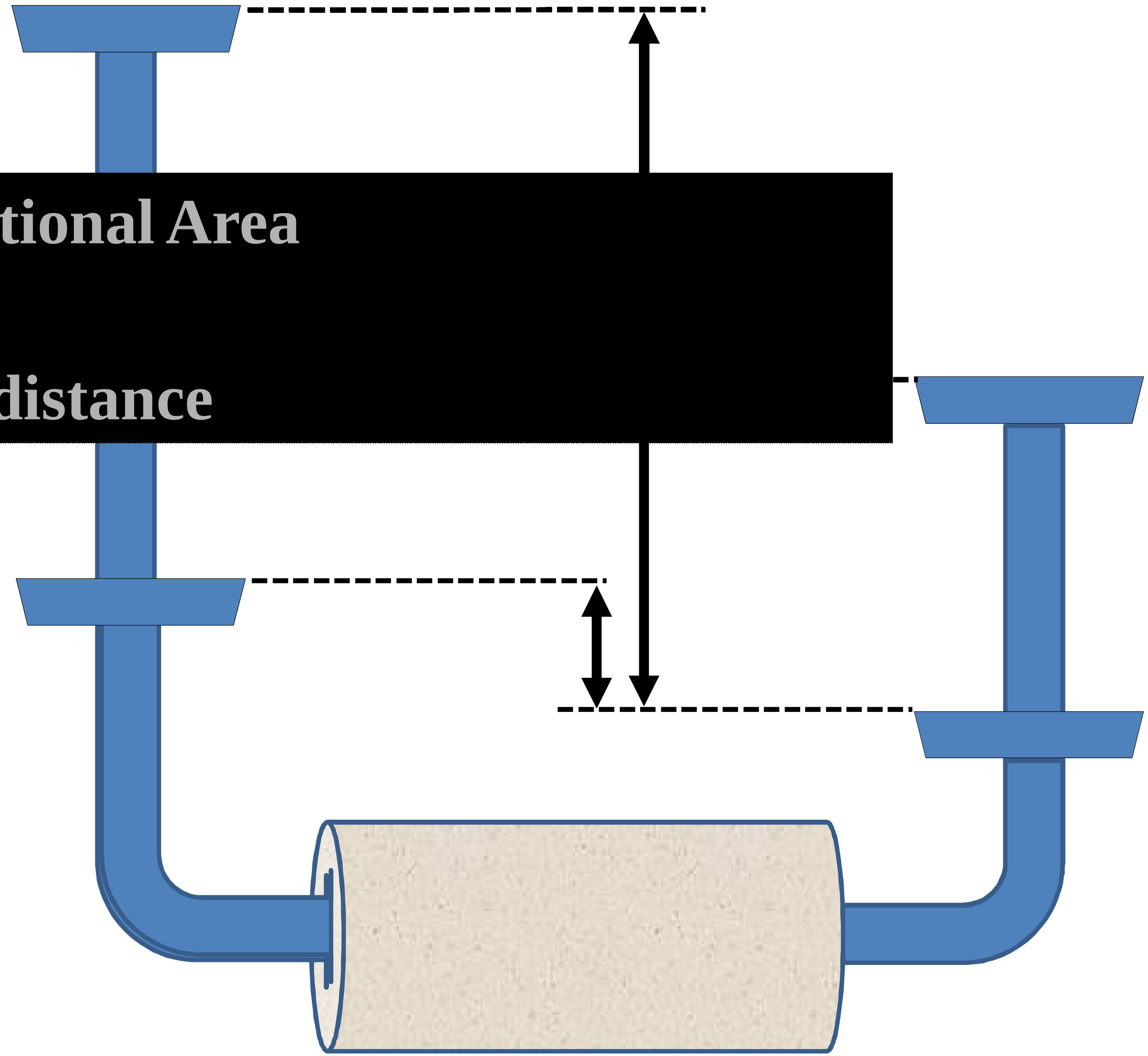
3: $Q \propto 1$ / flow distance



Darcy's 2nd observation

1: $Q \propto$ cross-sectional Area

3: $Q \propto$ 1 / flow distance

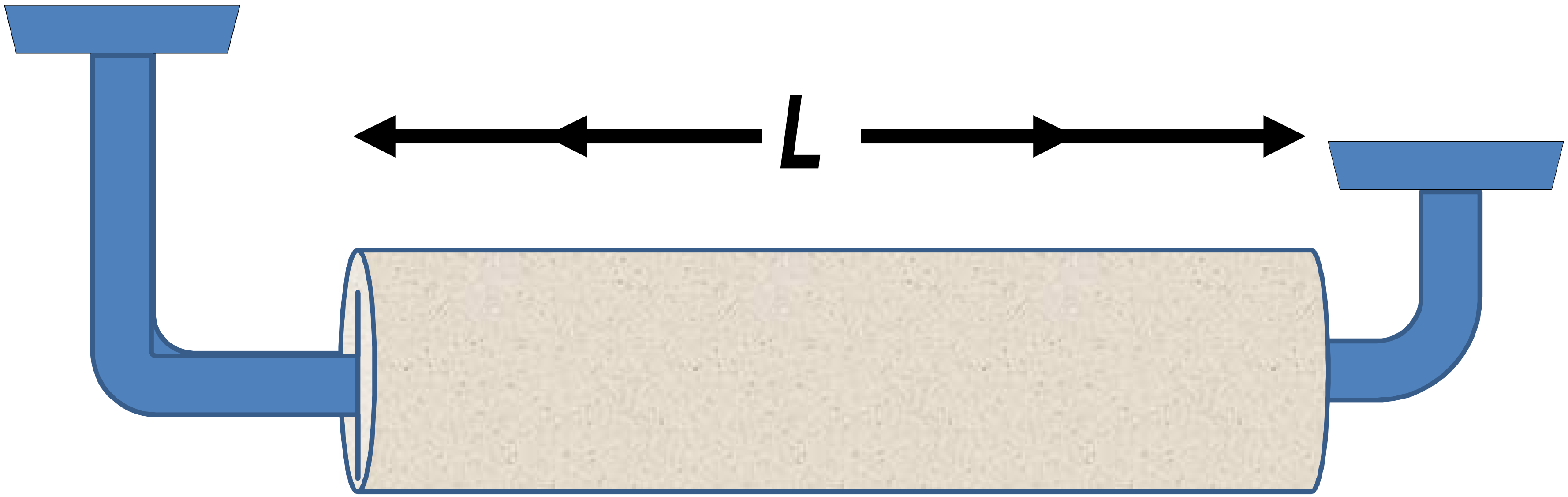


Darcy's 3rd observation

1: $Q \propto$ cross-sectional Area

2: $Q \propto$ drop in (pressure + elevation)

3: $Q \propto 1$ / flow distance



Darcy's Law as he wrote it

$$Q = KA\frac{(h_1 + z_1) - (h_2 + z_2)}{L}$$

Water pressure

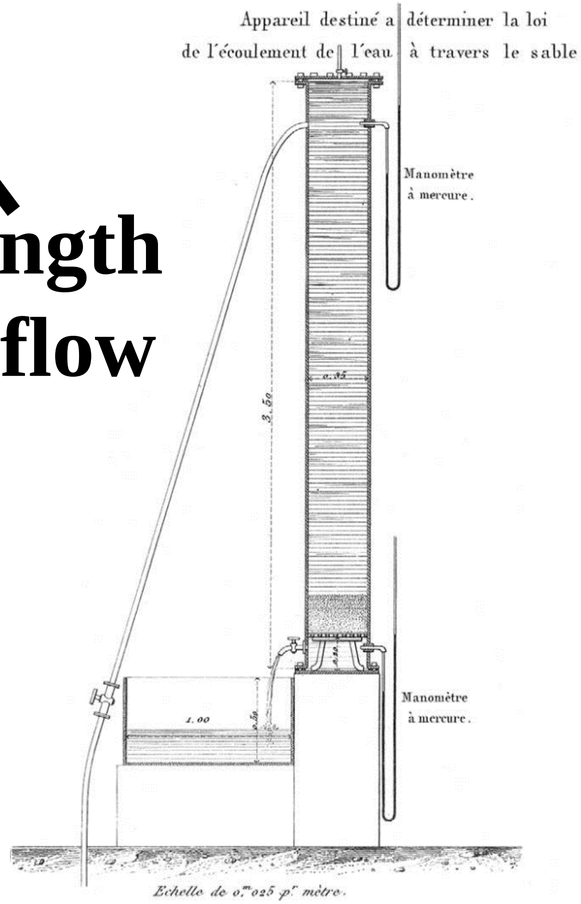
Height

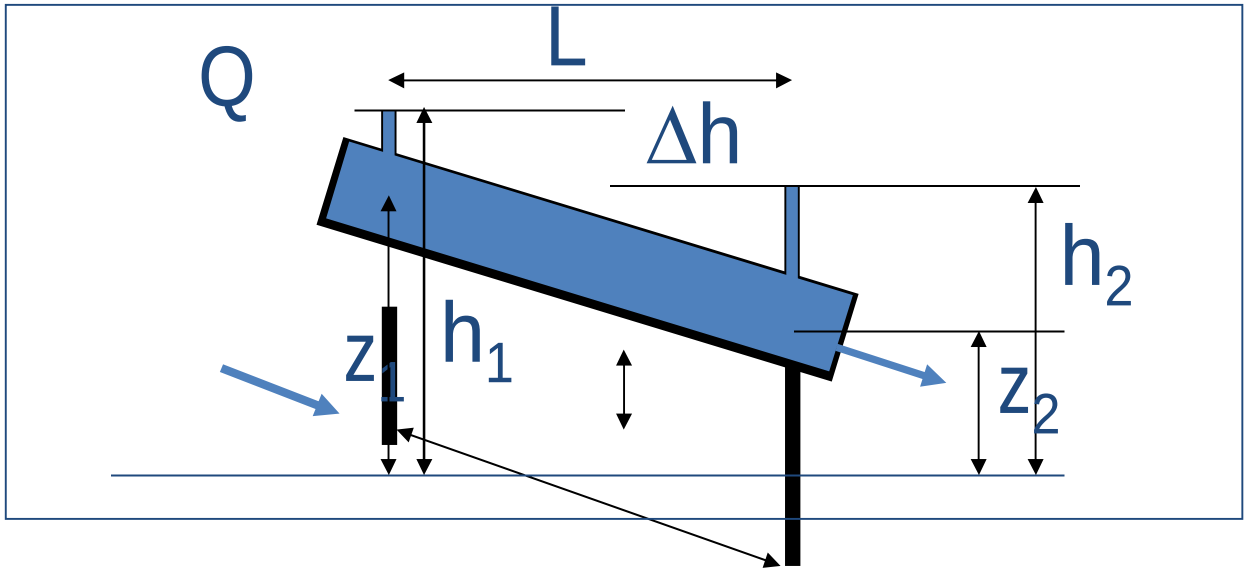
Length of flow

[redacted]

Water volume / unit time

Proportionality coefficient: Hydraulic conductivity





Equation describing a microscopical behavior at macroscopic scale

Q = flow rate
A = cross-sectional area (perpendicular to flow)
Δh = hydraulic head difference
L = distance between measurement points

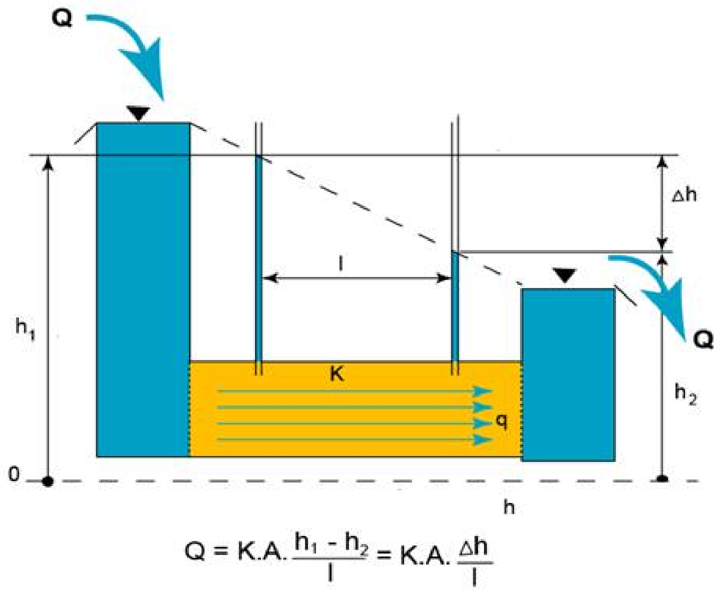
$$Q = KA\frac{\Delta h}{L}$$

Darcy's law

Darcy's law describes the flow of water in porous media

$$Q\ [m^3/s] = k\ [m/s]\ .\ A\ [m^2]\ .\ i$$

Q [m^3/s] volume of water flowing through a given volume of soil
A [m^2], cross-sectional area perpendicular to flow
k [m/s] coefficient of hydraulic conductivity
i [-] hydraulic gradient (measure of the change in groundwater head over a given distance)



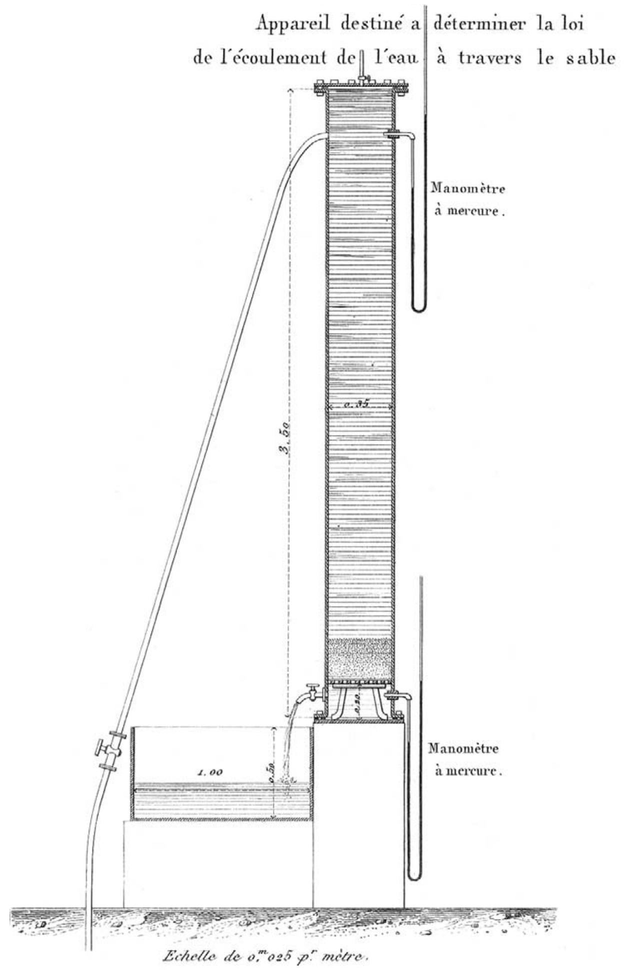
Units in Darcy's Law

$$Q = KA\frac{(h_1 + z_1) - (h_2 + z_2)}{L}$$

$$\frac{L^3}{T} = \frac{L}{T} L^2 \frac{L - L}{L}$$

Velocity

dimensionsless



Key implications of Darcy's law

The flow is **<u>linearly proportional</u>** to the gradient.

This puts Darcy's law into the same class as several other equations:

$$\varepsilon = \frac{-1}{E} \nabla \sigma$$

Hooke's law (elasticity)

$$q = -K \nabla h$$

Darcy's law (hydraulic conductivity)

$$j = -\sigma \nabla V$$

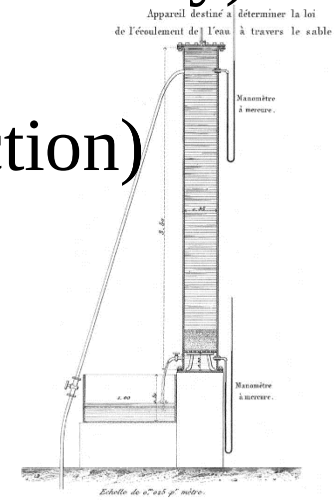
Ohm's law (electrical conductivity)

$$q_h = -\kappa \nabla T$$

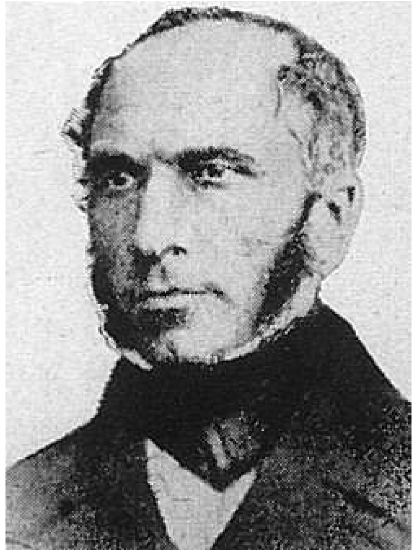
Fourier's law (heat conduction)

$$f = -D \nabla C$$

Fick's law (diffusion)



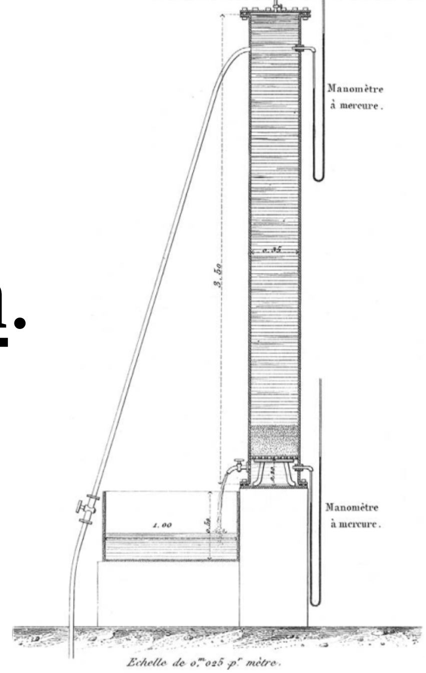
Key implications of Darcy's law



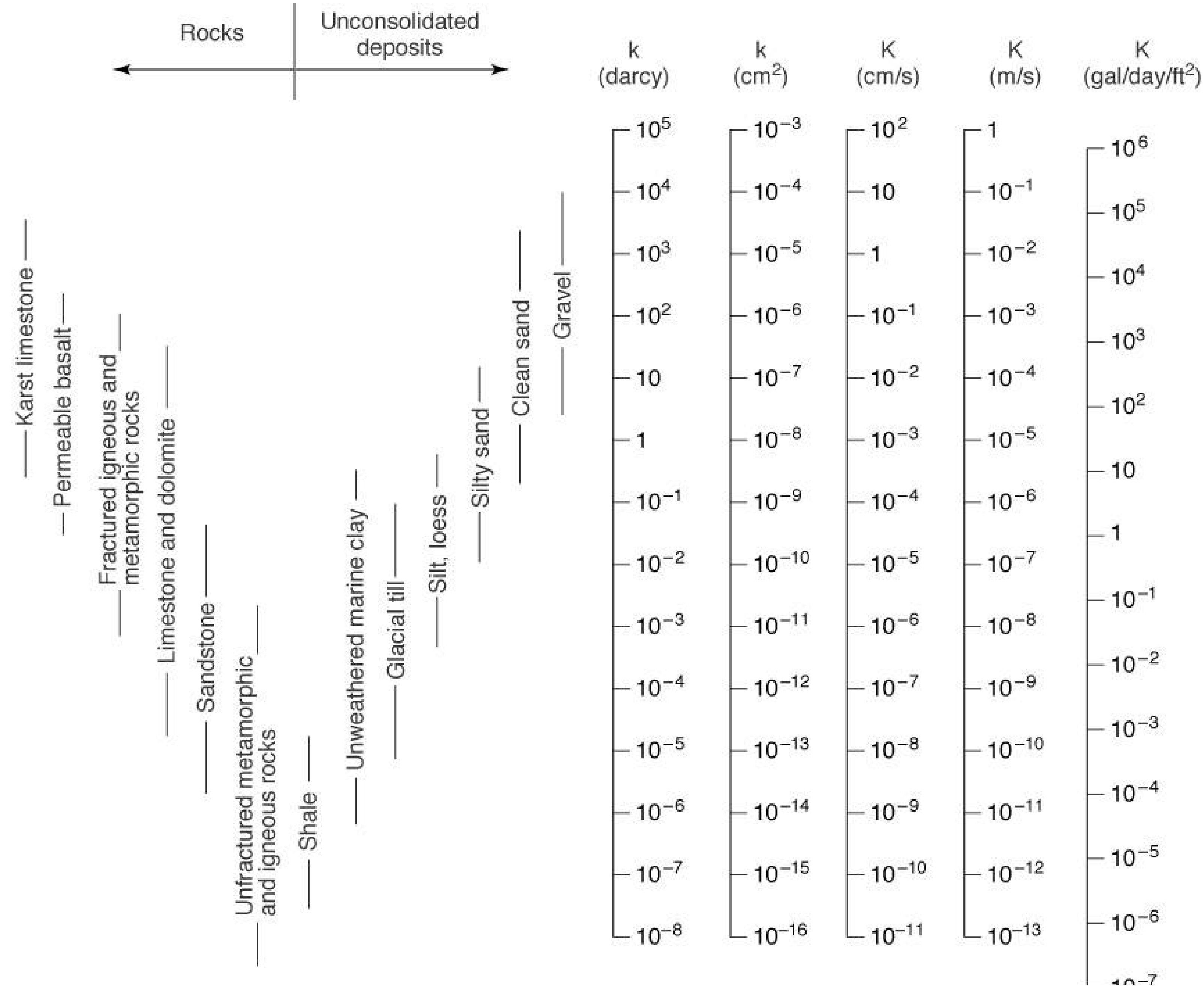
For flow through a **uniform medium**, the hydraulic gradient is **constant**.

The flow is **linearly proportional** to the gradient, as in Hooke's law, Fick's law, Fourier's law, etc.

K is a **property of the medium**.

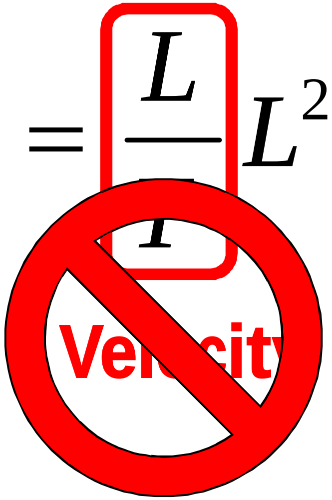


Hydraulic Conductivity

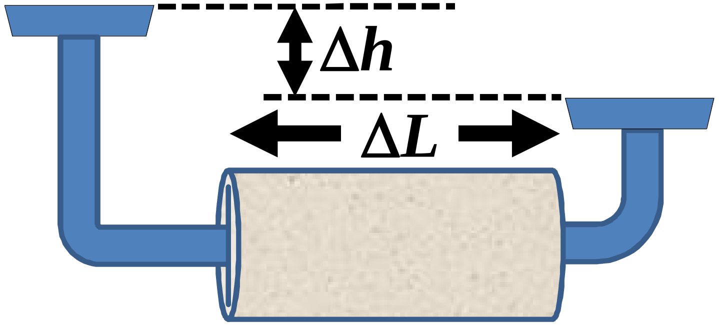


Is this velocity how fast the water moves?

$$\frac{L^3}{T} = \frac{L}{T} L^2 \frac{L - L}{L}$$



$$Q = KA\frac{\Delta h}{\Delta L}$$



No.

Water flows only through the <u>pores</u>.

Water flows through an area $A\, n_e$

Water flows at mean velocity $\bar{u} = \dfrac{Q}{A \cdot n_e} = \dfrac{K}{n_e}\dfrac{\Delta h}{\Delta L}$

$$Q = K\,A\,i$$

$$Q/A = V_D = K\,i$$

V_D velocità Darciana (fittizia)

$V_D/n_e = V_s$ velocità di deflusso (seepage velocity) V_s

v=Q/A detta portata specifica

Darcy's law

$$v = K\frac{\Delta H}{\Delta L} = Ki$$

Darcy found that the flow of water through a bed of "a given nature" is:
- proportional to the difference in the height of the two ends,
- inversely proportional to the length of the flow path
- proportional to the x-sectional area of the pipe
- flow is further related to a coefficient dependent on the nature of the media

$$v \propto \Delta h$$

$$v \propto K$$

$$v \propto \frac{1}{\Delta l}$$

$$Q = -KA\frac{\Delta h}{L}$$

(–) indicates that *V* occurs in the direction of the decreasing head.

segno negativo proviene da considerazioni legate al concetto matematico di gradiente. Si considera sempre il flusso nella direzione di gradiente idraulico negativo

q [m/s] = flow rate / cross-sectional area = - Q / A = k i = v_D [m/s

The specific discharge q =Q /A [m/s] or Darcy velocity

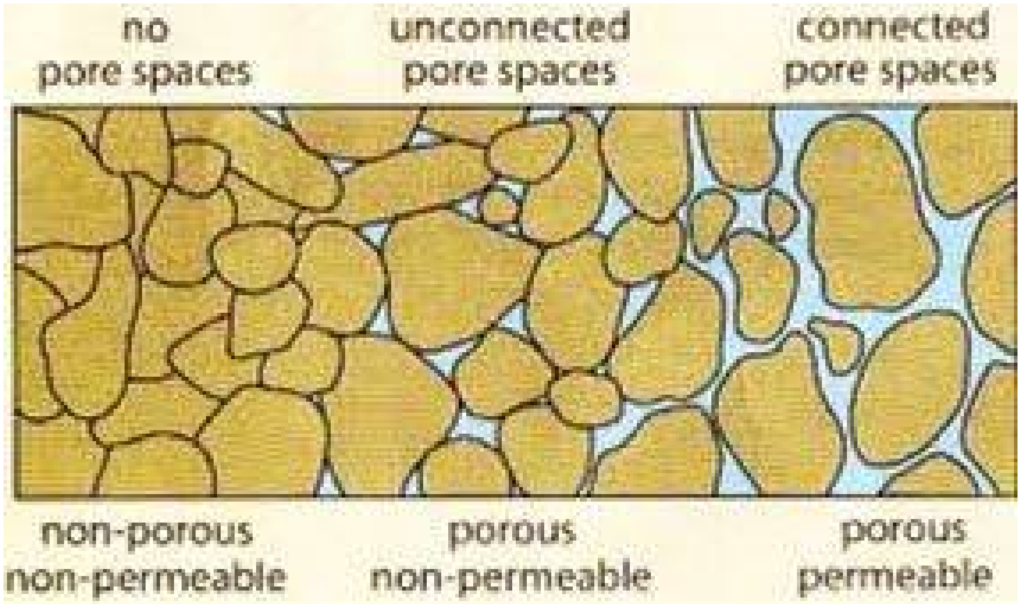
Specific discharge has units of velocity.

Darcy velocity is a fictitious velocity since it assumes that flow occurs across the entire cross-section of the soil sample.

Flow actually takes place only through interconnected pore channels.

Velocità Darciana & Velocità reale di filtrazione

- La velocità Darciana è una velocità fittizia dato che assume che il flusso avvenga sfruttando tutta la luce della sezione di mezzo poroso trasversale al flusso.
- Il flusso in realtà ha luogo attraverso pori interconnessi.



La velocità darciana è definita come flusso attraverso area unitaria del mezzo poroso, e nell'esperimento di Darcy, l'area della sezione è tutta quella del campione. Se immaginiamo il mezzo poroso come una scatola, il flusso darciano è la velocità con la quale il fluido scorre da una faccia a quella opposta della scatola, senza fornire indicazioni su quanto succede all'interno.

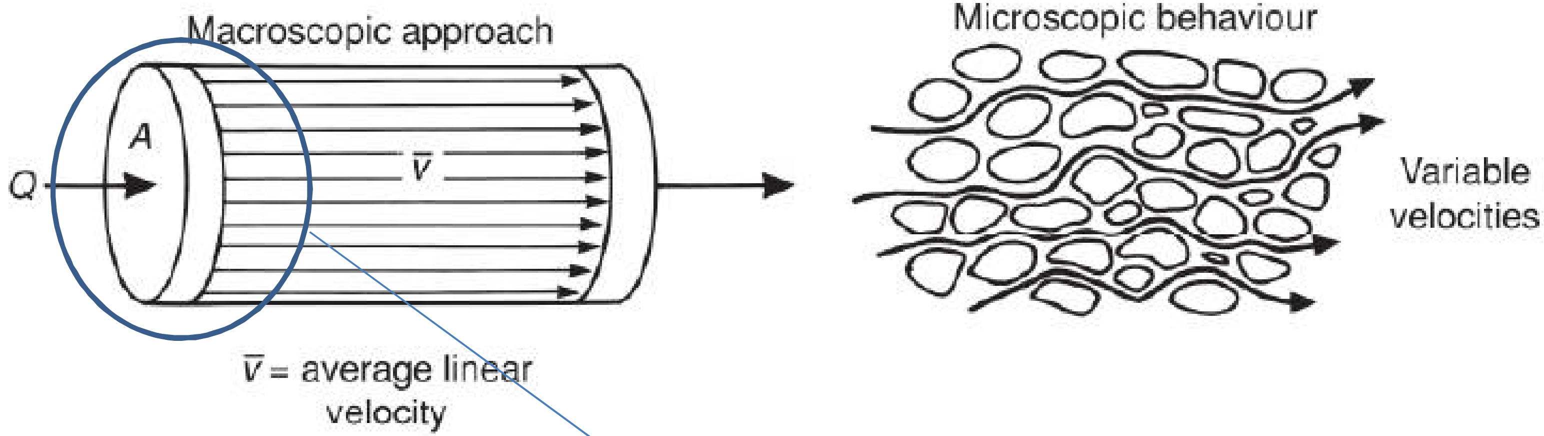
Dato che il campione è poroso l'acqua si sposta attraverso i canalicoli ed attorno alle particelle solide con una velocità maggiore, poiché la sezione libera è minore. Se si vuole ricavare la velocità effettiva di spostamento dell'acqua dobbiamo quindi inserire un termine che tenga conto della percentuale di spazio a disposizione per il flusso, si utilizza a questo scopo il valore di porosità efficace (ne), ottenendo la velocità effettiva:

$$v_e = Ki/n_e \qquad v_e > v_D$$

essendo ne < 1, la velocità effettiva è maggiore di quella ottenuta dalla formula di Darcy, cioè:

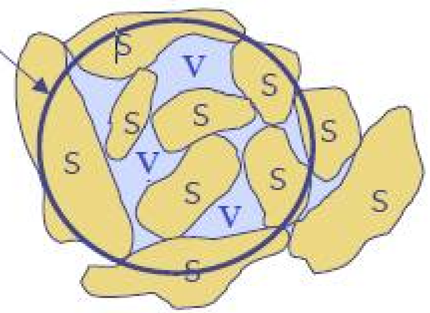
Darcy velocity –seepage velocity

In the definition of Darcy velocity there is the assumption that the flow takes place on all surface of porous medium (A_T).



In reality water flows just through interconnected voids (A_v), consequently the real seepage velocity will be higher

$$A_T = A_v + A_s$$



$$v_s = v_D / n_e$$

Velocità reale di filtrazione- Advezione

La velocità reale di filtrazione (seepage velocity) è quella attraverso quale si esplica il fenomeno di advezione

L'advezione è quel fenomeno per il quale l'inquinante viene trasportato dalla massa d'acqua in movimento; Il trasporto avviene lungo la direzione del flusso idrico.

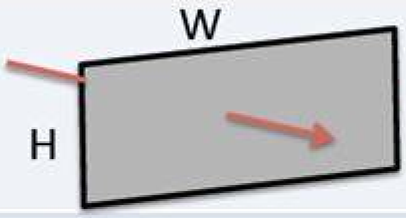
- **Advection** is mass transport due simply to the flow of the water in which the mass is carried.
- The direction and rate of transport coincide with that of the groundwater flow.

The average pore velocity is obtained dividing the flow rate by the effective porosity n_e

$$v = \frac{q}{n_e}$$

From Darcy's law**:** $\mathbf{v = q / n_e = - (K / n_e)\, dh/dx}$

Difference between Darcy Velocity (also called Specific Discharge) and Seepage Velocity (also called Interstitial Velocity).

	Darcy Velocity (Equation 1) **(also called Specific Discharge)**	**Seepage Velocity (Equation 2)** **(also called Interstitial Velocity)**
Description	Darcy velocity is averaged over entire transect area	Seepage velocity is velocity in open pore space
Diagram		
Area Used for any Flow Calculation	Use entire transect area: Area = W • H	Use only open porosity area: Area = W • H • n
Flow Calculation	$K * i$ = *Darcy Velocity (q)* Flow = $K \bullet i \bullet W \bullet H$	$\frac{K \bullet i}{n}$ = *Seepage Velocity* (V_s) Flow = $\frac{K \bullet i}{n} \bullet W \bullet H \bullet n$
	Flow = $K \bullet i \bullet W \bullet H$	Flow = $\frac{K \bullet i}{n} \bullet W \bullet H \bullet n$

Darcy & Seepage Velocity

- From the Continuity Equation:
- $$Q = A_T v_D = A_v v_s$$
  - Where:

    Q = flow rate

    A_T = total cross-sectional area of material

    A_v = area of voids

    v_s = seepage velocity

    v_D = Darcy velocity

Darcy & Seepage Velocity

- Therefore: $v_S = v_D\, (A_T/A_V)$
- Multiplying both sides by the length of the medium (L)

  $$v_S = v_D\, (AL\, /\, A_V L\,) = v_D\, (V_T\, /\, V_V\,)$$
- Where:

  V_T = total volume

  V_v = void volume
- By Definition, $V_v\, /\, V_T = n$, the soil porosity
- Thus $\boldsymbol{v_S = v_D\, /\, n_e}$

Being ne<1 seepage velocity is always higher than Darcy’s velocity

Example

The real velocity, if confused with the Darcy velocity, will be underestimated

Groundwater seepage velocity is very important in the study of pollution phenomena.

For example the velocity of an alluvial aquifer with a flow rate of **1.5 m^3/sec** that flows through a cross sectional area of **180000 m^2** will be, using Darcy velocity, equal to **8.3 x 10^{-6} m/s**, value not acceptable for this kind of aquifer.

Considering the effective porosity of alluvial aquifers (0.09) the seepage velocity will be **9.3 x 10^{-5}** m/s, more coherent with the values measured with tracers

---

The real velocity, if confused with the hydraulic conductivity, will be overestimated

Example: Sandy Aquifer

K= 60 m/d

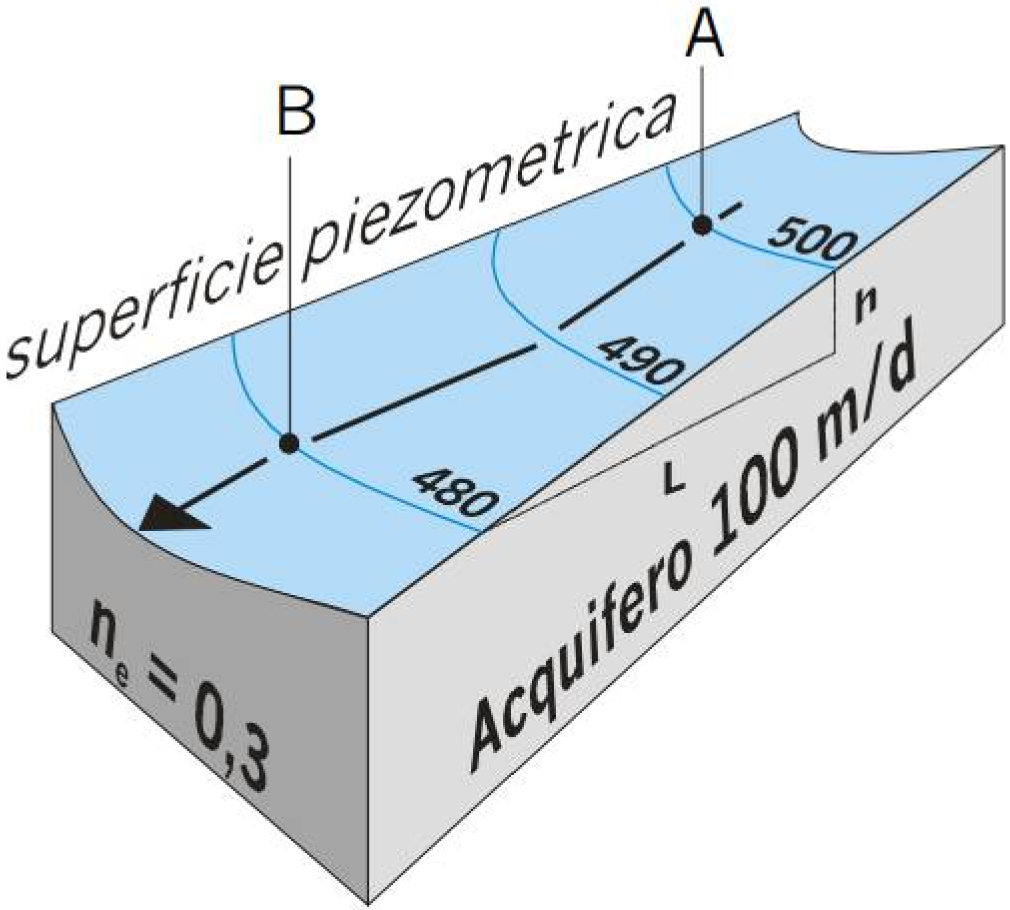
dh/dl = 1m/1000m

n_e= 0.20

$$v = \frac{K}{n_e} \times \frac{dh}{dl} = \frac{60m}{\text{day}} \times \frac{1}{0.20} \times \frac{1m}{1000m} = \frac{60m^2}{200m\,\text{day}} = 0.3m\ \text{day}^{-1}$$

K=60 m/d v= 0.3 m/d

Utilizzo della legge di Darcy: Calcolo della velocità effettiva



K=100 m/d

V_e= 6,6 m/d

Dovendo calcolare la velocità della falda tra due punti A e B sulla sua superficie, si applica la legge di Darcy:

h/L = i (gradiente della falda) se L = 1000 m; i = 20/1000 = 0,02

v = K h/L (portata specifica, specific discharge)

v = 100 x 0,02 = 2

la velocità effettiva è ve = Ki / ne

ve = 2 /0,3 = 6,6 m/d

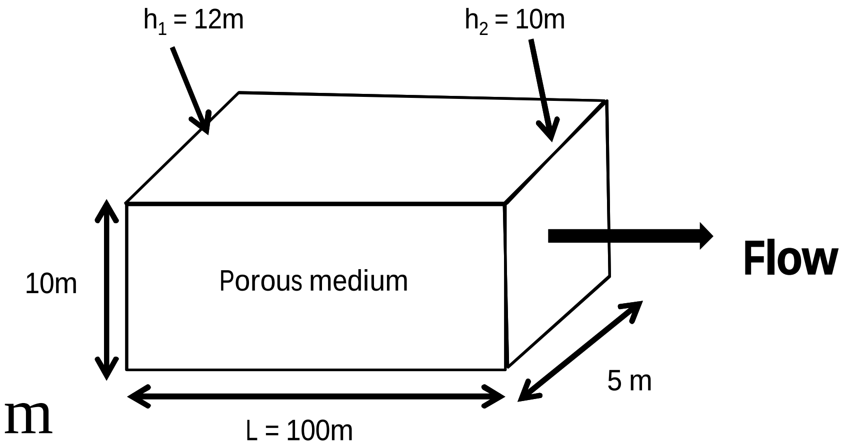
Il tempo impiegato da una particella fluida (o da un tracciante che si sposta solo per advezione) da A a B è:

t = s/v t = 1000/6,6 = 150 giorni

Esercizio

$K = 1x10^{-5}$ m/s
$n_e = 0.3$

Trovare *q*, *Q*, and v_e



$dh = (h_1 - h_2) = (12 \text{ m} - 10 \text{ m}) = 2 \text{ m}$

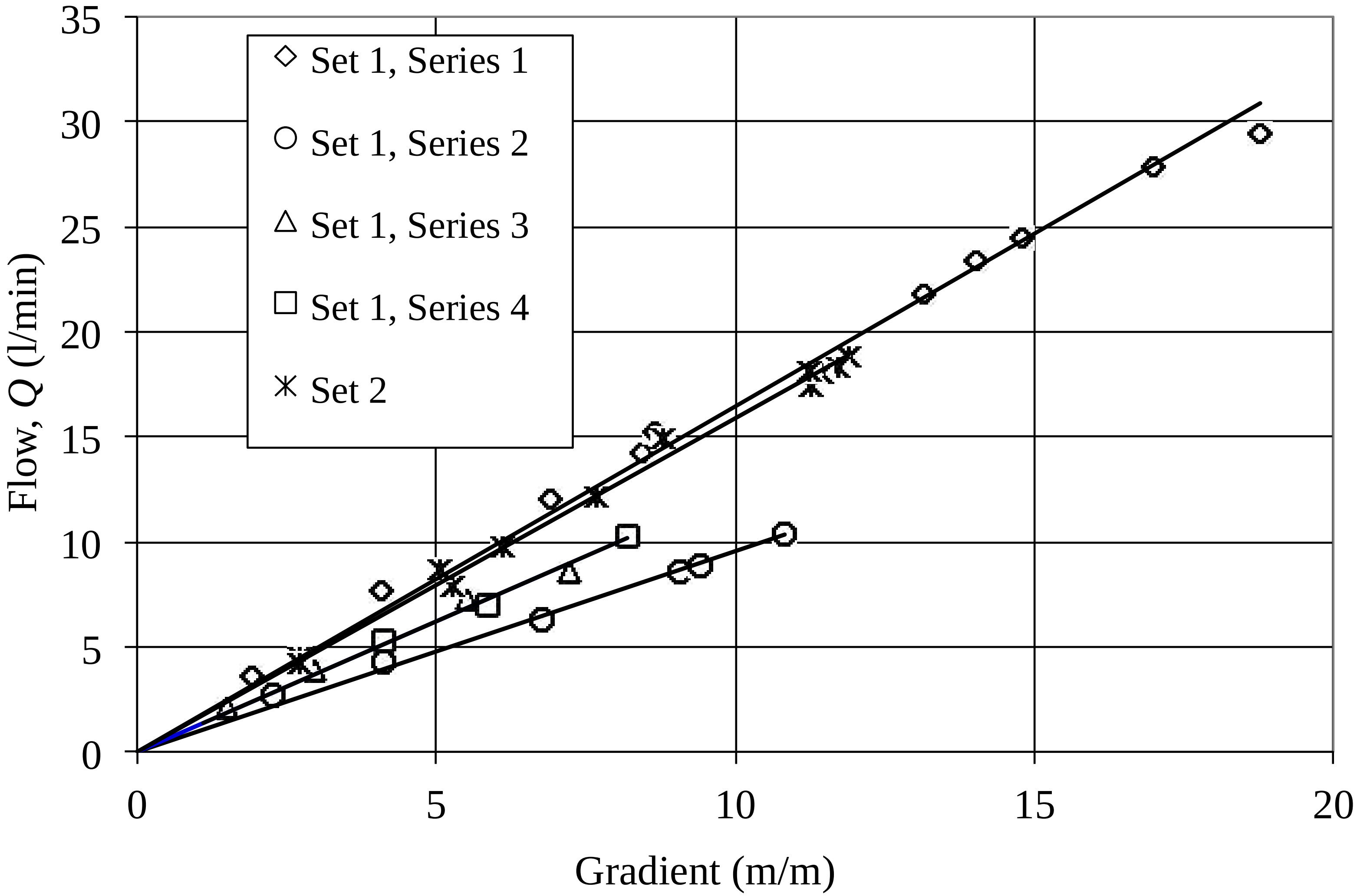
$i = dh/dx = (2 \text{ m})/100 \text{ m} = 0.02 \text{ m/m}$

$q = Ki = (1x10^{-5} \text{ m/s}) \text{ x } (0.02 \text{ m/m}) = 2x10^{-7} \text{ m/s}$

$Q = qA = (2x10^{-7} \text{ m/s}) \text{ x } 50 \text{ m}^2 = 1x10^{-5} \text{ m}^3\text{/s}$

$v_e = q/n_e = 2x10^{-7} \text{ m/s} / 0.3 = 6.6x10^{-7} \text{ m/s}$

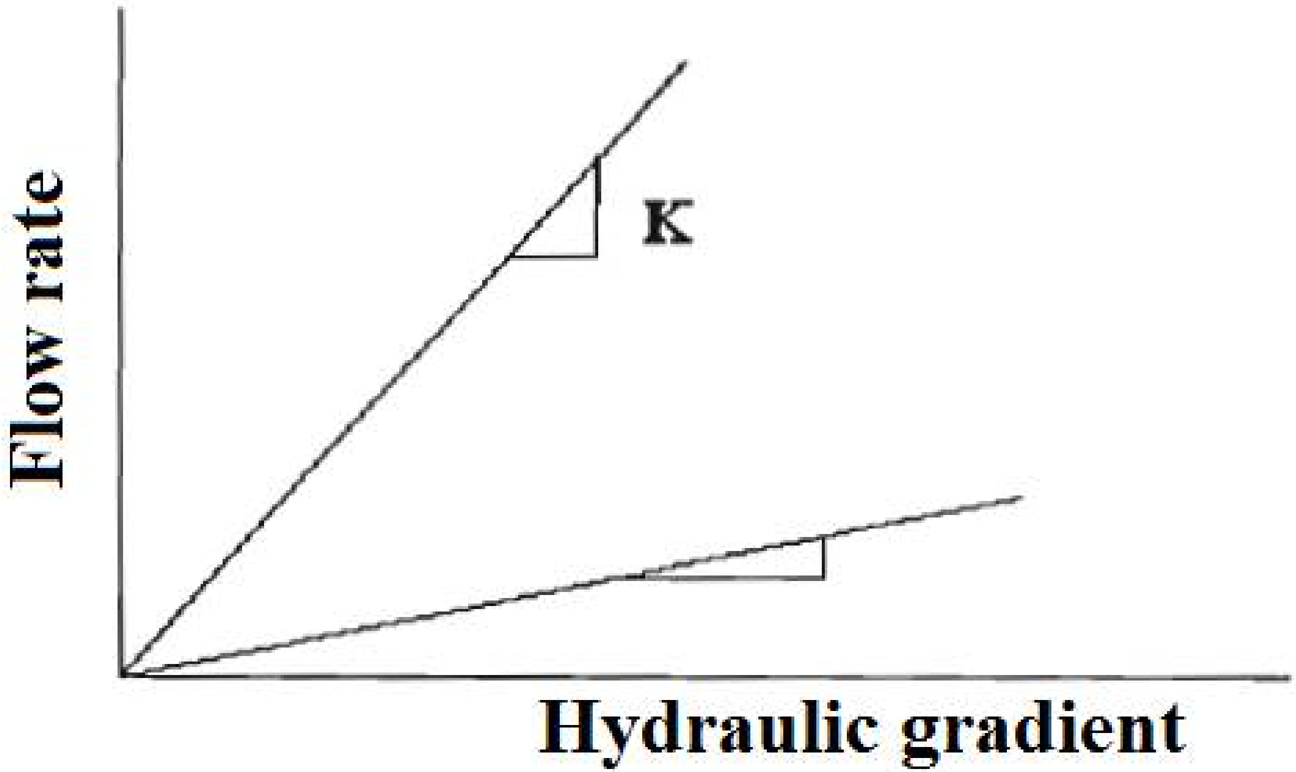
Darcy's Data



$$q = \frac{Q}{A} = -K\frac{dh}{dl}$$

Limitations of the Darcian approach

Linear relation flow rate - gradient



Darcy's law was established in certain circumstances:

- ❑ laminar flow in saturated granular media
- ❑ steady-state flow conditions
- ❑ fluid homogenous, isotherm and incompressible
- ❑ negligible kinetic energy

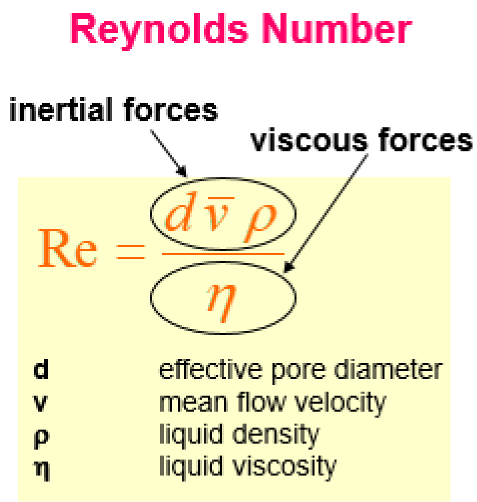
The most restrictive hypothesis of Darcy's law is the one that considers the flow *laminar*, and the fluid movement as dominated by viscous forces.

This occurs when the fluids are moving slowly, and the water molecules move along parallel streamlines. When the velocity of flow increases (for instance in the vicinity of a pumping well), the water particles move chaotically and the streamlines are no longer parallel. The flow is *turbulent*, and the inertial forces are more influential than the viscous forces.

The ratio between the inertial forces and the viscous forces driving the flow is computed by the Reynolds number, which is used as a criterion to distinguish between the laminar flow, the turbulent flow and the transition zone. For porous media, the Reynolds number is defined as:

$$R_e = \frac{\rho \cdot q \cdot d}{\mu} = \frac{q \cdot d}{\nu}$$

R_e is dimensionless



ρ the fluid density (M/L3 ; kg/m3)
q specific discharge (L/T ; m/s)
d usually, the mean grain diameter or the mean pore dimension (L; m)
μ dynamic viscosity (M/T.L ; kg/s.m)
ν kinetic viscosity (L2/T ; m2/s)

Validity of Darcy's Law

- We ignored kinetic energy (low velocity)
- We assumed laminar flow
- We can calculate a Reynolds Number for the flow

$$R_e = \frac{\rho q d}{\mu}$$

q = Specific discharge

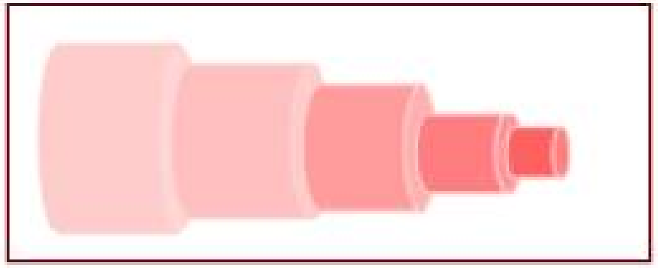
d = effective grain size diameter

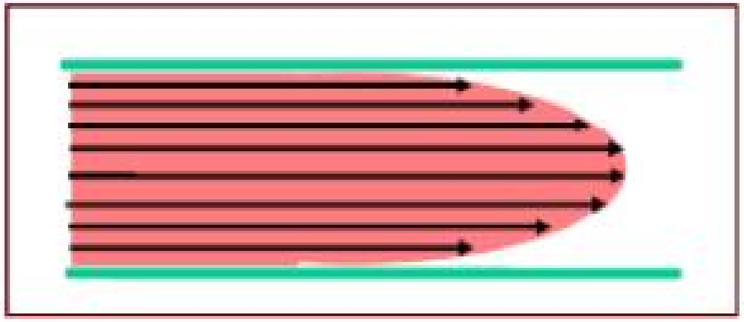
- Darcy's Law is valid for $R_e < 1$ (maybe up to 10)

RICHIAMI

IL MOTO LAMINARE

Moto laminare: le particelle di liquido si muovono secondo lamine di scorrimento (cilindri di scorrimento), parallele una all'altra. Le lamine, di spessore infinitesimo, scorrono con velocità diversa



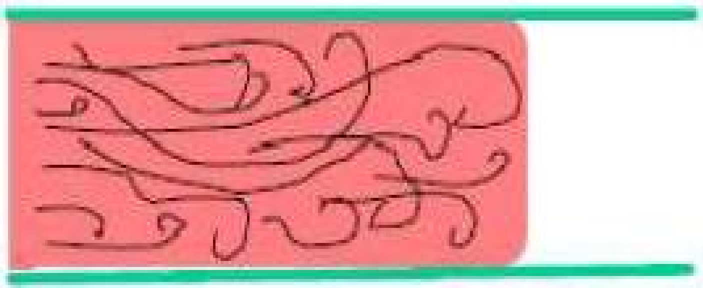


La lamina a contatto con la parete è stazionaria, quella a contatto con questa si muove lentamente, la terza più velocemente e così via fino alla lamina centrale che ha la massima velocità.

Il flusso laminare ha quindi un fronte di avanzamento parabolico.

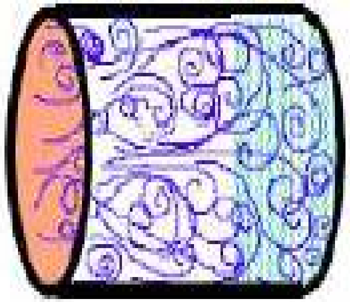
COSA E' IL MOTO TURBOLENTO?

Moto turbolento: le particelle di liquido si muovono con moto vorticoso che determina la comparsa di rumori



QUINDI

1. MOTO DISORDINATO, VORTICI

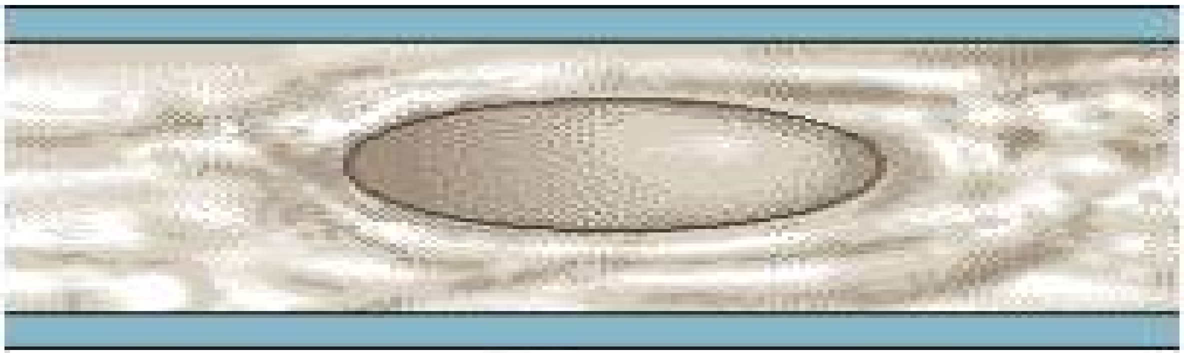


2. LA VELOCITA' NON HA PIU' UN PROFILO REGOLARE

3. LA PORTATA NON E' PIU' PROPORZIONALE ALLA DIFFERENZA DI PRESSIONE MA INVECE

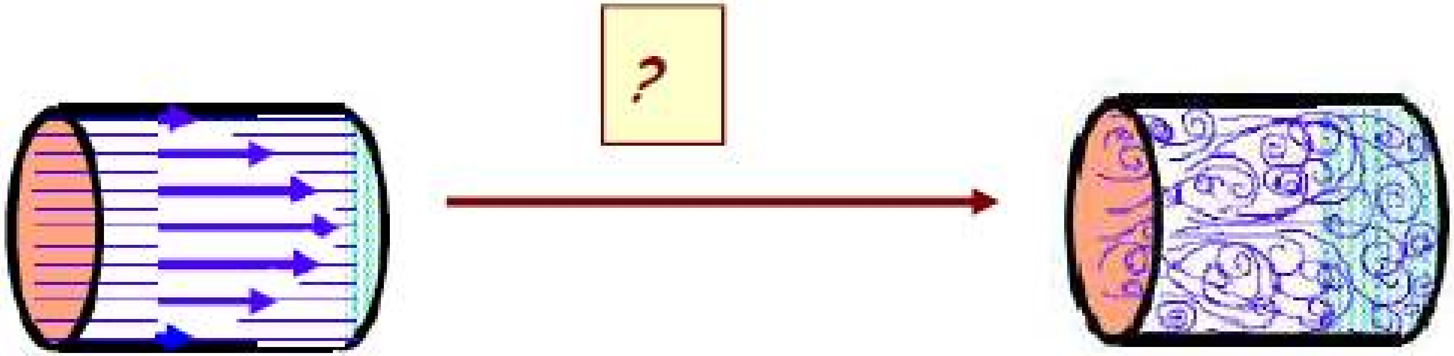
× flusso turbolento:

flusso irregolare
con regioni simili a **vortici**
[es. acqua in prossimità
di rocce e strettoie,
formazione di rapide]



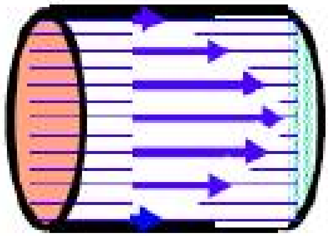
Alta velocità con turbolenza

COSA DETERMINA IL PASSAGGIO DAL MOTO LAMINARE AL MOTO TURBOLENTO?

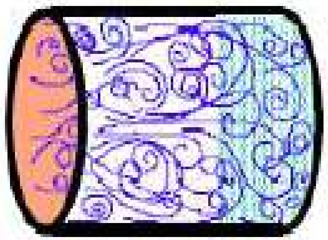


AL DI SOPRA DI UNA CERTA ***VELOCITA' CRITICA*** v_c, IL MOTO NON PUO' PIU' ESSERE DESCRITTO DAL REGIME LAMINARE.

IL MOTO SI FA DISORDINATO E SI CREANO DEI VORTICI.



$$v > v_c$$



$$v_c = \frac{\Re\,\eta}{d\,r}$$

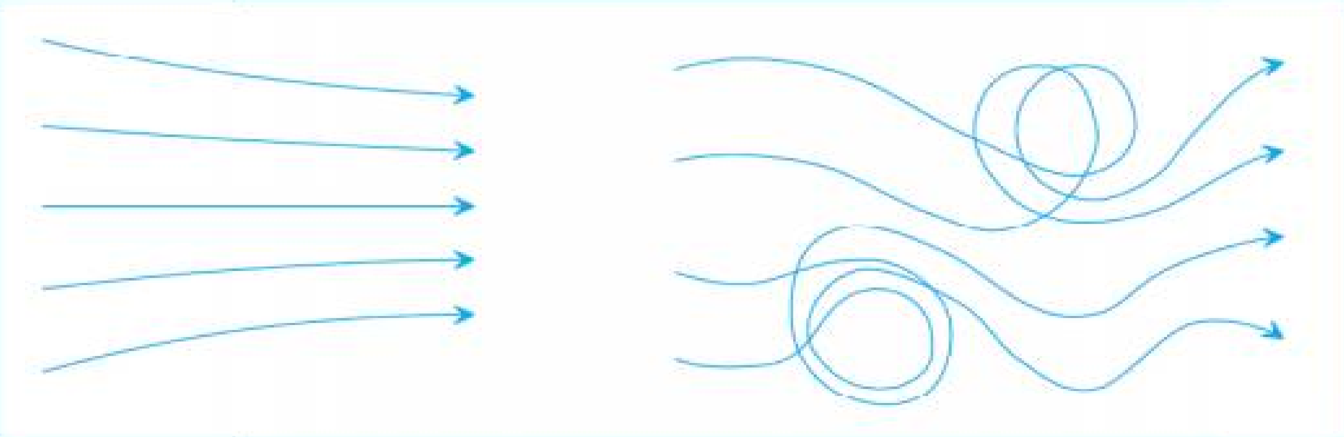
DOVE

$\Re$ = numero di Reynolds

d: DENSITA'

η: VISCOSITA'

r: RAGGIO



Flusso laminare (a sinistra) e turbolento

Il Numero di Reynolds determina se il flusso del fluido sarà turbolento (**R_e alto**) o laminare (**R_e basso**)

$$R_e = \frac{\rho v d}{\mu}$$

R_e = adimensionale
ρ= densità del fluido (M/L^3, Kg/m^3)
v = velocità di deflusso (darciana) (L/T, m/s)
d = diametro del tubo di flusso (L, m)
μ= viscosità (M/T*L, Kg/s*m)

In acque sotterranee la turbolenza è stata riscontrata per numeri di Reynolds **tra 60 e 600**.

Bear (1972) definì che la legge di Darcy è valida fino a che il numero di Reynolds non eccede valori variabili tra 1 e 10.

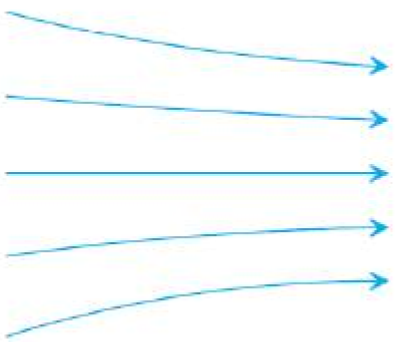
Al di sotto di R_e=1 tutti i flussi attraverso mezzi granulari sono di tipo laminare.

For high velocities: Reynolds number

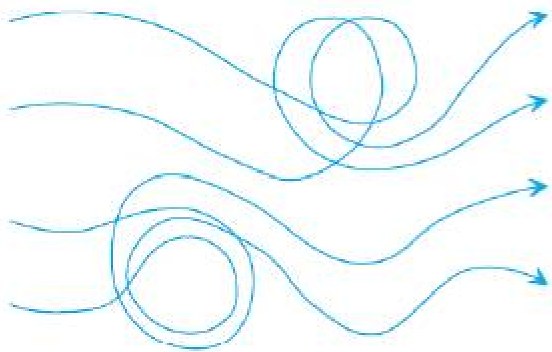
It has been observed that the linear relation between the hydraulic gradient and the flow rate is no more valid when the flow rate increases

Making an analogy between flow between voids in a porous medium and flow in a tube, it is possible to give a limit in terms of Reynolds Number

Laminar flow



Turbulent flow



Reynolds number (dimensionless)

$$R_e = \frac{\rho v d}{\mu}$$

ρ= fluid density (M/L3, Kg/m3)
v = Darcy velocity (L/T m/s)
d= characteristical dimension of porous medium.
(L, m)
μ= viscosity (M/T*L, Kg/s*m)

It determines if the flow is laminar (Re low) or turbulent (Re high)

Reynolds number

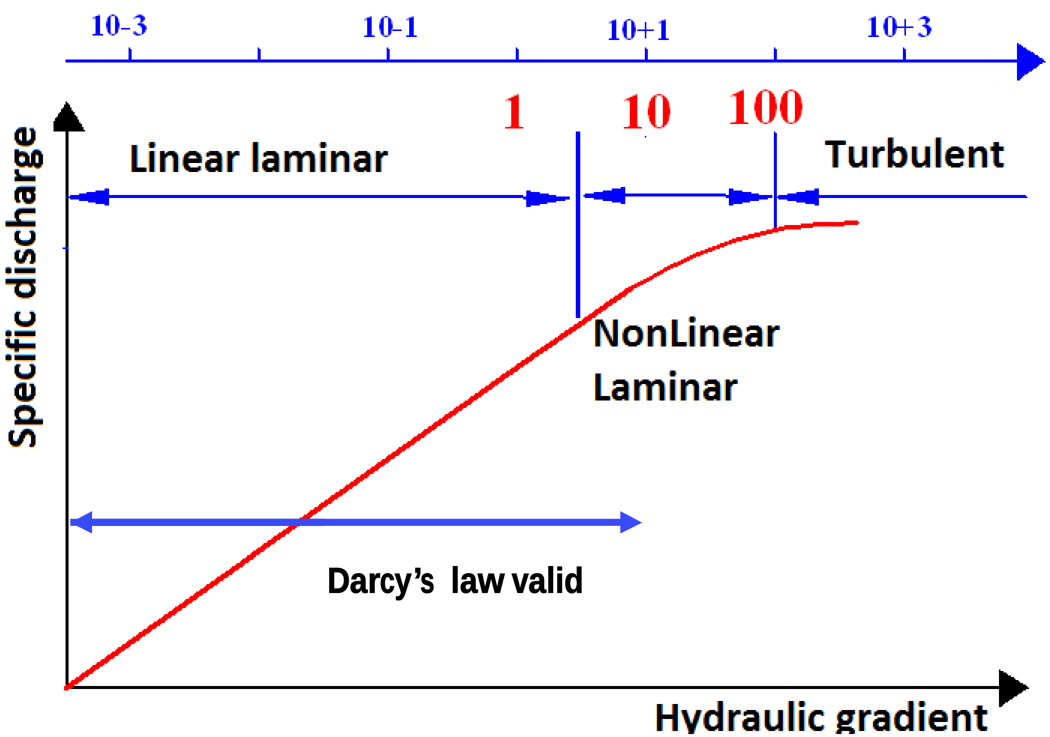
Reynolds number can be calculated in the same manner for a porous medium, with the difference that there is a characteristical dimension of this porous medium

Most of the authors use the grain diameters (in case of granular material) $Re = \frac{\rho V d}{\mu}$

Others prefer using the permeability:

$d = \sqrt{K/\Phi}$ $\quad d = \sqrt{K}$ $\quad \phi$ porosity

According to Bear (1972), Darcy' law which supposes a laminar flow is valid for Reynolds number less than 1, but the upper limit can be extended up to about 10



Turbulence in groundwater has been found for Reynolds numbers between 60 and 100.

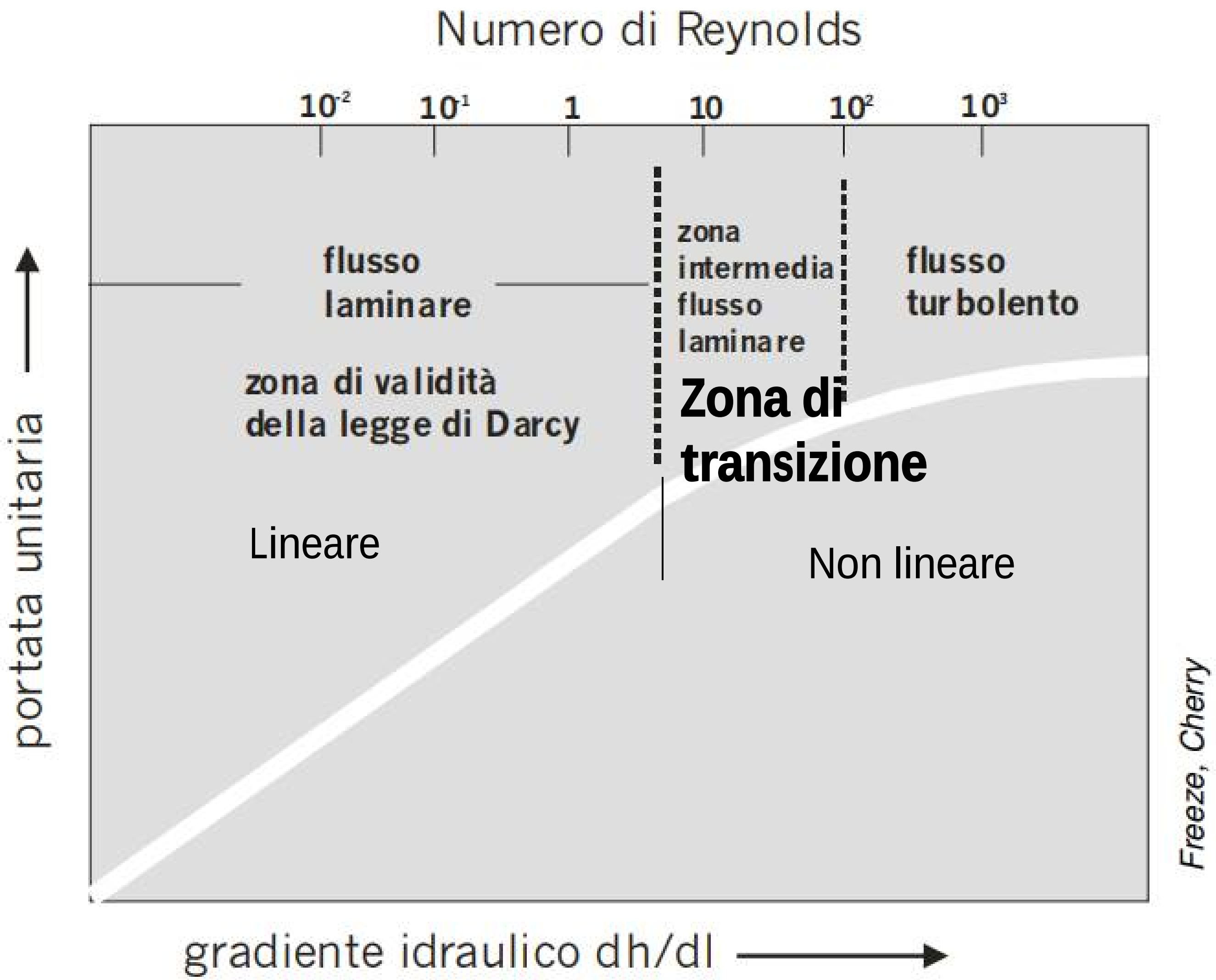
Flow rates exceed the upper limit of validity of Darcy's law in the case of karstic limestones and dolomites and cavernous structures; in these cases, a non-linear relation between flux and gradient must be used.

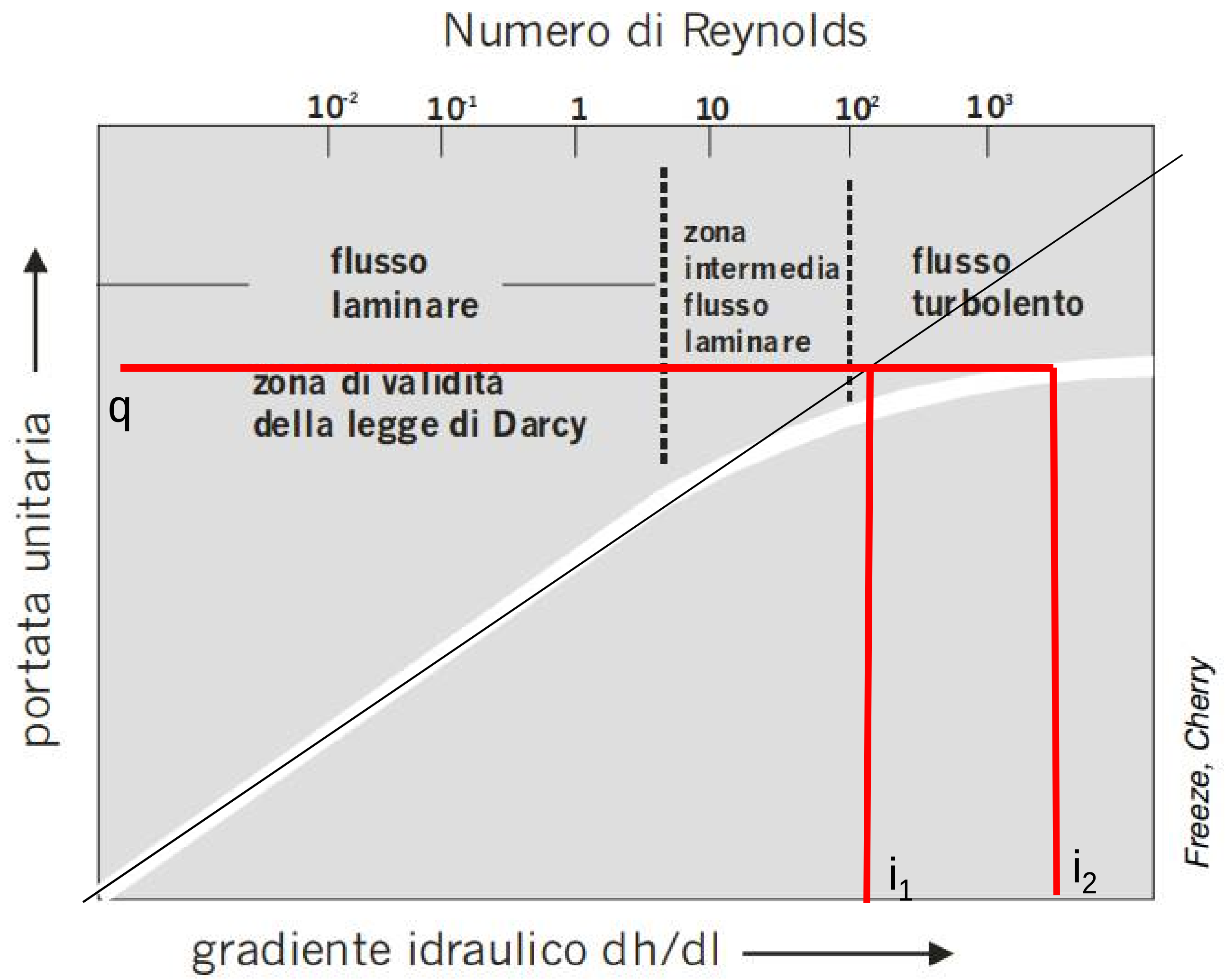
Between the laminar and the turbulent flow there is a transition zone, where the flow is laminar but non-linear. In a general way, Darcy's law can be written:

$$q = -K \cdot \left(\frac{dH}{dl}\right)^m$$

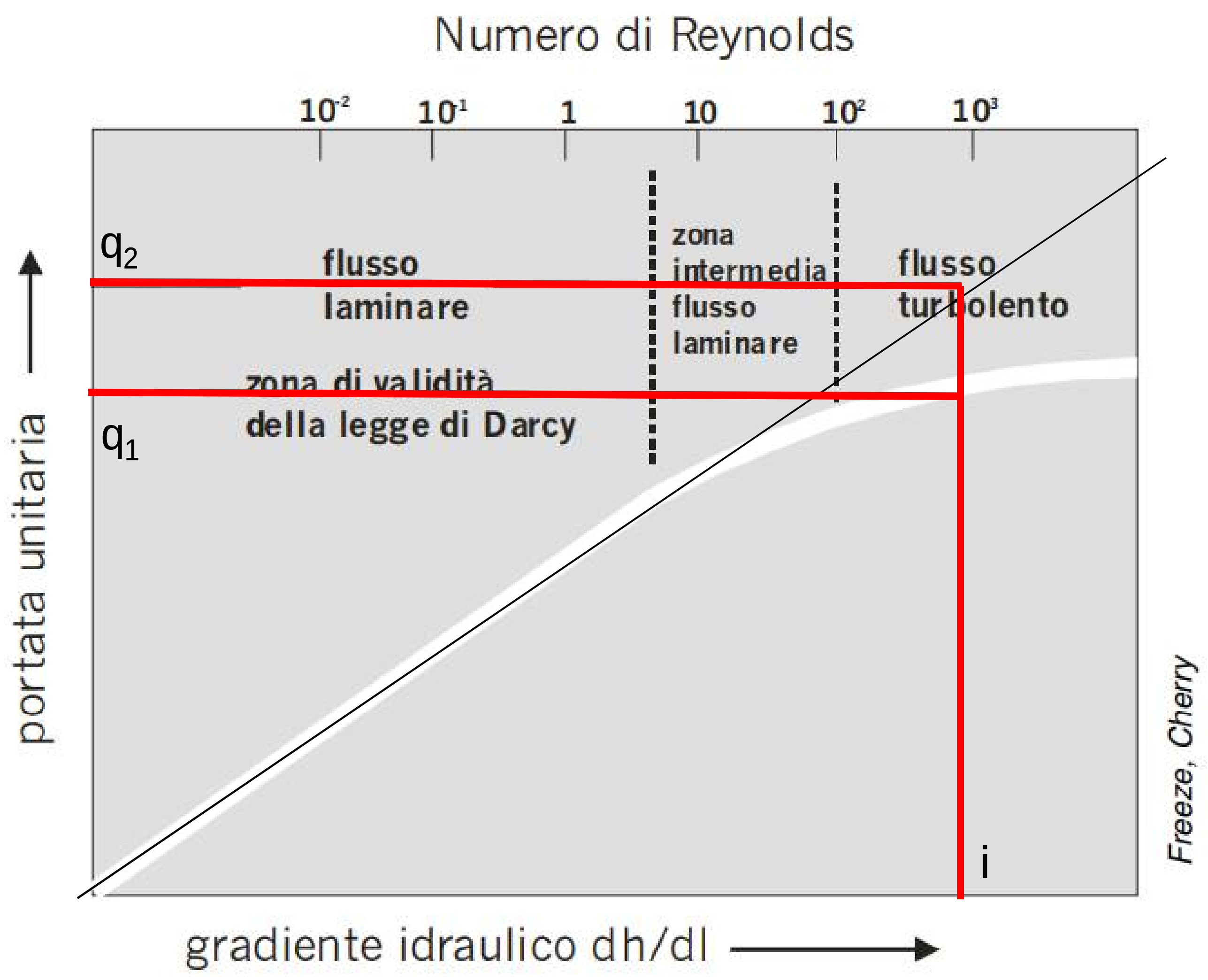
m = 1, the flow law is linear laminar

Campo di applicazione della legge di Darcy



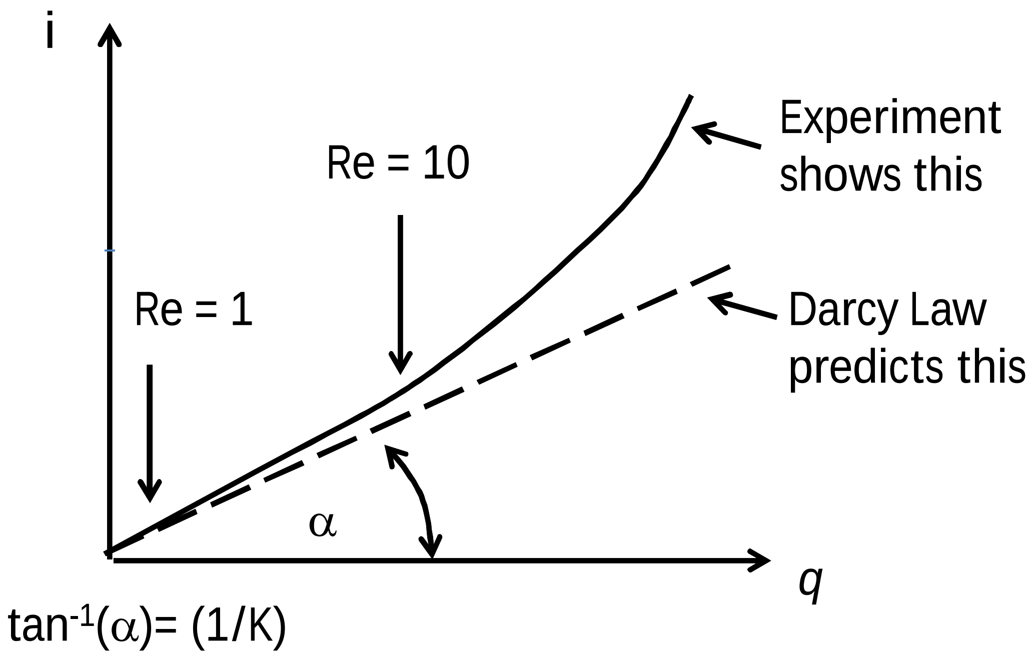


La stessa portata q ha bisogno di un gradiente idraulico maggiore (i_2) per essere trasportata in condizioni di flusso non lineare (di transizione o turbolento) rispetto alle condizioni lineari (i_1)(darciane) (per via della dissipazione di energia dovuta al moto turbolento)



Lo stesso gradiente i riesce a trasportare una portata minore (q_1) in condizioni di flusso non lineare (di stransizione o turbolenteo) rispetto alle condizioni lineari (q_2)(darciane) (per via della dissipazione di energia dovuta al moto turbolento)

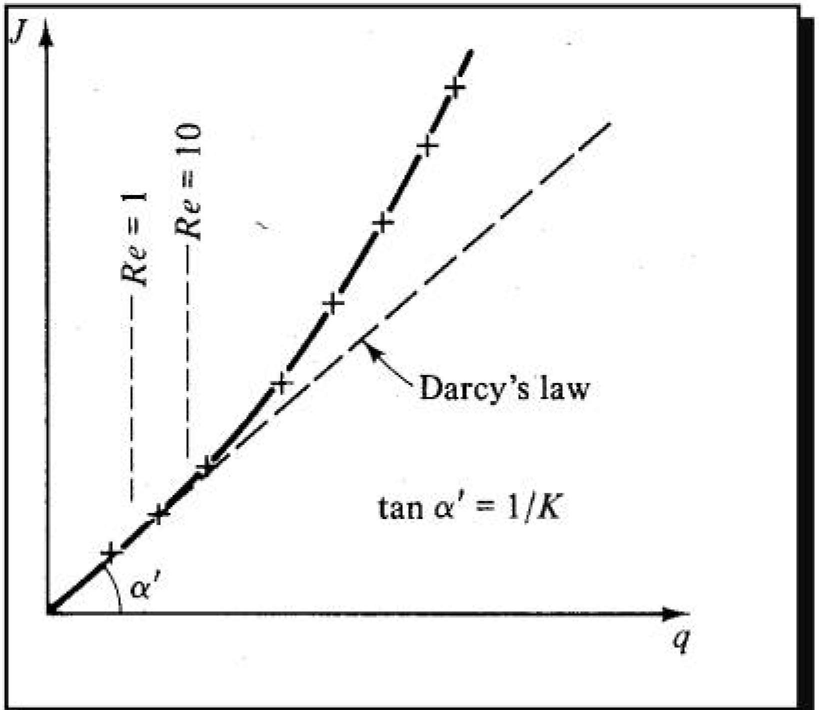
Specific Discharge vs Head Gradient



La Legge di DARCY

Campo di validità: al crescere della velocità del fluido, la relazione fra portata defluente e perdita di carico diviene non più lineare.

Numero di Reynolds dei granuli: $\mathrm{Re}_f = \dfrac{q \cdot d}{\nu}$ **dove d è il diametro medio del mezzo poroso**



La linearità viene meno per *Re* >10 (cala l'effetto delle forze viscose rispetto a quelle inerziali).

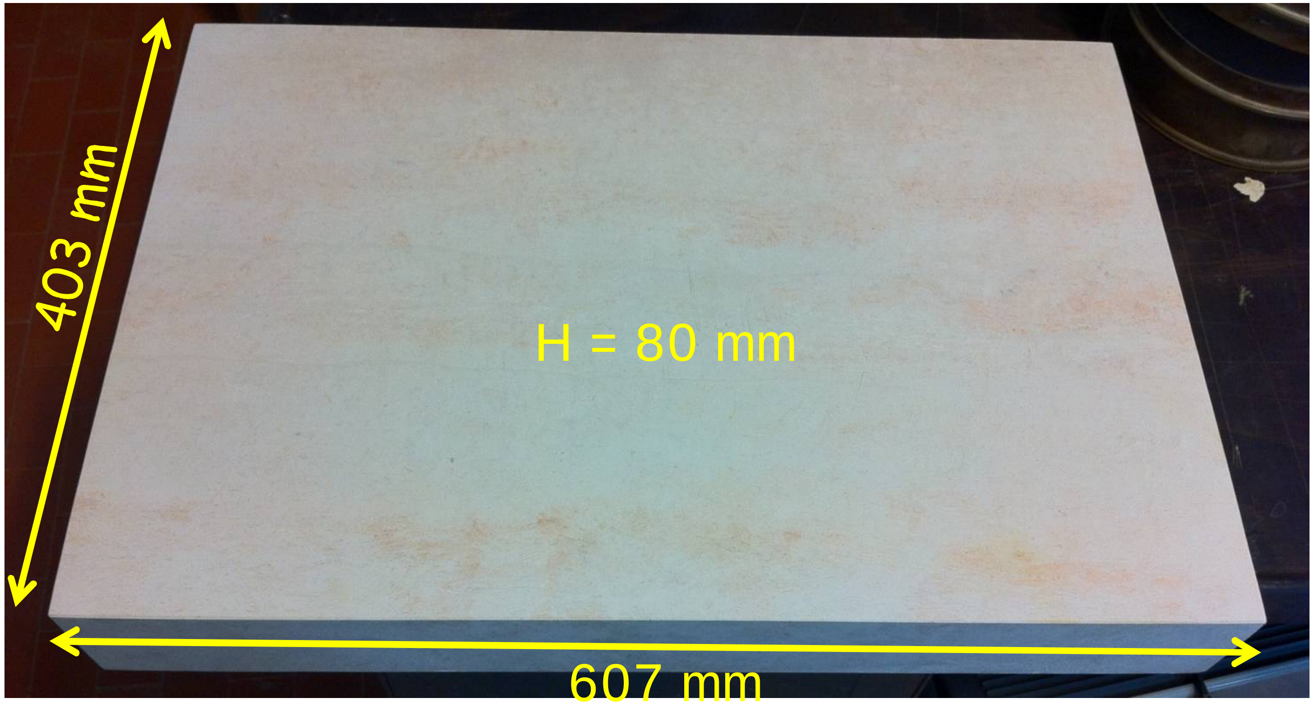
Ciò si verifica in vicinanza di grandi pozzi di emungimento/ricarica, sorgenti, ecc.

In genere ci si trova nel campo di validità della Legge di Darcy

Analysis on artificial rock samples

Preparation of block sample

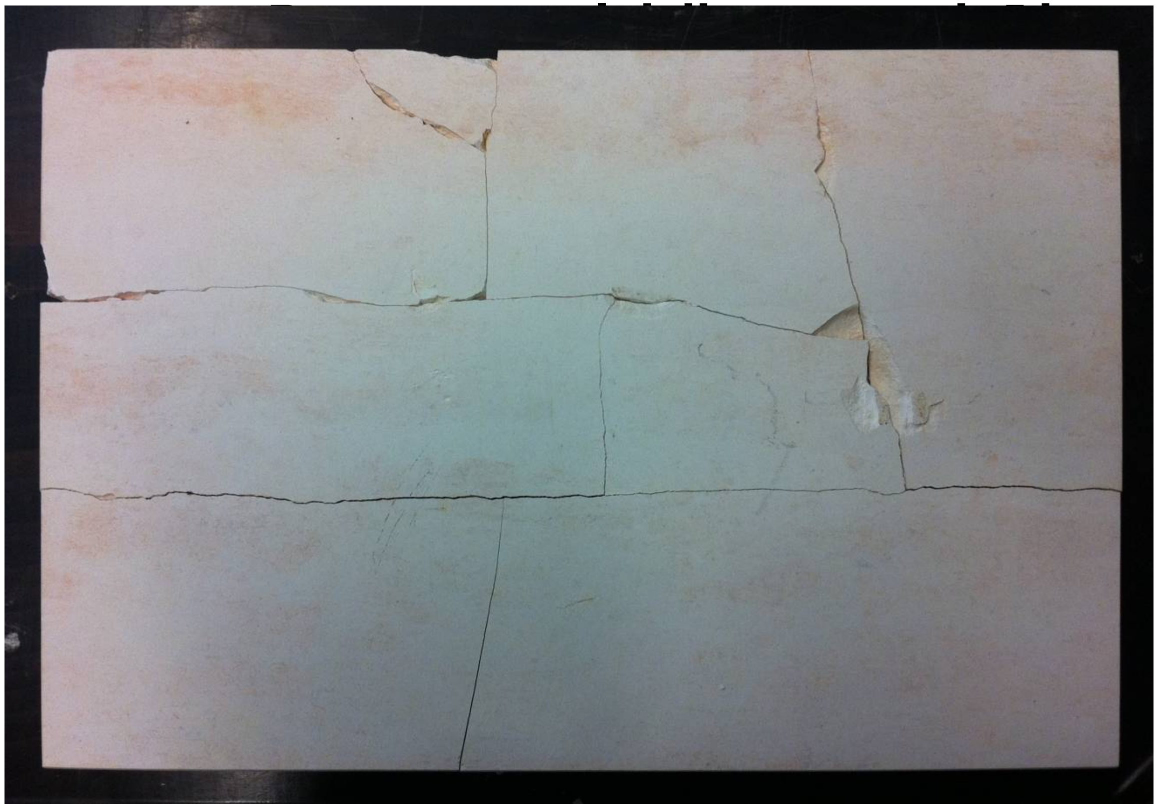
Limestone block (pietra di Cisternino)



Analysis on artificial rock samples

Preparation of block sample

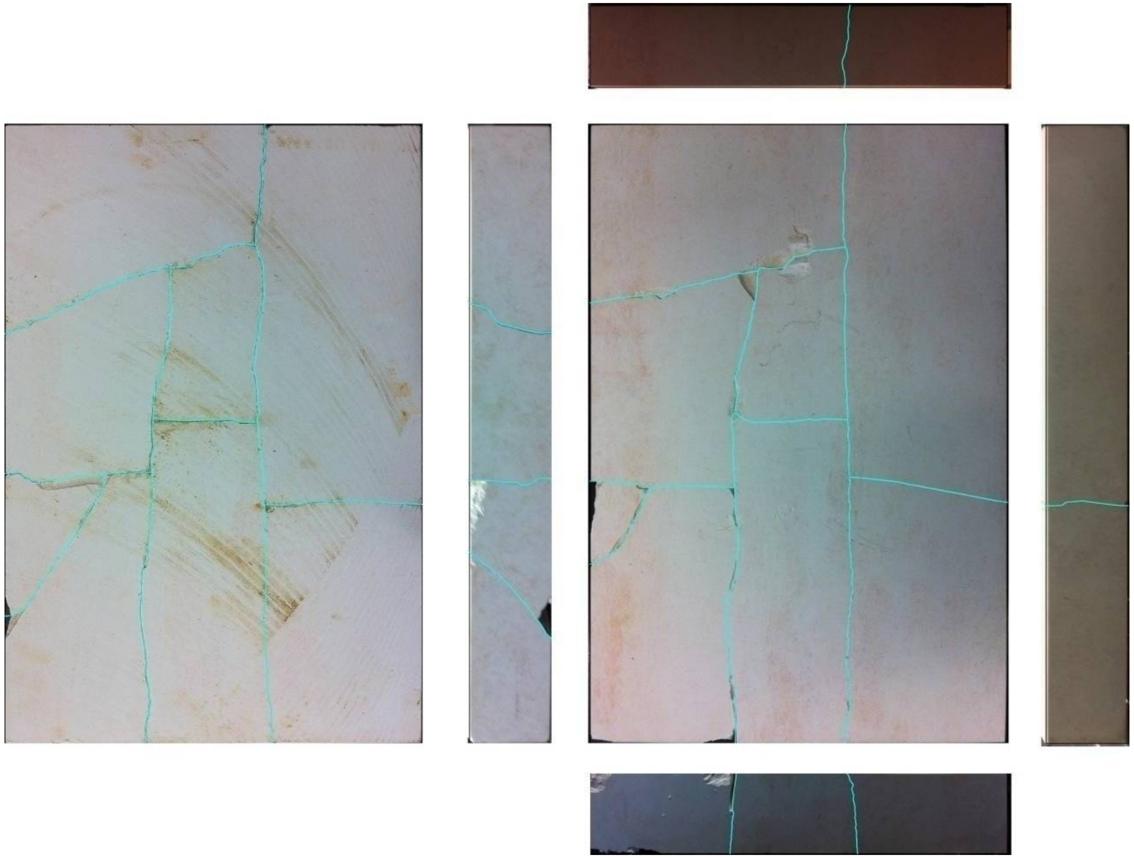
Fracture network made artificially by means of 5 kg mallet blows



Analysis on artificial rock samples

Characterization of discontinuities

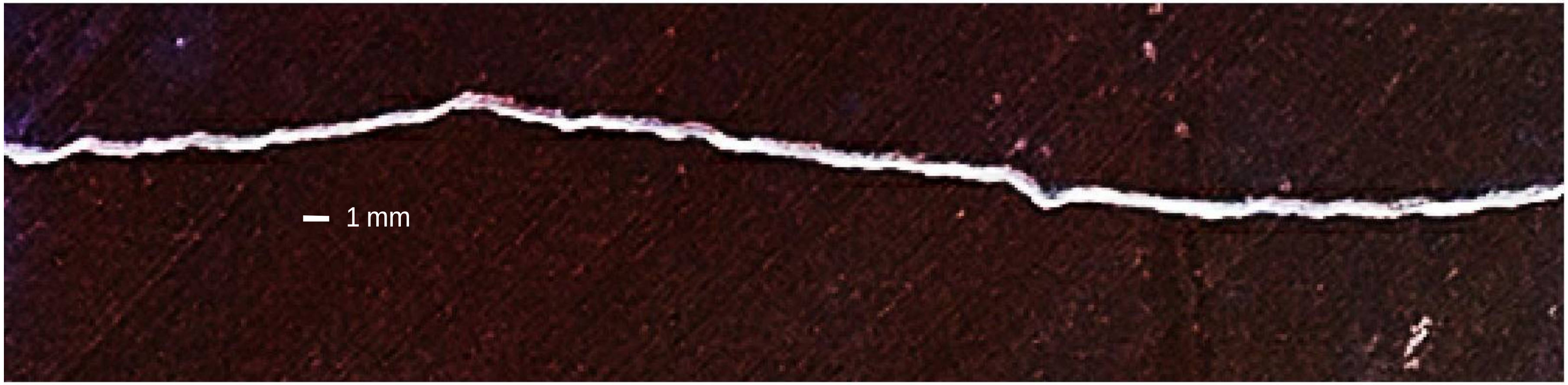
Perspective rectification



Analysis on artificial rock samples

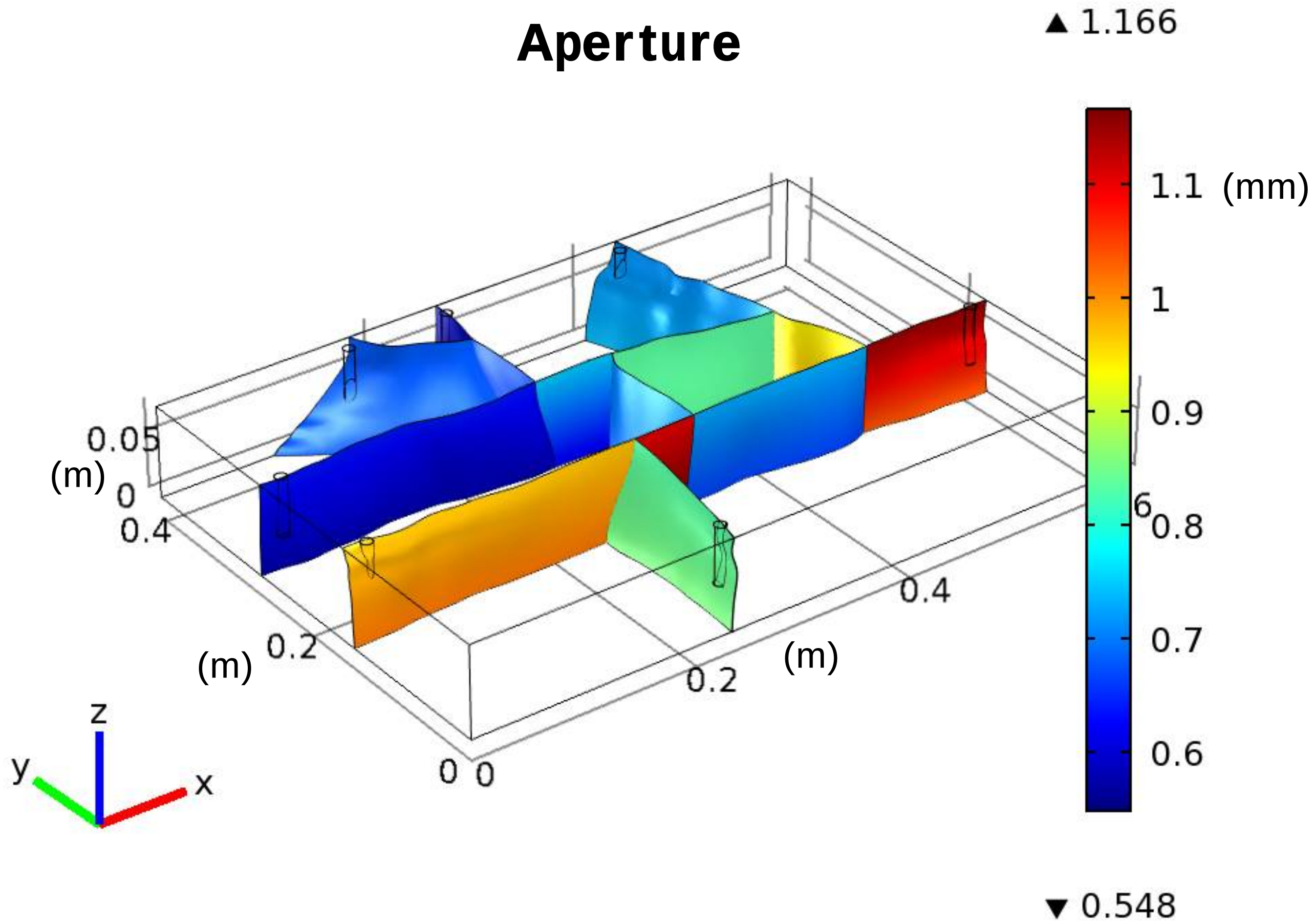
Characterization of discontinuities

Characterization of profile by means of photogrammetric techniques



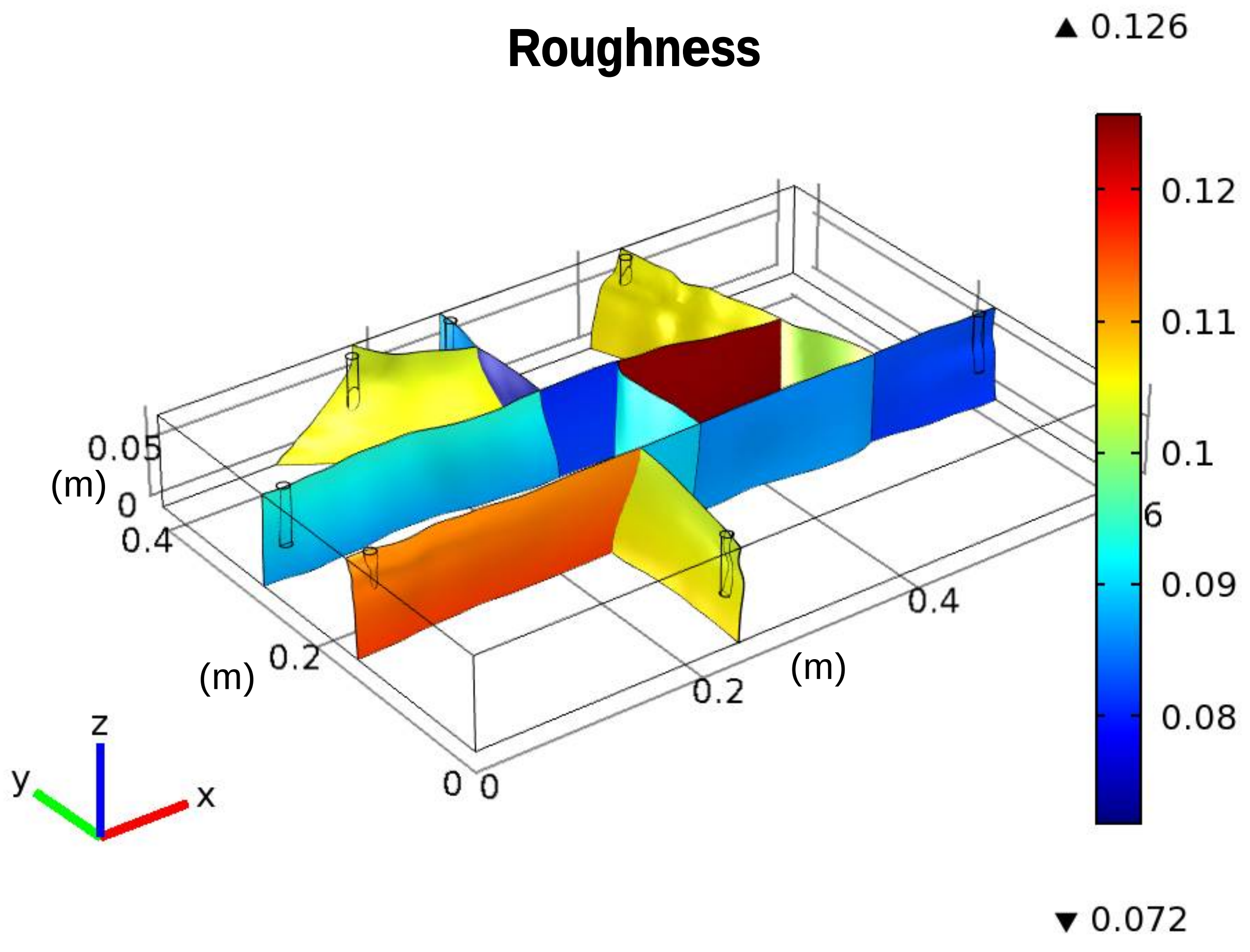
Analysis on artificial rock samples

Characterization of discontinuities



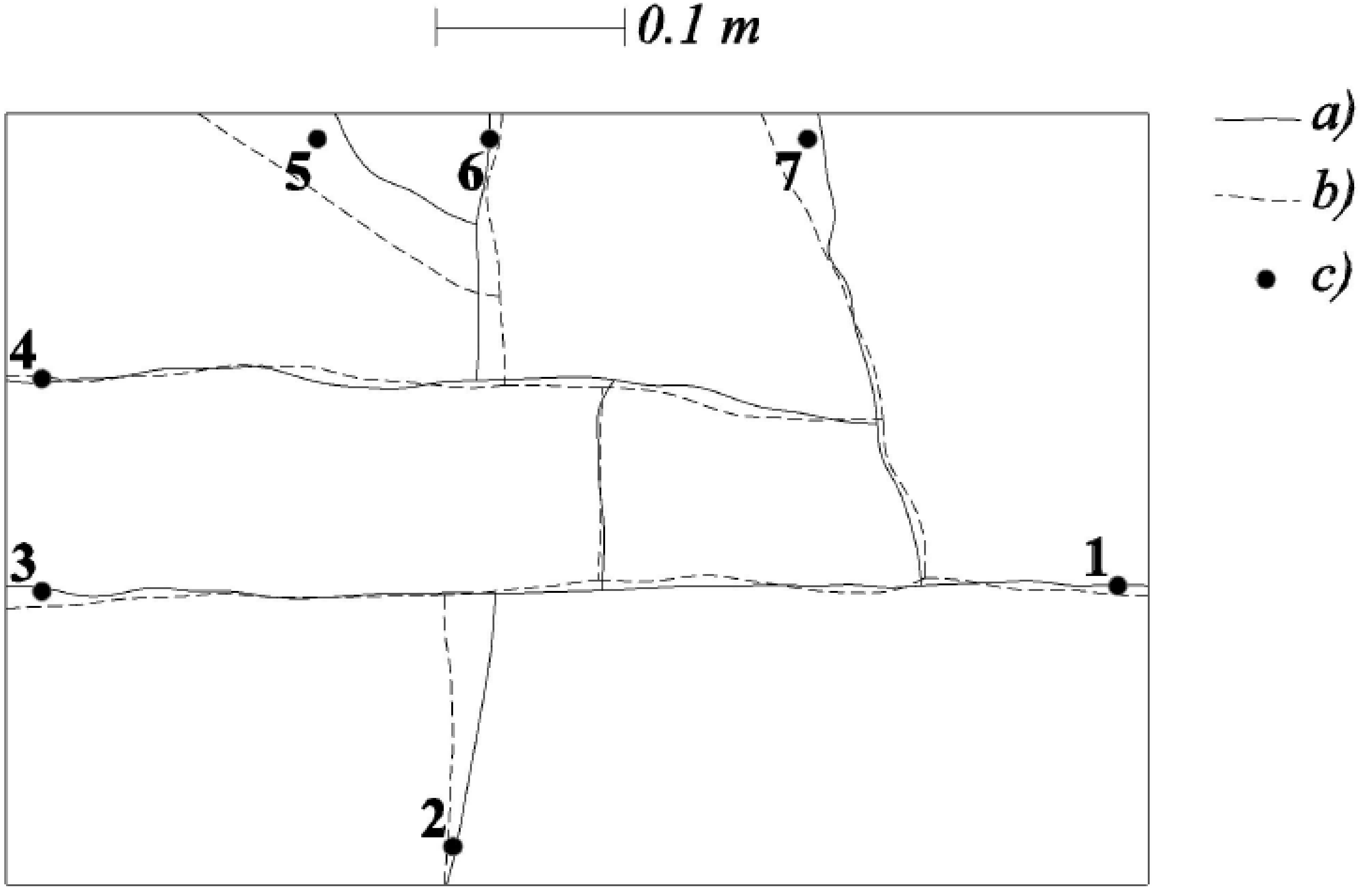
Analysis on artificial rock samples

Characterization of discontinuities



Analysis on artificial rock samples

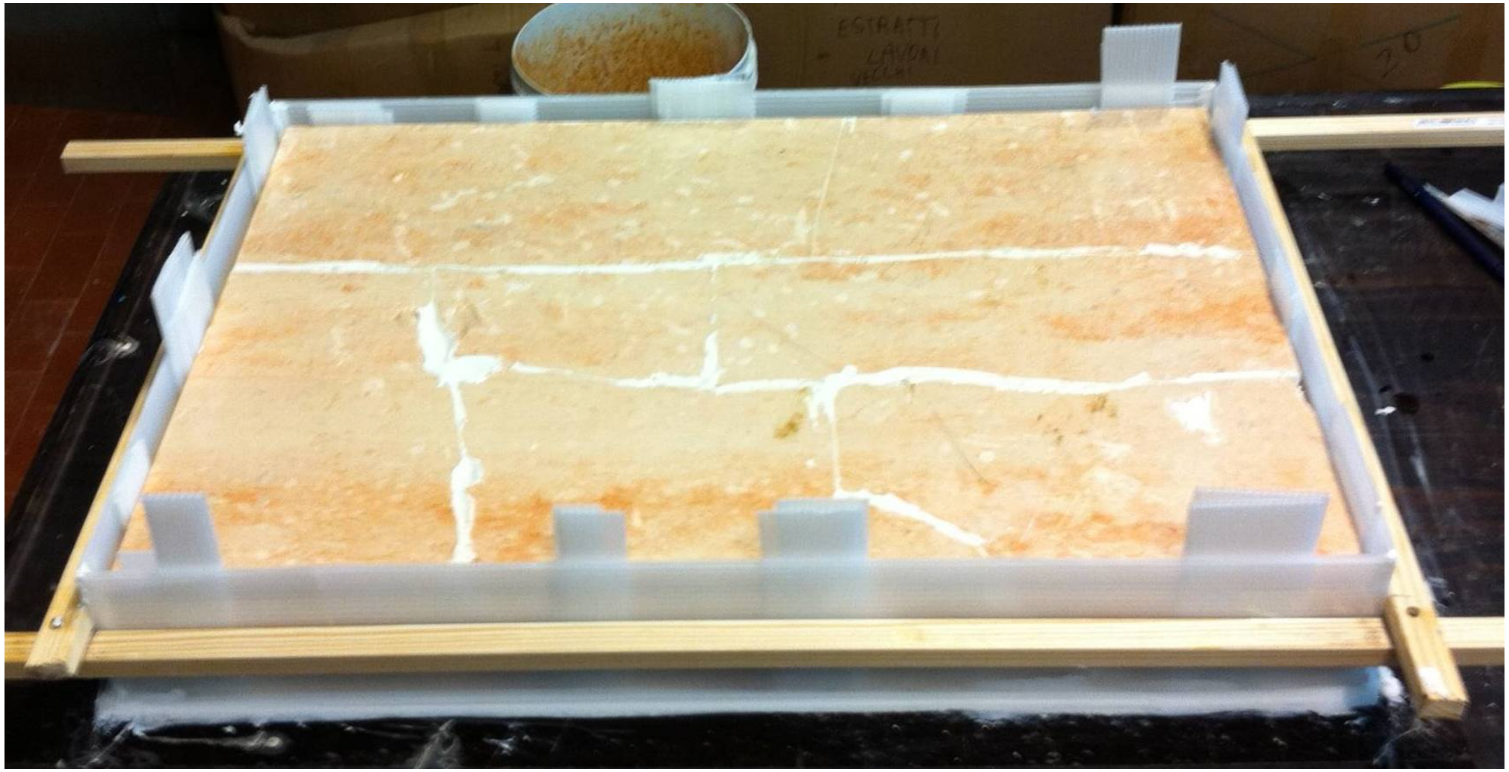
Representation of discontinuities



Analysis on artificial rock samples

Preparation of sample

Surface of block sample sealed with transparent epoxy resin.



Analysis on artificial rock samples

Preparation of sample

Opening of ports in correspondance of fractures

Analysis on artificial rock samples

Preparation of sample

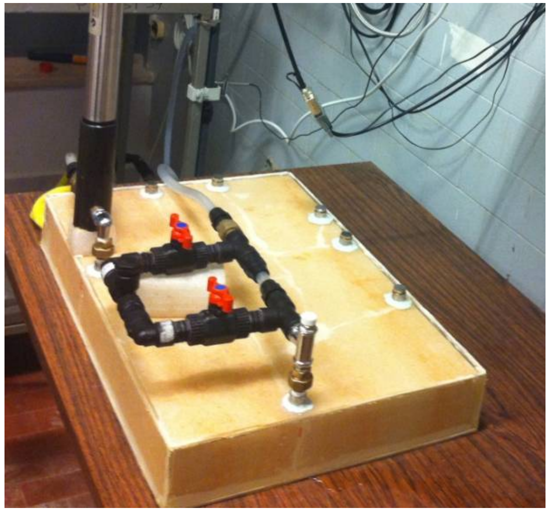
Opening of ports in correspondance of fractures

Hydraulic tests on a fractured rock sample

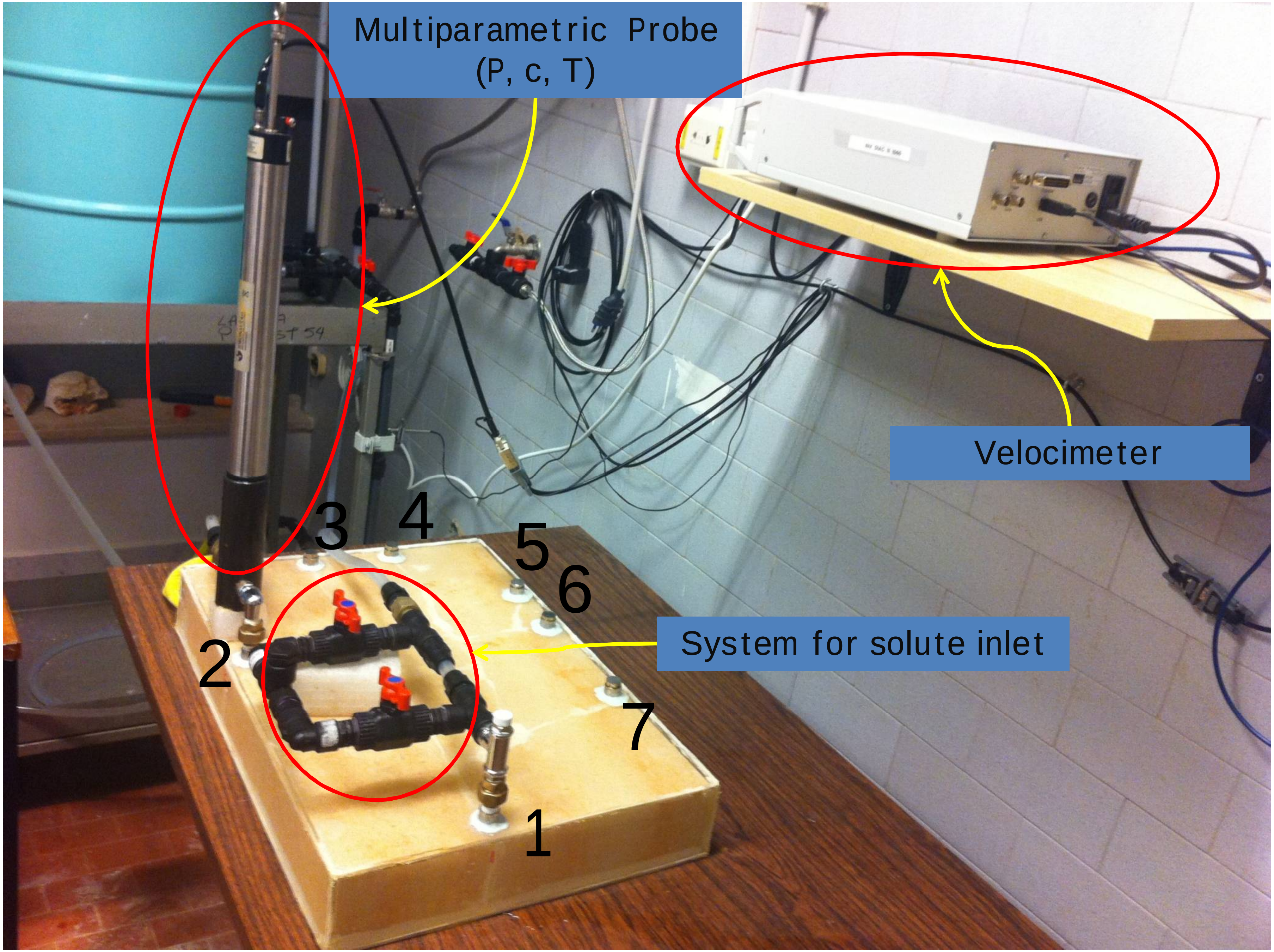
<u>Hydraulic circuit</u>

Water tank

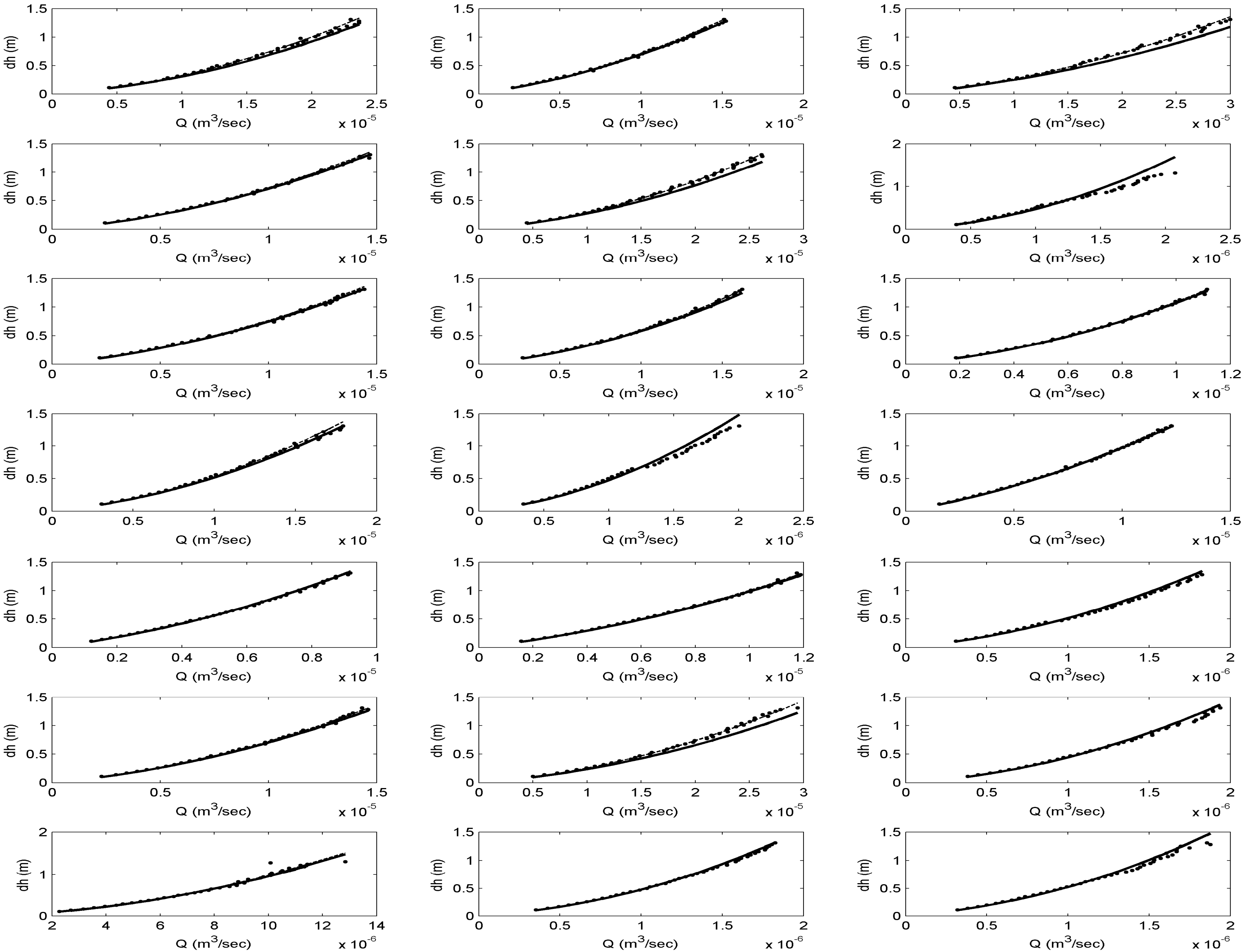
Water tank



Water moves from the upstream to the downstream tank and returns to the upstream tank by means of a transfer pump.

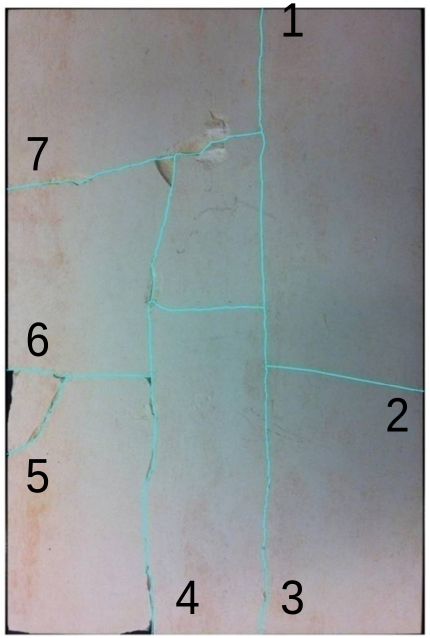

Multiparametric Probe
(P, c, T)
Velocimeter
System for solute inlet
1
2
3
4
5
6
7

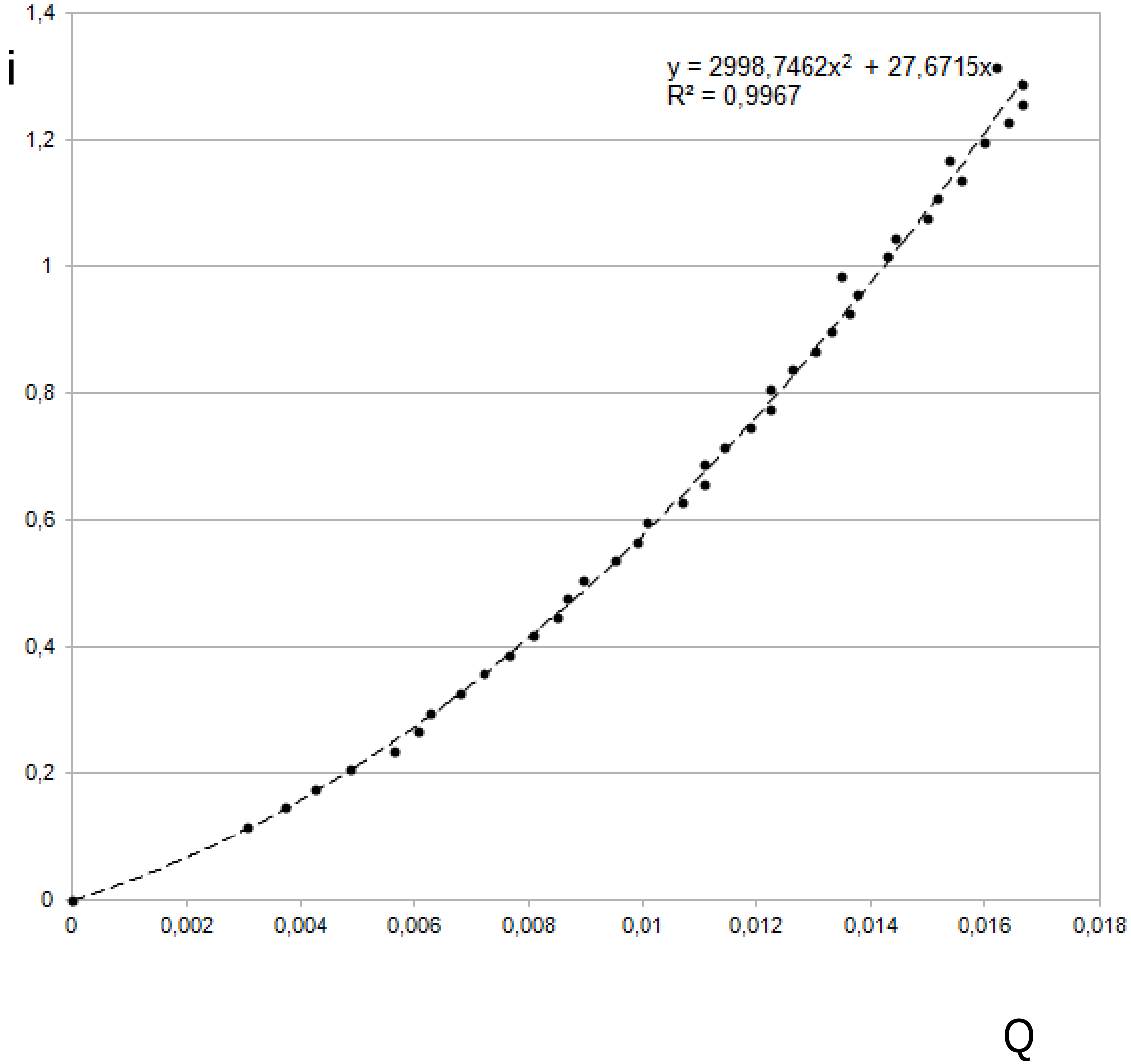
Flow rate *Q* versus hydraulic head difference dh for all the experiments. Dots represents the experimental values, dashed line represents the fitting of experimental values, marked line represents the relationship without the effect of circuit (C=0).



Results

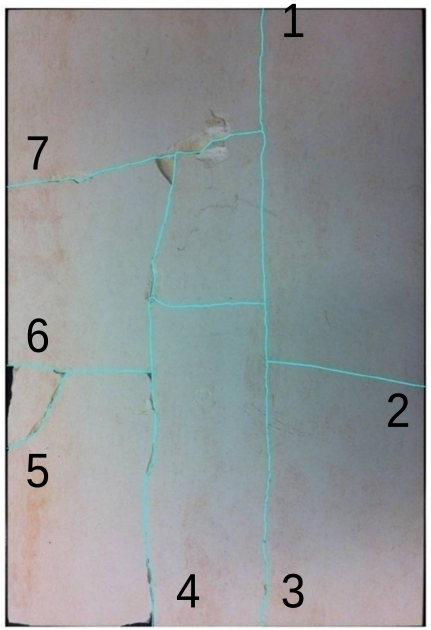
Ports 1-2

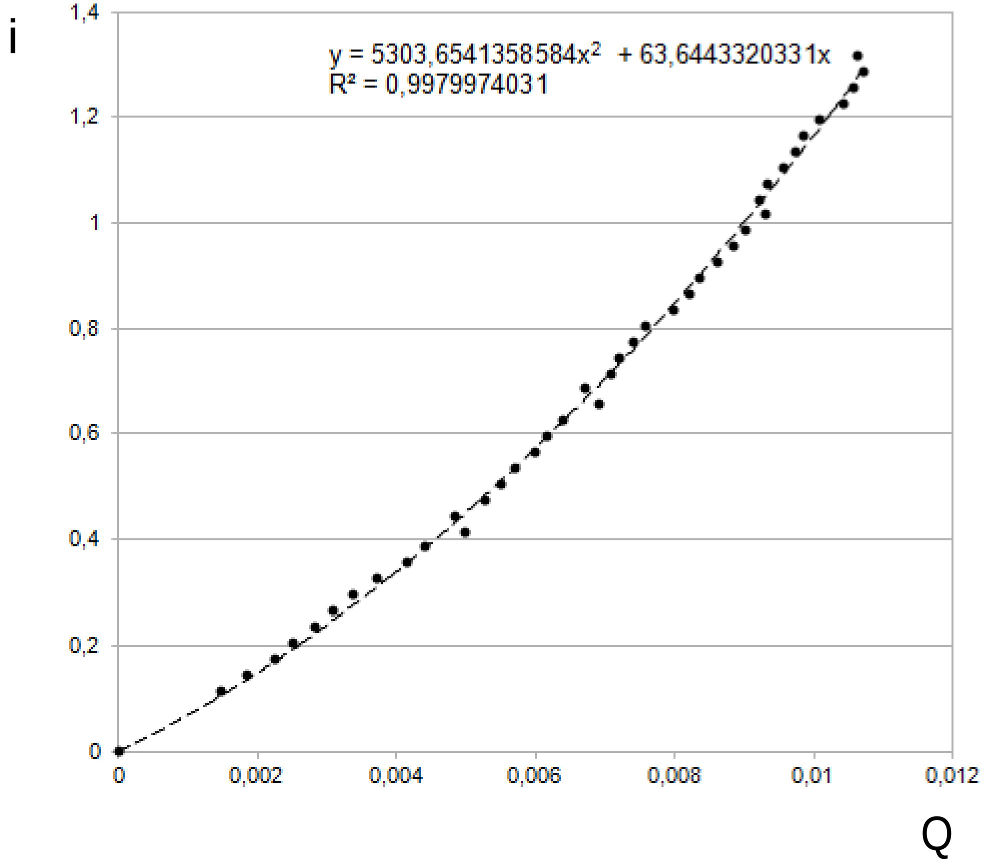




Results

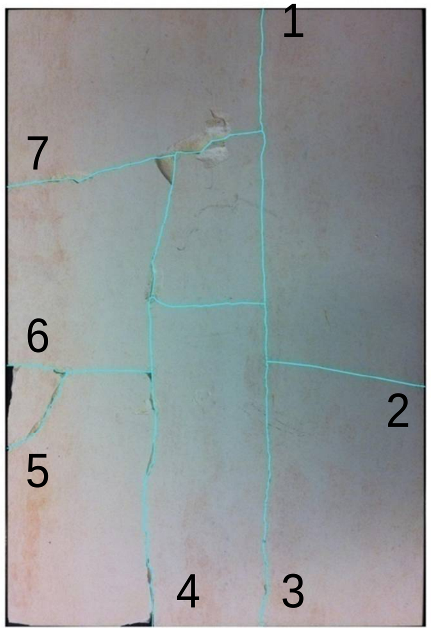
Ports 1-3

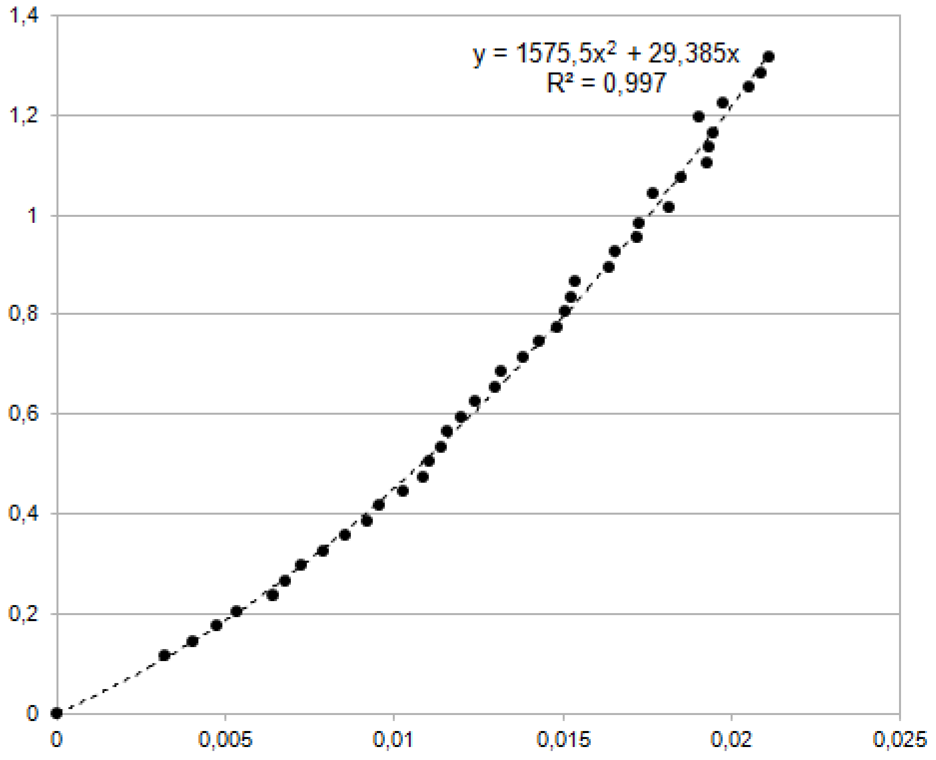




Results

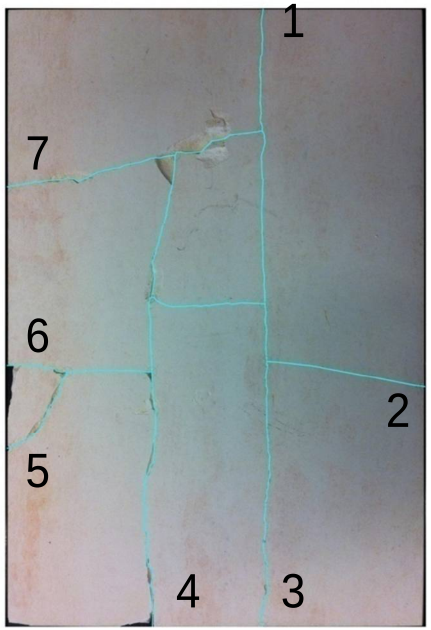
Ports 1-4

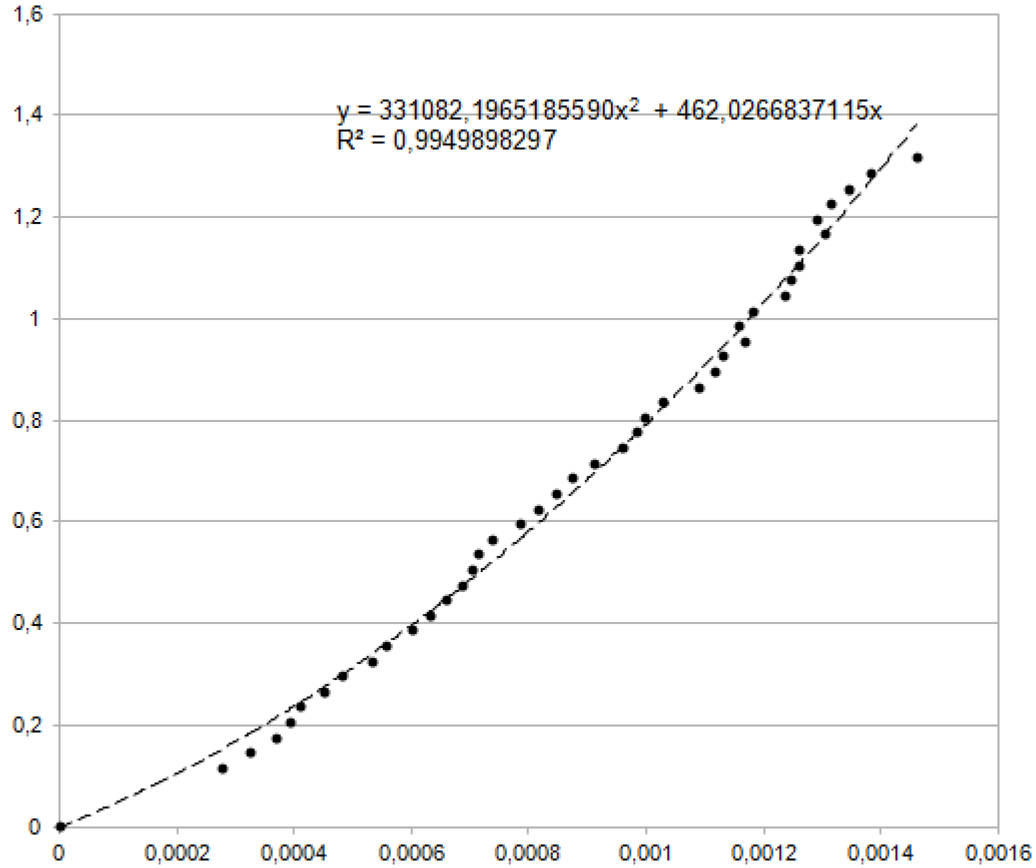




Results

Ports 1-7





EVIDENCE OF NON LINEAR BEHAVIOR

“ STRONG INERTIA” REGIME

TRANSITIONAL

CURVES FITTED BY FORCHHEIMER EQUATION

extension of Darcy's law

The pressure loss is described
by the sum of a linear and a square term in seepage velocity

$q = Q/A = K\,i$

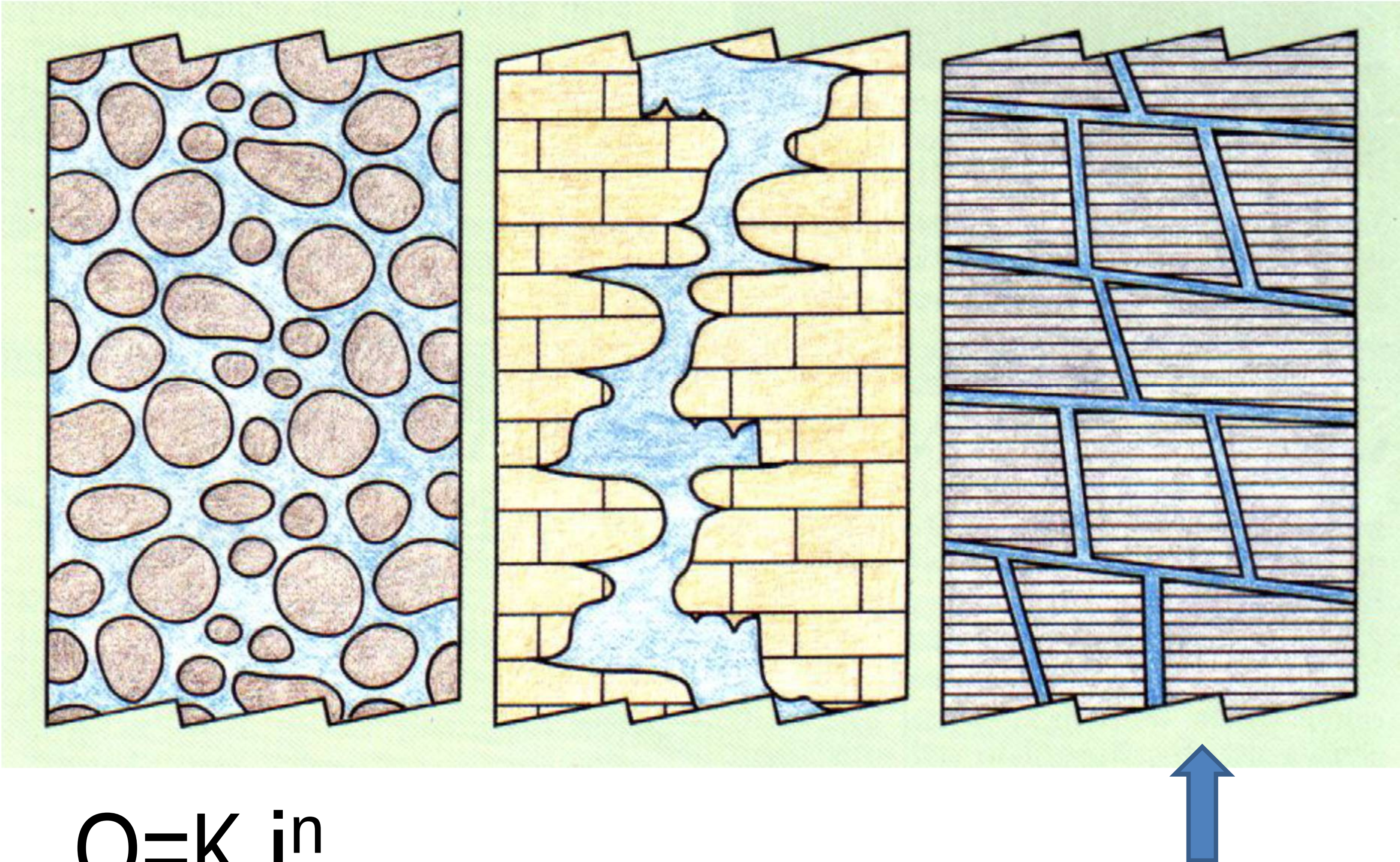
$$i = aq + bq^2$$

Forchheimer equation

$$i = q/K$$

The linear term is proportional to fluid viscosity μ and inversely proportional to permeability k

The square term is proportional to inertial resistance β and the fluid density ρ

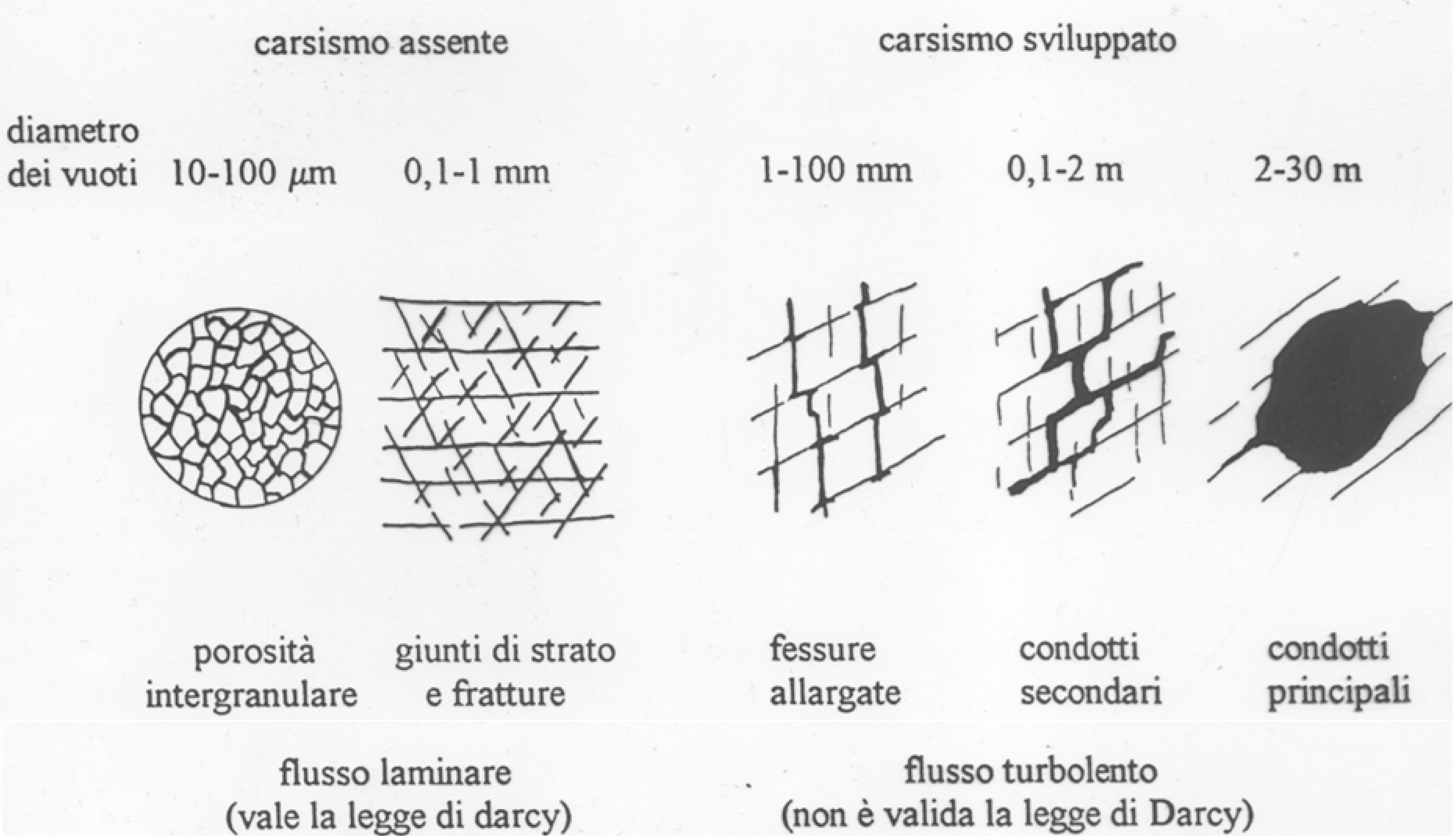


$Q=K\ i^n$

$n=1$

Izbash equation $Q=K\ i^n$

$n>1$


carsismo assente
carsismo sviluppato
diametro dei vuoti
10-100 μm
0,1-1 mm
1-100 mm
0,1-2 m
2-30 m
porosità intergranulare
giunti di strato e fratture
fessure allargate
condotti secondari
condotti principali
flusso laminare
(vale la legge di darcy)
flusso turbolento
(non è valida la legge di Darcy)

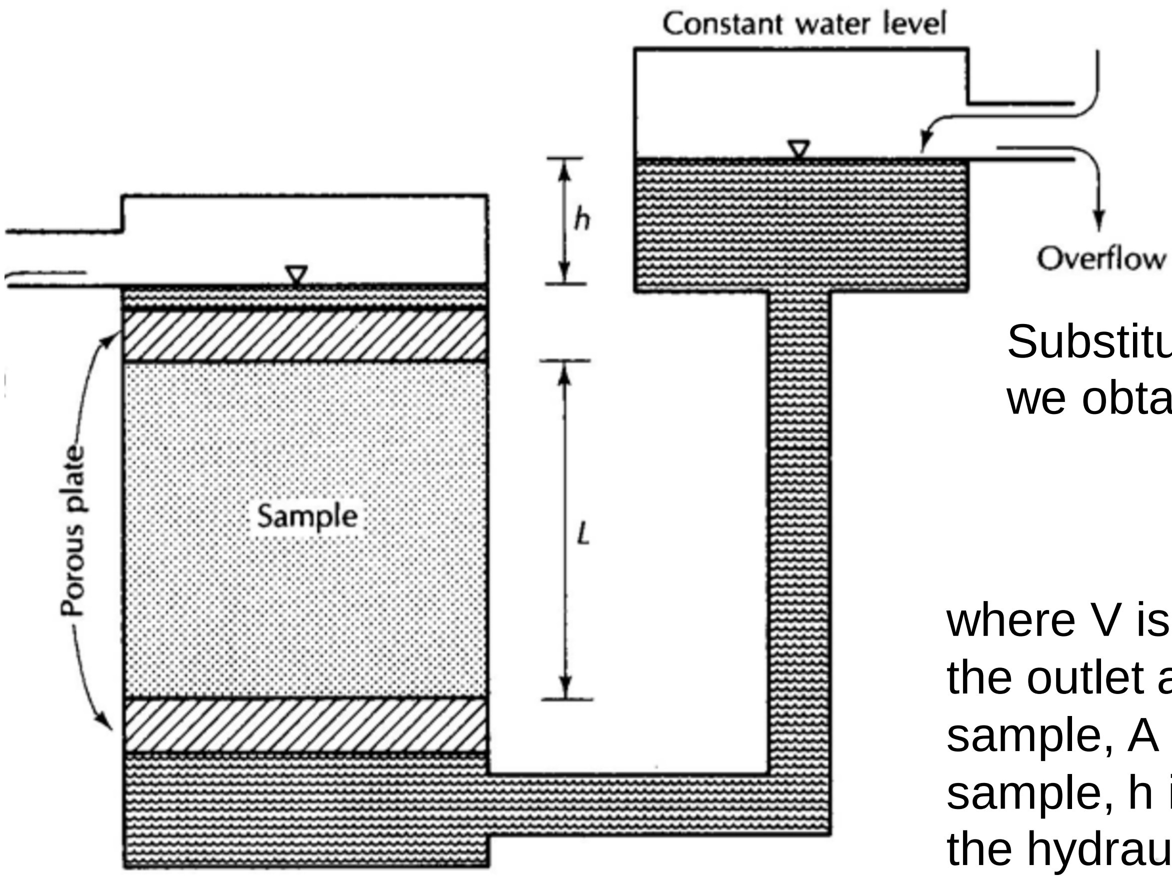
Measuring Hydraulic Conductivity

A permeameter is a simple column with an inlet and outlet.

For media of relatively high hydraulic conductivity (e.g., medium to coarse sand), a constant-head method is used to determine the hydraulic conductivity. Water is continuously added to the reservoir to keep the water level constant, and the flow from the outlet of the permeameter is collected over time. Applying Darcy's law we have:

best for samples with K>0.01cm/min

$$Qt = KA\frac{\Delta h}{L}t$$



Substituting V for Qt and rearranging, we obtain:

$$K = \frac{VL}{Ath}$$

where V is the volume of water collected at the outlet at time t, L is the length of the sample, A is the cross-sectional area of the sample, h is the hydraulic head, and K is the hydraulic conductivity k> 10^{-5} m/s permeametro a carico costante

Prova a carico costante

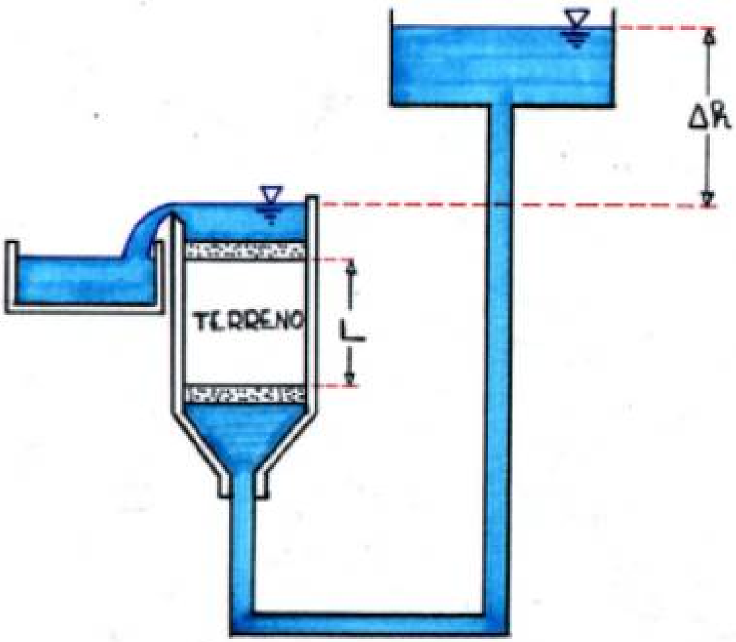


Figura 8.1

$$q = v \cdot A$$

$$q = k \frac{\Delta h}{L} \cdot A$$

Si misura la portata, mentre L, A, Δh sono noti. Troviamo così:

$$k = \frac{qL}{A \Delta h}$$

Per un'argilla, ad esempio

$$k = 10^{-9} \frac{m}{s} \quad L = 0{,}2\,m \quad \Delta h = 2\,m \quad A = 10^{-2}\,m^2$$

$$q = 10^{-9} \frac{2}{0{,}2} 10^{-2} \quad = 10^{-10} \frac{m^3}{s}$$

È più l'acqua che evapora che quella che filtra!
Allora concludiamo che questa prova va bene per permeabilità medio–alte.

Per i materiali che presentano una permeabilità bassa, come l'argilla, si fa una prova a carico variabile.

10^{-8} <k< 10^{-5} m/s permeametro a carico variabile

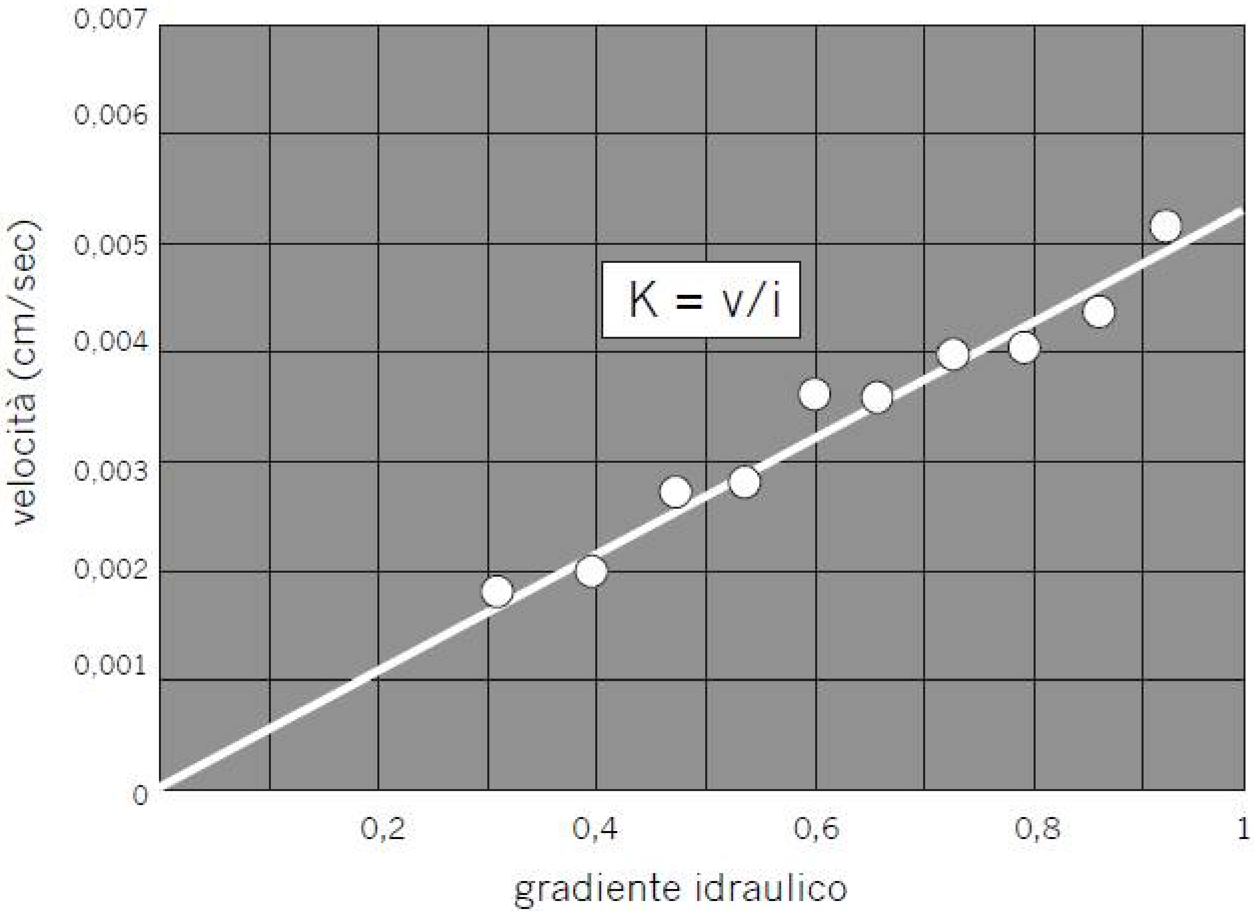


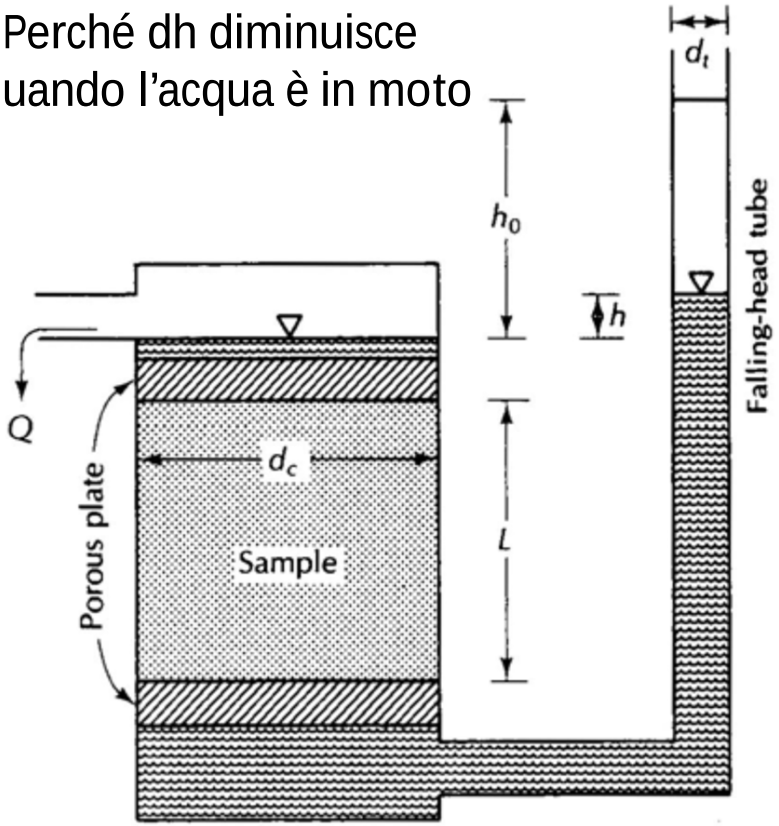
Grafico velocità, gradiente ricavato dalla prova con permeametro a carico costante.

La conducibilità idraulica per il campione, si ottiene da una qualunque coppia di valori v – i lungo la retta.

For media of relatively low hydraulic conductivity, the fluid discharge from the system is small. In this case, it is easier to perform a “falling head” measurement

To calculate hydraulic conductivity in falling-head method, a mass balance approach is taken. The rate of water discharges from the tube into the permeameter is equal to the product of the area of the tube and the rate of fluid movement through the tube:

- Perché dh diminuisce quando l’acqua è in moto



$$Q_{tube} = -A_{tube}\frac{dh}{dt}$$

where dh/dt is the rate of fluid movement through the tube, which is also the rate of movement of the air water interface through the tube, which is also the rate of change in the hydraulic head driving the flow into the permeameter

On the other hand, the rate at which water discharges from the permeameter is given by:

$$Q_{perm} = KA_{perm}\left(\frac{h}{L}\right)$$

where h refers to the difference in hydraulic head at the inlet and the outlet, and L is the length of the sample.. Notice that h decreases with time, hence the flow also decreases with time.

At any one instant, the flow entering the permeameter must equal the flow leaving the permeameter, hence:

$$-A_{tube}\frac{dh}{dt} = KA_{perm}\left(\frac{h}{L}\right)$$

rearranging:

$$Kdt = -\frac{A_{tube}}{A_{perm}}L\frac{dh}{h}$$

integrating from time zero to time t, during which the head decreases from h_o to h

$$\int_0^t Kdt = -\frac{A_{tube}}{A_{perm}}L\int_{h_o}^{h}\frac{dh}{h}$$

gives:

$$Kt = -\frac{A_{tube}}{A_{perm}}L\ln\frac{h}{h_o}$$

rearranging:

$$K = \frac{A_{tube}L}{A_{perm}t}\ln\frac{h_o}{h}$$

Measuring K: Constant and falling head tests

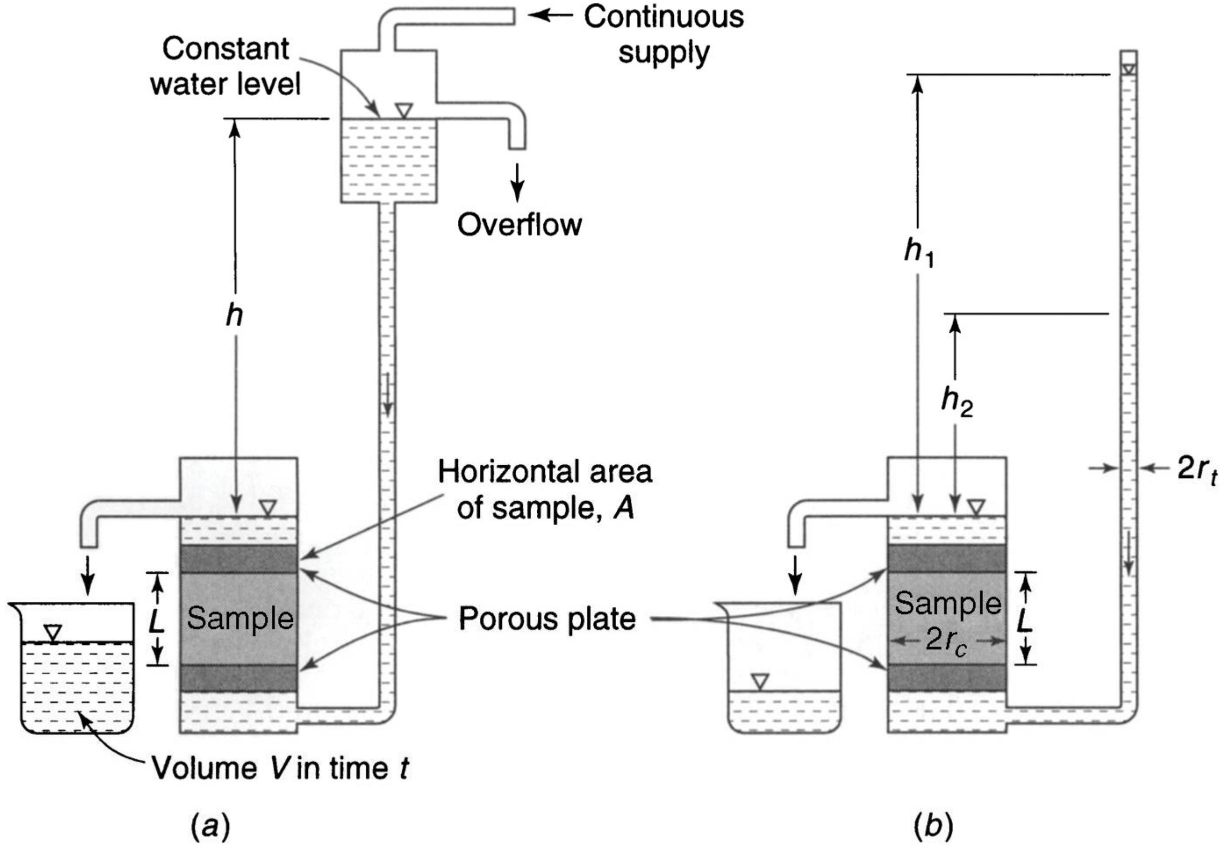


Figure 3.3.1

Flow in tube:

$$Q = \pi r_t^2 \frac{dh}{dt}$$

Flow in sample:

$$Q = \pi r_c^2 K \frac{h}{L}$$

Assume flow in tube
=
flow in sample

Equating and integrating: $$K = \frac{r_t^2 L}{r_c^2 t} \ln\left(\frac{h_1}{h_2}\right)$$

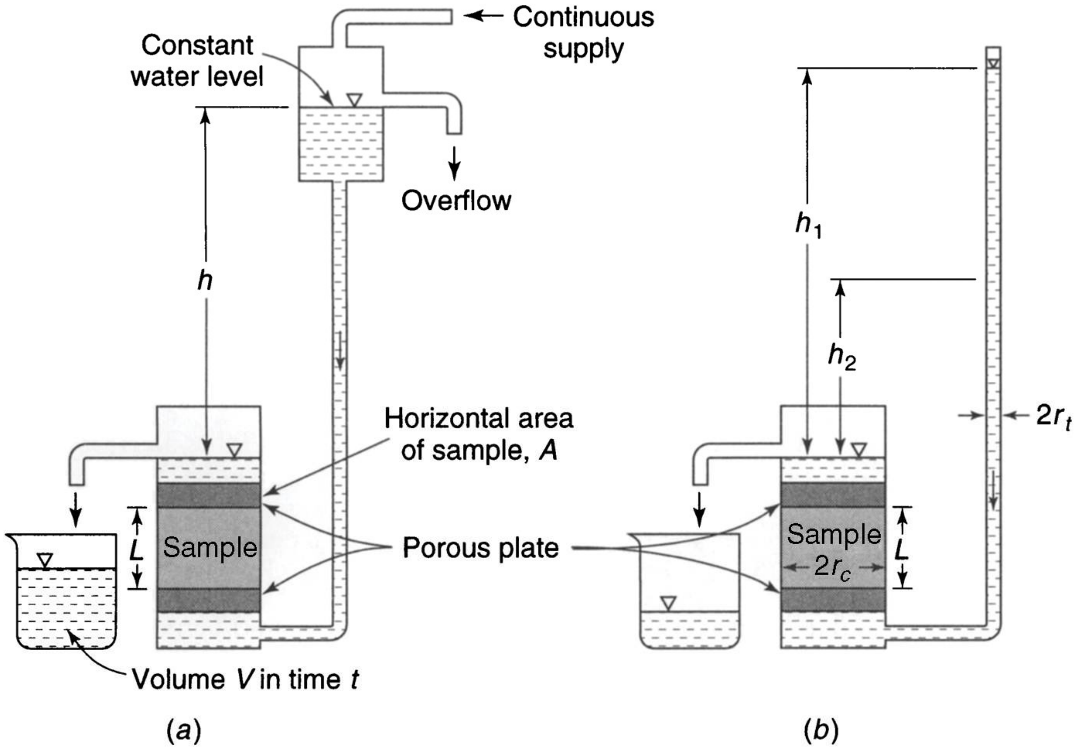
Where t is the time for the water level to fall from h_1 to h_2

Measuring K

A field sample from an unconfined aquifer is packed into the test cylinder in the testing equipment shown in figure (a) below. The length and diameter of the sample are 1.0 m and 0.08 m respectively. A test is run over a 5 minute period with a constant head difference of 0.224 m. At the end of the test 0.0652 litres of water is collected at the outlet. Determine the hydraulic conductivity of the aquifer sample. Express your answer in units of m day^{-1}.

$$Q = \pi r_c^2 K \frac{h}{L}$$

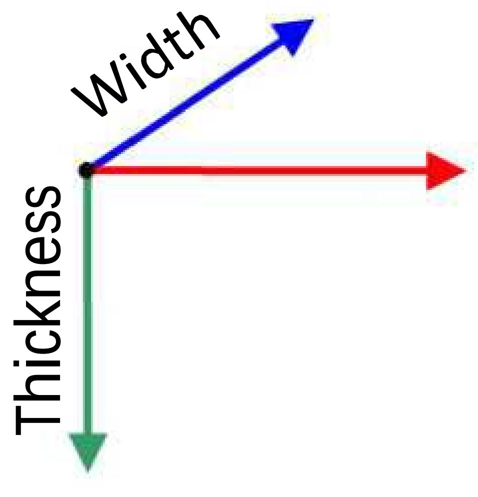
$$K = \frac{r_t^2 L}{r_c^2 t} \ln\left(\frac{h_1}{h_2}\right)$$



Example of Darcy's Law

Exercise 1

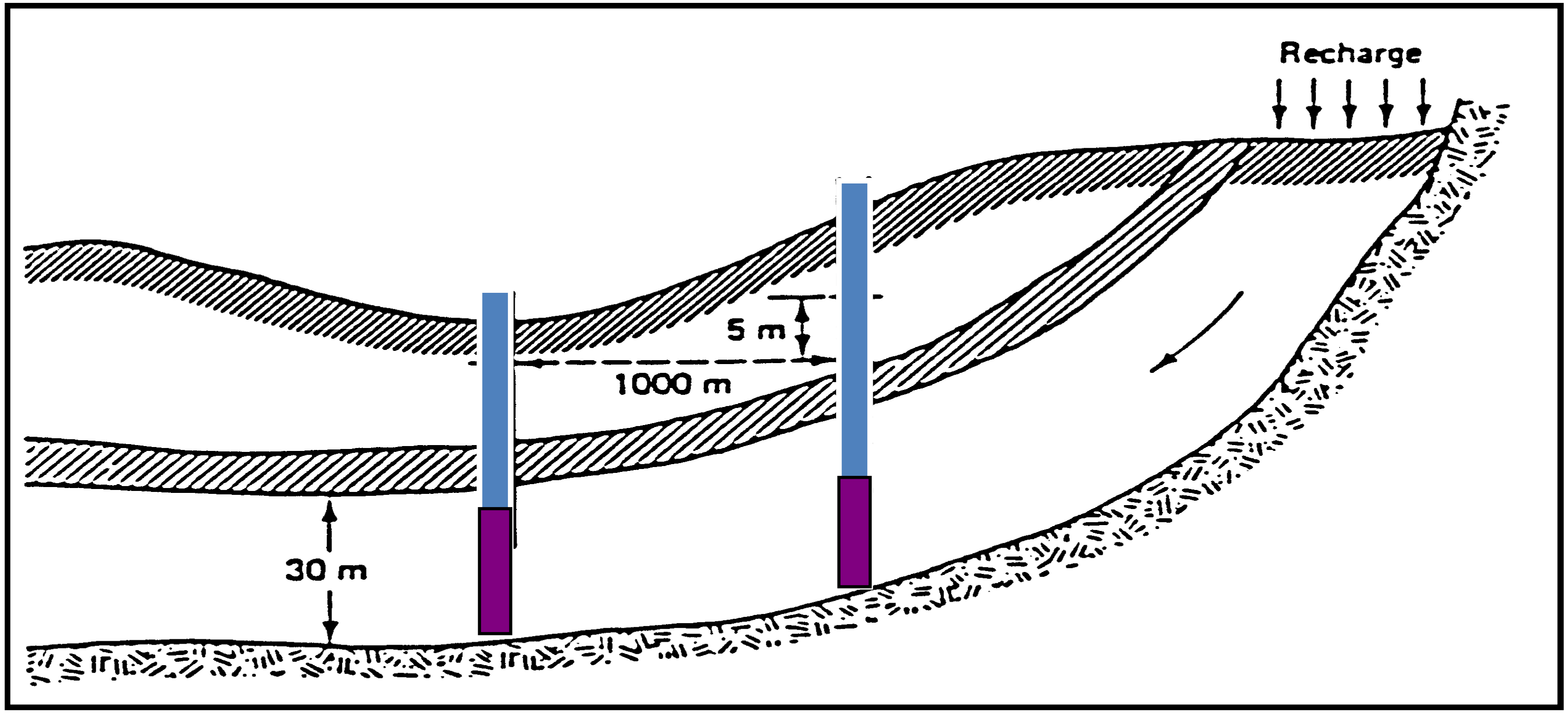
- A confined aquifer has a source of recharge.
- K for the aquifer is 50 m/day, and n_e is 0.2.
- The piezometric head in two wells 1000 m apart is 55 m and 50 m respectively.
- The average thickness of the aquifer is 30 m, and the average width of aquifer is 5 km.



Compute:

Exercise 1

- a) the rate of flow through the aquifer
- (b) the average time of travel from the head of the aquifer to a point 4 km downstream
- **assume no dispersion or diffusion*



Exercise 1

The solution

- Cross-Sectional area= 30 x5000 = 15 x 10^4 m^2
- Hydraulic gradient = (55-50)/1000 = 5 x 10^{-3}
- Rate of Flow for K = 50 m/day

  Q = 50 m/day x 5 x 10^{-3} x15 x 10^4 m^2= 37,500 m^3/day
- Darcy Velocity: V = Q/A = (37,500m^3/day) / (15 x 10^4 m^2) = <u>0.25m/day</u>

Exercise 1

And

- <u>Seepage Velocity:</u>

  $V_s = V/n_e = (0.25) / (0.2) =$
  1.25 m/day

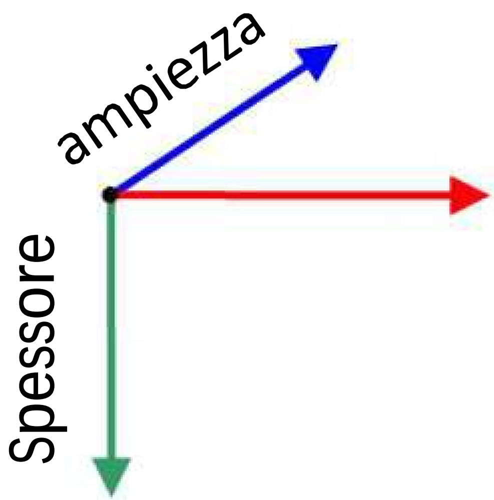
- <u>Time to travel 4 km downstream:</u>

  T = 4(1000m) / (1.25m/day) =
  3200 days or 8.77 years

- *This example shows that water moves very slowly underground*

Esercizio 2

In un acquifero sabbioso la pendenza della superficie piezometrica è di 3,6 m per 1 km.
La conducibilità idraulica per una sabbia grossolana è di 0,51 cm/s
Calcolare la velocità di Darcy e la portata che transita nell'acquifero di spessore 22 m e ampiezza 430 m



Esercizio 2

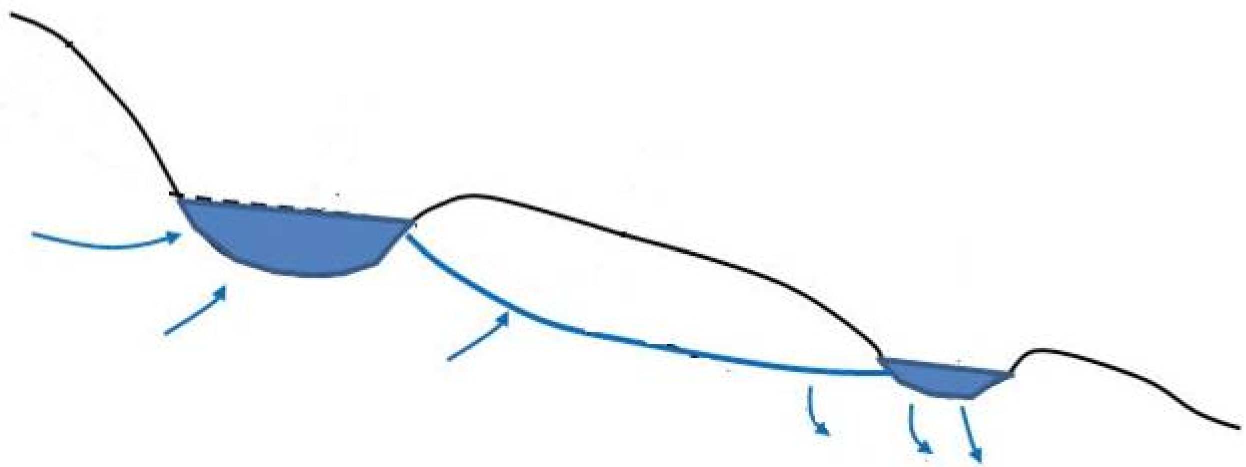
i= 3.6 m/1000 m= 0.0036

v =Ki= 0.51 x 0.0036=0.00184 cm/s =1.59 m/g

Q =Vx A = 1.59 m/g x430 m x22 m= 15041 m^3/g

Esercizio 3

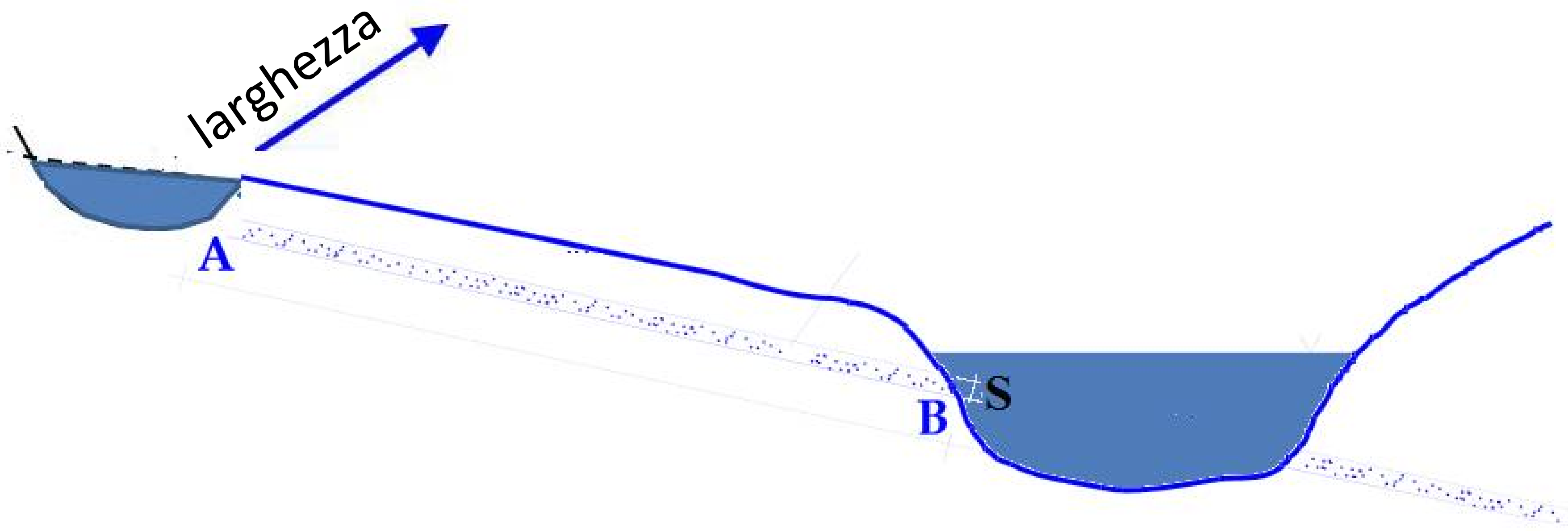
L'acqua fluisce dal lago a monte fino a quello a valle attraverso il sottosuolo.
Con i seguenti dati:
Differenza di elevazione tra i due laghi Δh: 25m
Lunghezza del tragitto L=1,5 km
Area della sezione di flusso A=120 m^2
Conducibilità idraulica K=0,15 cm/s
Porosità efficace dell'acquifero n_e=0,25
Calcolare la portata che transita da un lago all'altro e il tempo che ci mette.



Esercizio 3

Calcolare l’area della sezione del flusso:

A= S (spessore dello strato permeabile) x larghezza acquifero



Esercizio 3

Sui calcola la velocità di Darcy V

$V=Ki= 0,0015\ m/s\ (25/1500)=2,5 \times 10^{-5}\ m/s$

Si calcola la portata

$Q=KiA= 120 \times 2,5 \times 10^{-5}\ m/s= 3 \times 10^{-3}\ m^3/s$

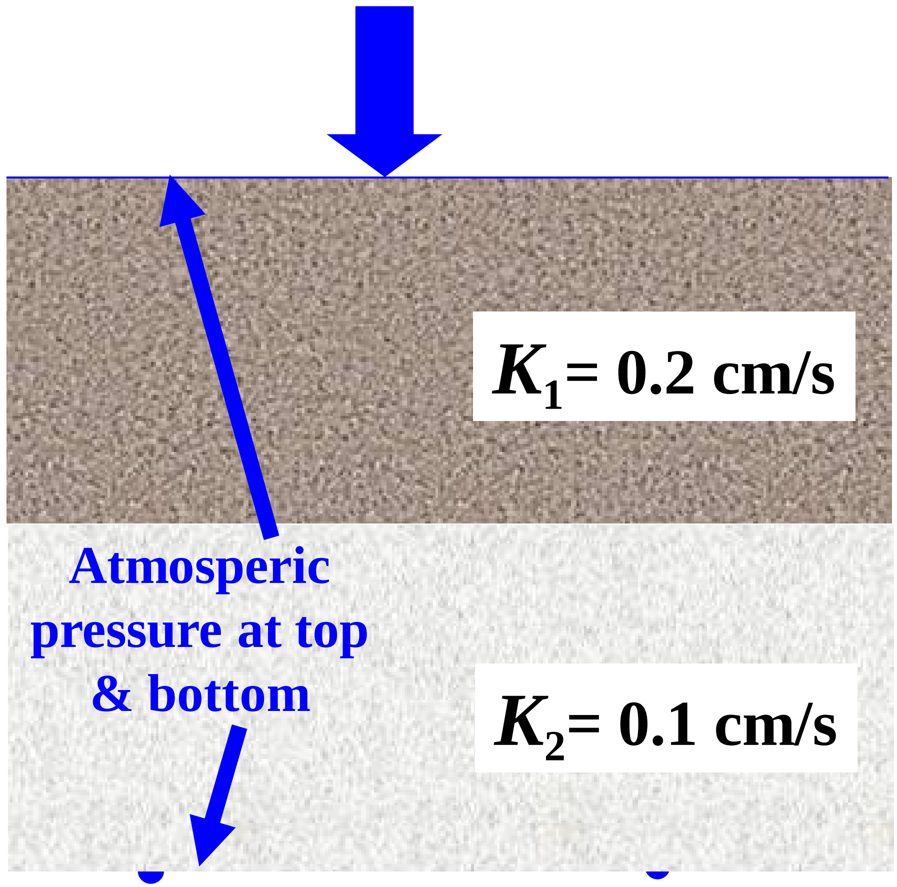
Si calcola la velocità di flusso

$V_p= V/n_e= 2,5 \times 10^{-5} /0,25 = 1 \times 10^{-4}\ m/s$

Si calcola il tempo t

$T=L/V_p= 1500\ m/ (1 \times 10^{-4}\ m/s)= 1,5 \times 10^{7}\ s = 173,6$ giorni

Darcy in layered systems



Steady-state flow

Unit gradient overall

$L_1 = L_2$

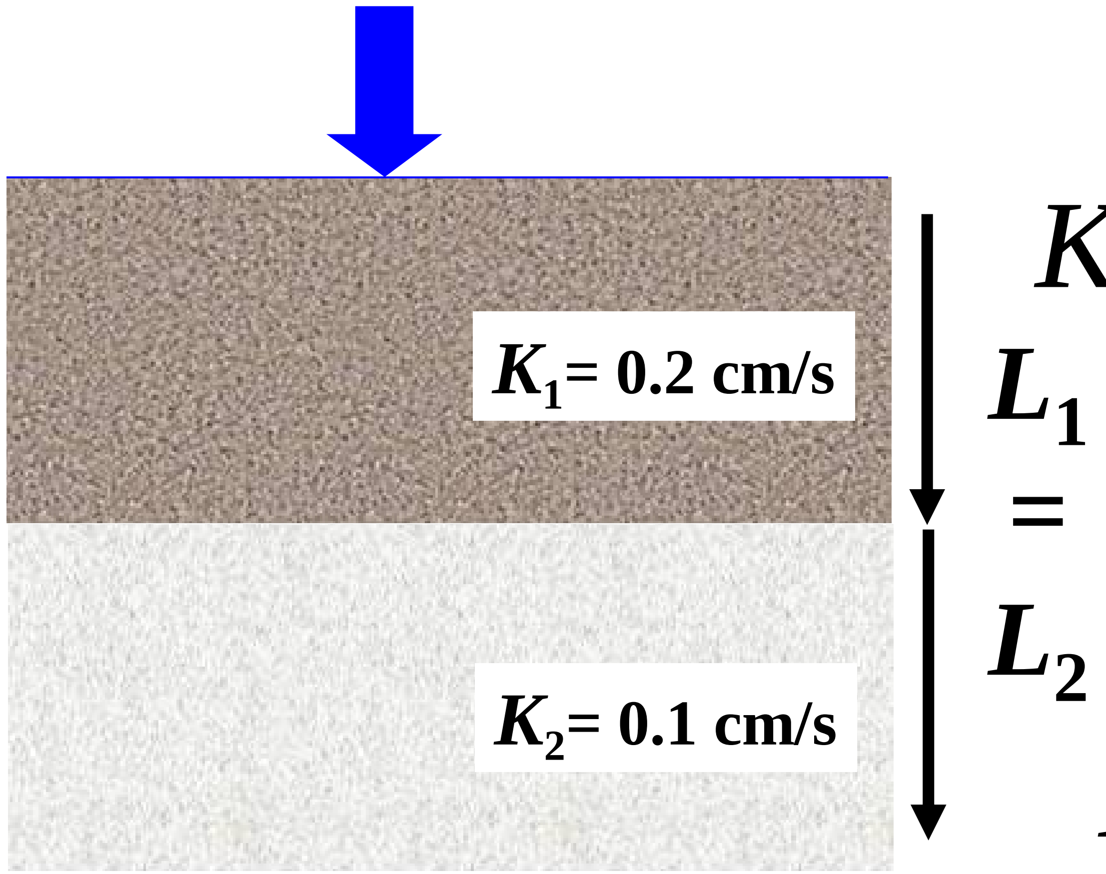
$\Delta h_1 = ?$

$\Delta h_2 = ?$

$$q = K \frac{\Delta h}{\Delta L}$$

$q_1 = ?$ $\quad q_2 = ?$

Darcy in layered systems



$$q_1 = q_2$$

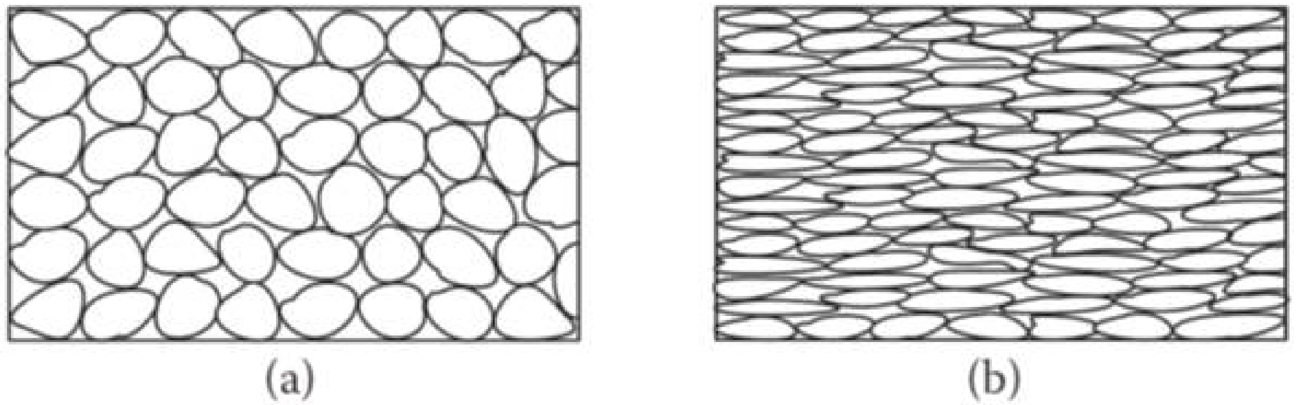
$$K_1 \frac{\Delta h_1}{L_1} = K_2 \frac{\Delta h_2}{L_2}$$

$L_1 = L_2$, **so**

$$K_1 \Delta h_1 = K_2 \Delta h_2$$

Hydraulic conductivity: Anisotropic systems

Hydraulic conductivity formations in three types of heterogeneous systems.

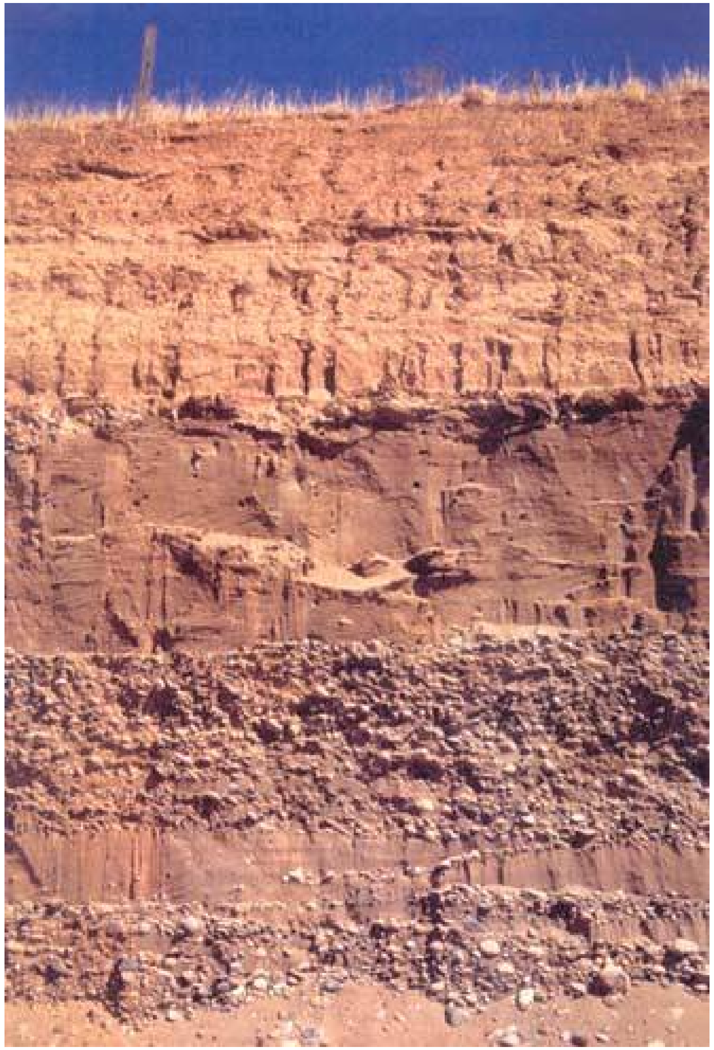


(a) Isotropic and (b) anisotropic sediment deposits.

Heterogeneity and Anisotropy

- Homogeneous
  - Properties same at every point
- Heterogeneous
  - Properties different at every point
- Isotropic
  - Properties same in every direction
- Anisotropic
  - Properties different in different directions
- Often results from stratification during sedimentation

$$K_{horizontal} > K_{vertical}$$



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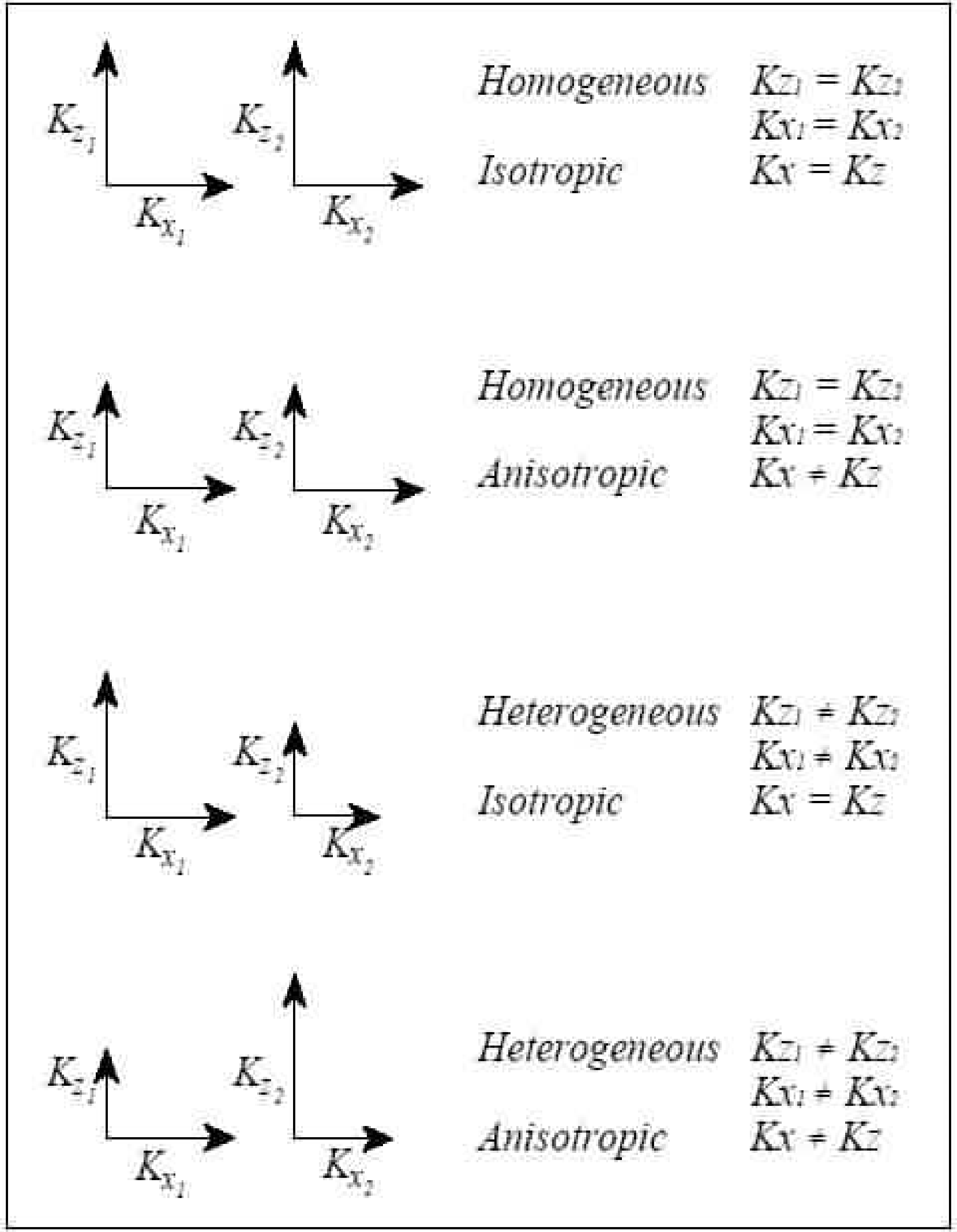
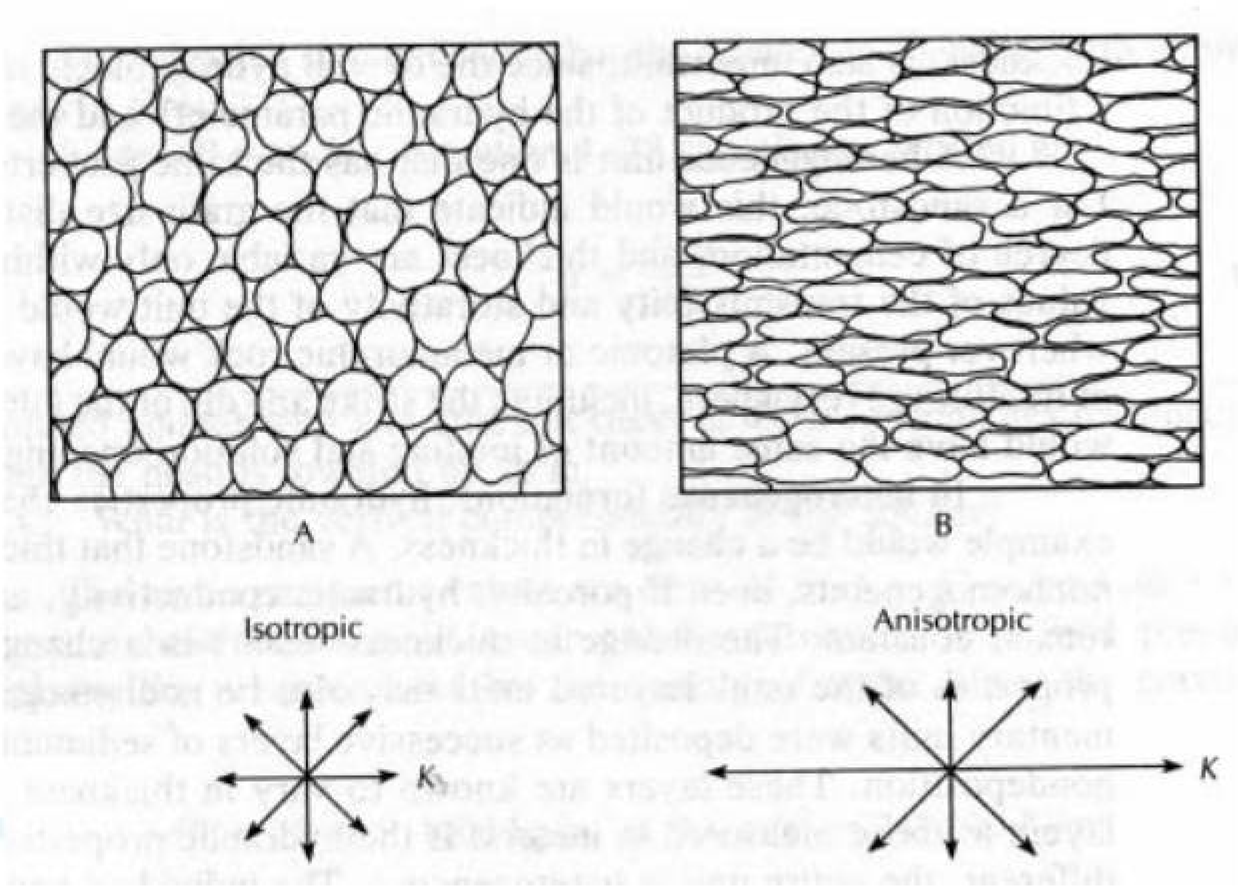
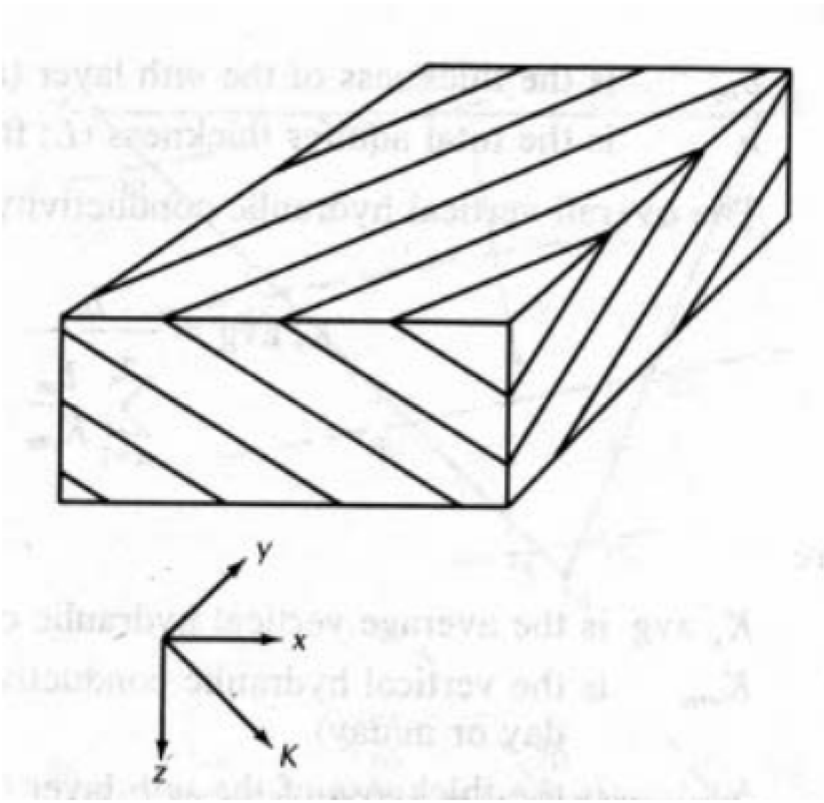


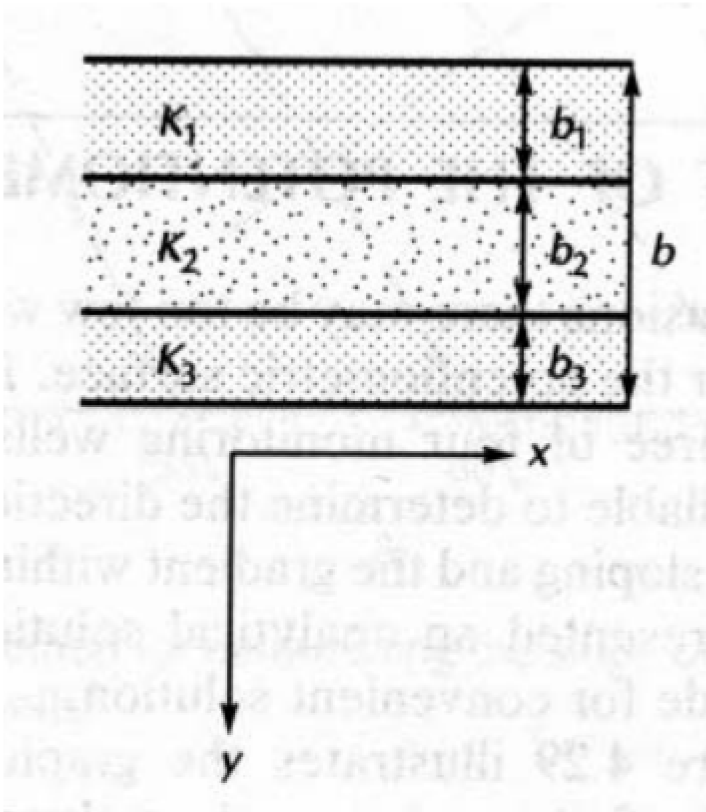
K_{z_1}
K_{x_1}
K_{z_2}
K_{x_2}
Homogeneous $Kz_1 = Kz_2$
$Kx_1 = Kx_2$
Isotropic $Kx = Kz$
Homogeneous $Kz_1 = Kz_2$
$Kx_1 = Kx_2$
Anisotropic $Kx \neq Kz$
Heterogeneous $Kz_1 \neq Kz_2$
$Kx_1 \neq Kx_2$
Isotropic $Kx = Kz$
Heterogeneous $Kz_1 \neq Kz_2$
$Kx_1 \neq Kx_2$
Anisotropic $Kx \neq Kz$



Grain shape and orientation can affect the isotropy or anisotropy of a sediment



Anisotropy of fractured rock units due to directional nature of fracturing



Heterogeneous formation consisting of three layers of differing hydraulic conductivity

Hydraulic Conductivities

Unconsolidated Deposits	Max (m/s)	Median (m/s)	Min (m/s)
Gravel	3×10^{-2}	**3×10^{-3}**	3×10^{-4}
Sand	6×10^{-3}	**3×10^{-5}**	2×10^{-7}
Silt / Loess	2×10^{-5}	**2×10^{-7}**	2×10^{-9}
Fractured Till	2×10^{-5}	**6×10^{-8}**	2×10^{-10}
Unfractured Till	2×10^{-6}	**2×10^{-9}**	2×10^{-12}
Lacustrine Clay	5×10^{-9}	**7×10^{-10}**	1×10^{-12}
Marine Clay	2×10^{-9}	**4×10^{-11}**	8×10^{-13}

Overall range is more than 10 orders of magnitude

Hydraulic Conductivities

Cemented Sedimentary Rocks	Max (m/s)	Median (m/s)	Min (m/s)
Karst / Reef Limestone	2×10^{-2}	**1×10^{-4}**	1×10^{-6}
Limestone / Dolomite	6×10^{-6}	**8×10^{-7}**	1×10^{-9}
Sandstone	6×10^{-6}	**4×10^{-8}**	3×10^{-10}
Siltstone	2×10^{-8}	**5×10^{-9}**	1×10^{-11}
Evaporite Anhydrite	2×10^{-8}	**9×10^{-10}**	4×10^{-13}
Shale / Mudstone	3×10^{-9}	**2×10^{-11}**	1×10^{-13}
Evaporite Salt	1×10^{-10}	**1×10^{-11}**	1×10^{-12}

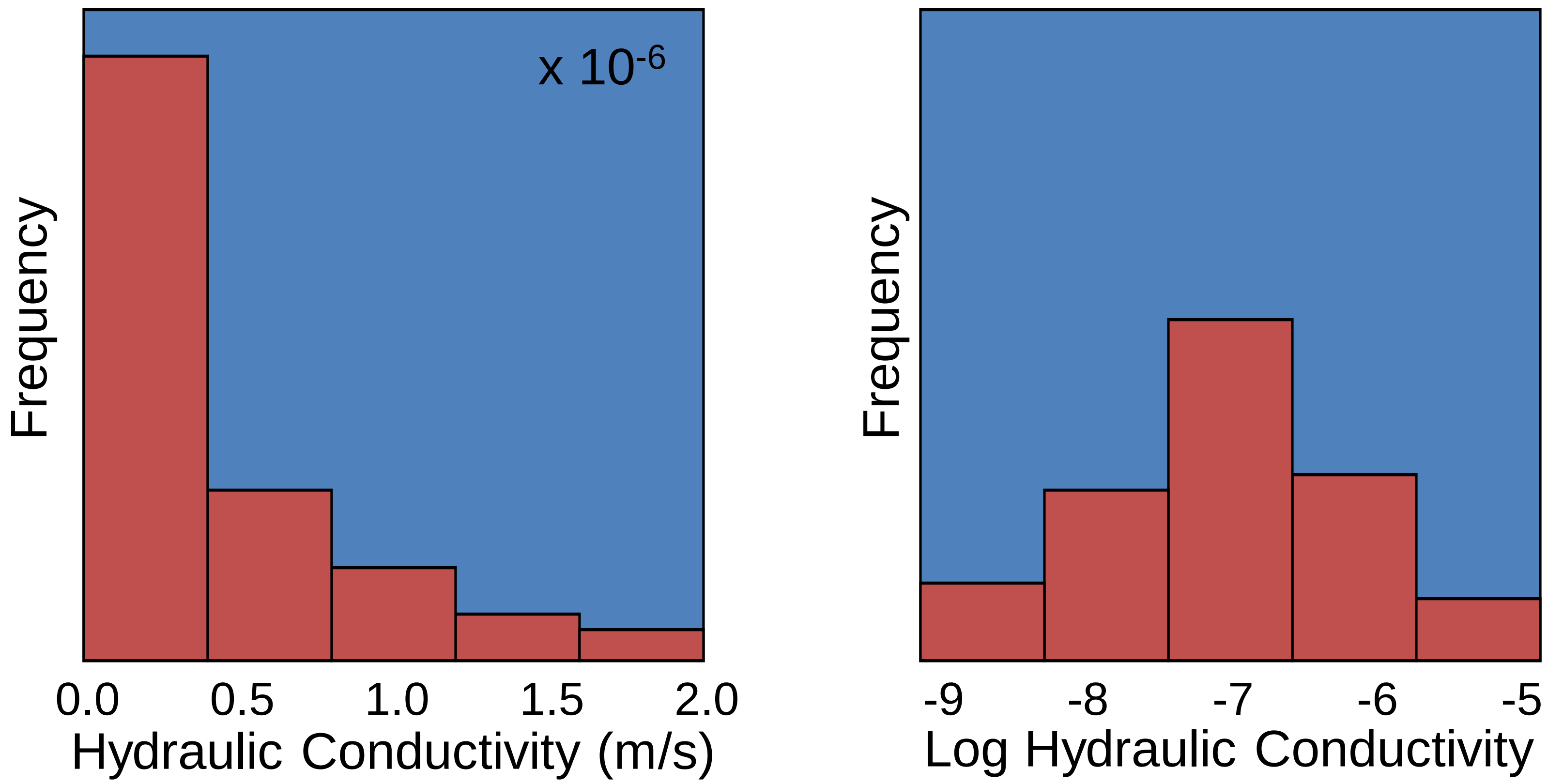
Overall range is more than 10 orders of magnitude

Hydraulic Conductivities

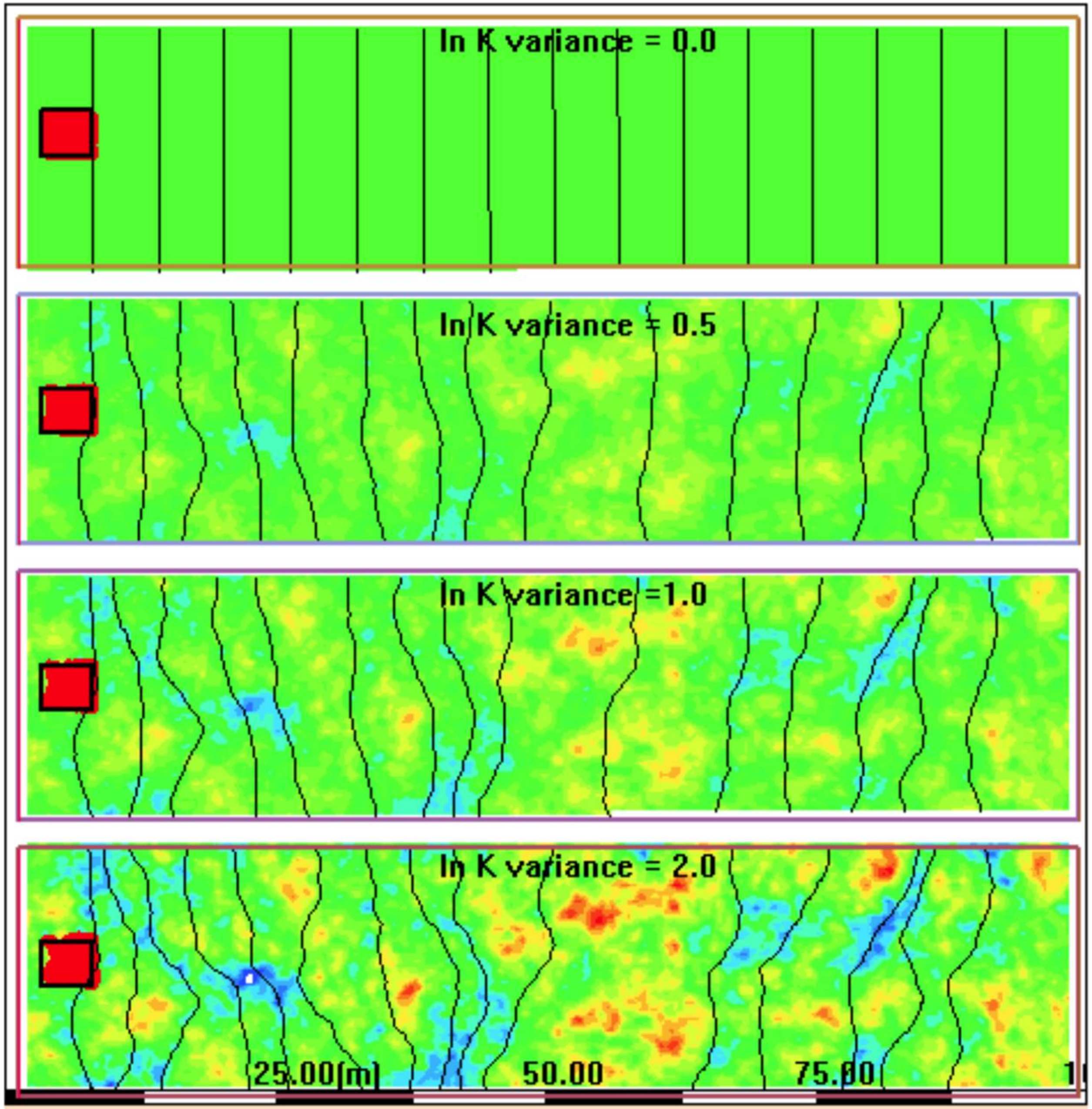
Crystalline Rocks	Max (m/s)	Median (m/s)	Min (m/s)
Fractured Extrusives (weathered flow tops)	2×10^{-2}	**9×10^{-4}**	4×10^{-7}
Weathered Intrusives	5×10^{-5}	**5×10^{-6}**	5×10^{-7}
Fractured Intrusives	3×10^{-4}	**1×10^{-8}**	8×10^{-9}
Fractured Metamorphics	3×10^{-4}	**1×10^{-8}**	8×10^{-9}
Massive Extrusives	4×10^{-7}	**3×10^{-9}**	2×10^{-11}
Massive Intrusives	2×10^{-10}	**2×10^{-12}**	3×10^{-14}
Massive Metamorphics	2×10^{-10}	**2×10^{-12}**	3×10^{-14}

Overall range is almost 12 orders of magnitude

Hydraulic Conductivity Distributions



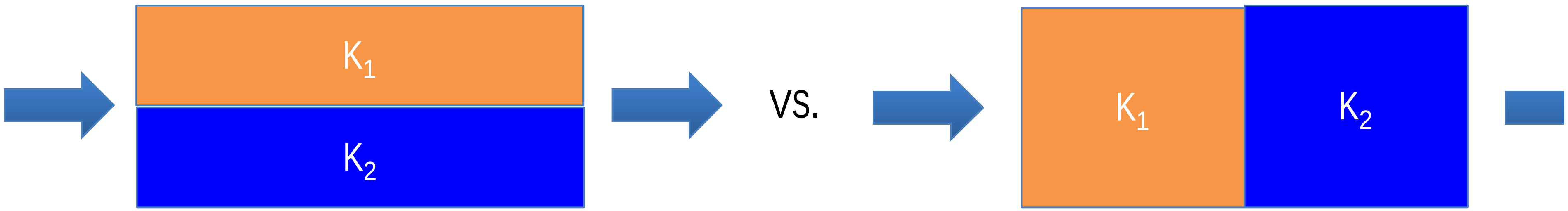
- Hydraulic conductivity is almost always log-normally distributed
- Ranges of 4 or 5 orders of magnitude for the same geologic unit are commonplace



Steady Flow, Time Elapsed = 0 days (0.00 years)

Heterogeneity

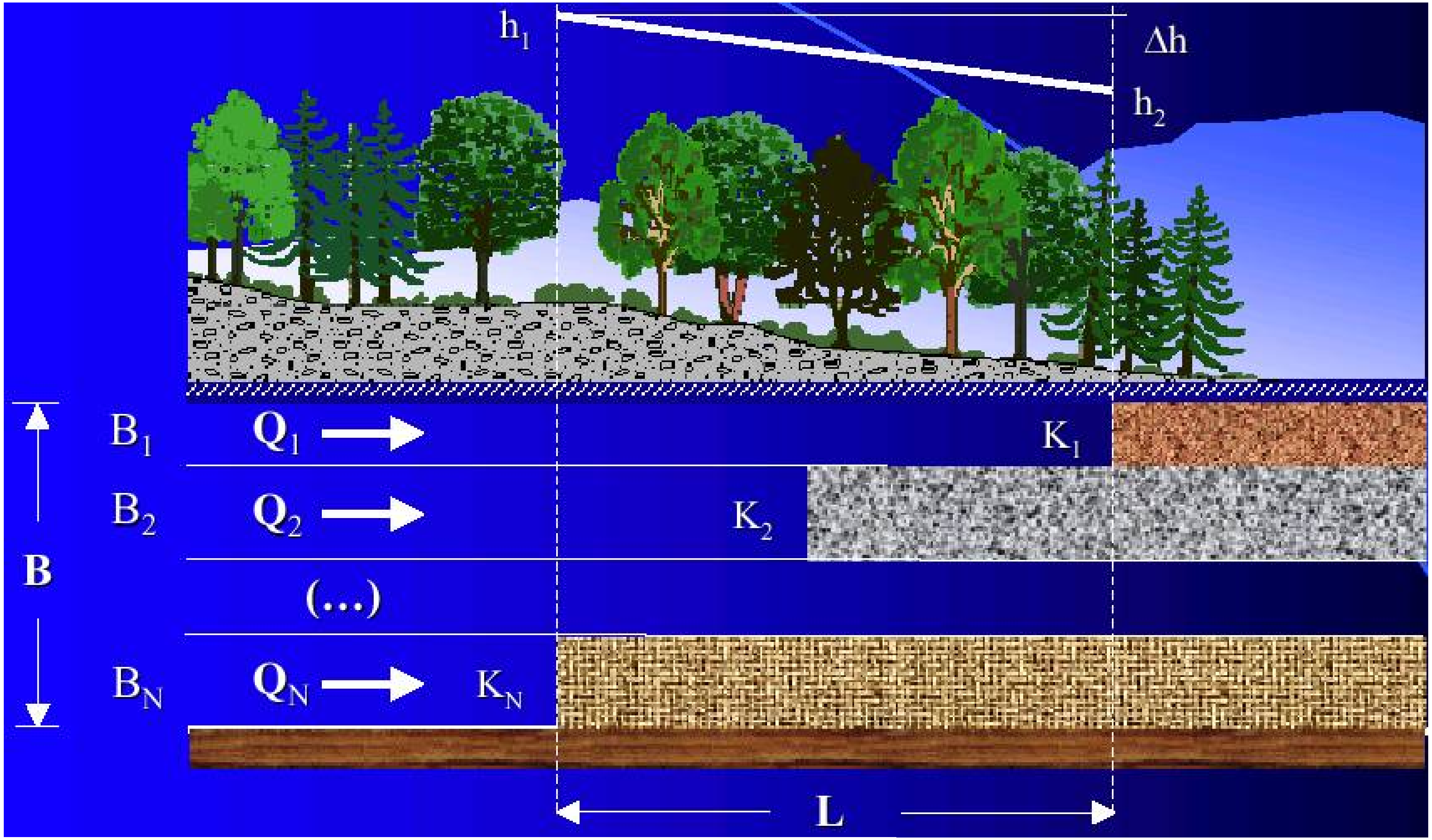
- Effective Hydraulic Conductivity – We like to replace heterogeneous blocks with analogous homogeneous ones



- Replace with

- Are they the same for the two – how would you do it?

Layered Porous Media (Flow Parallel to Layers)

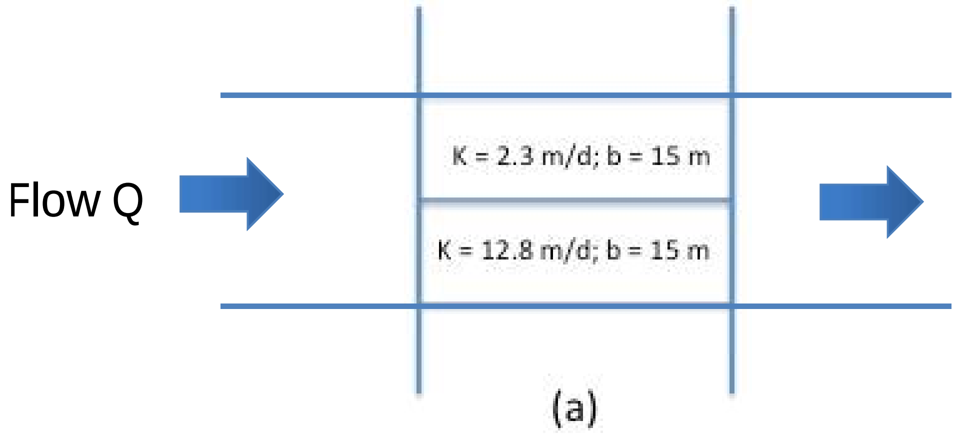


$$Q = \sum_{i=1}^{N} Q_i \qquad B = \sum_{i=1}^{N} B_i \qquad Q_i = K_i B_i \frac{\Delta h}{L} \qquad Q = \frac{\Delta h}{L} \sum_{i=1}^{N} K_i B_i = \frac{\Delta h}{L} \cdot K_{eq}^{P} B$$

$$K_{eq}^{P} = \frac{1}{B} \sum_{i=1}^{N} K_i B_i$$

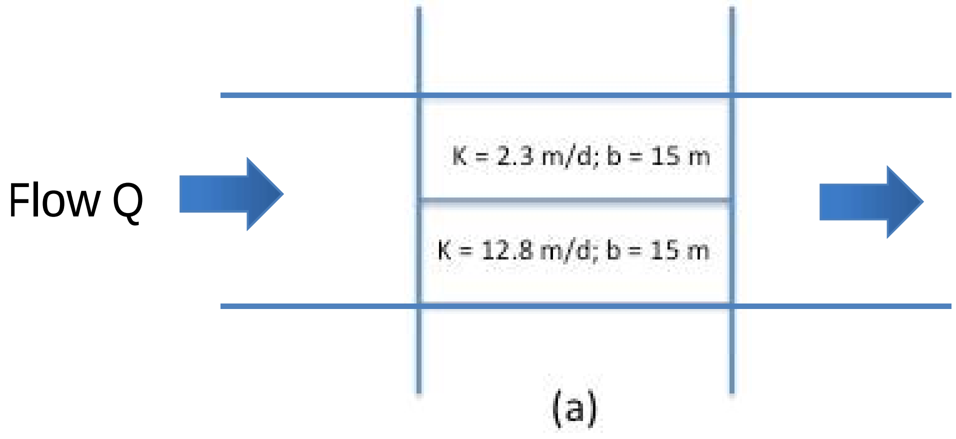
K_{eq} equivalent hydraulic conductivity

Example



- Find average K

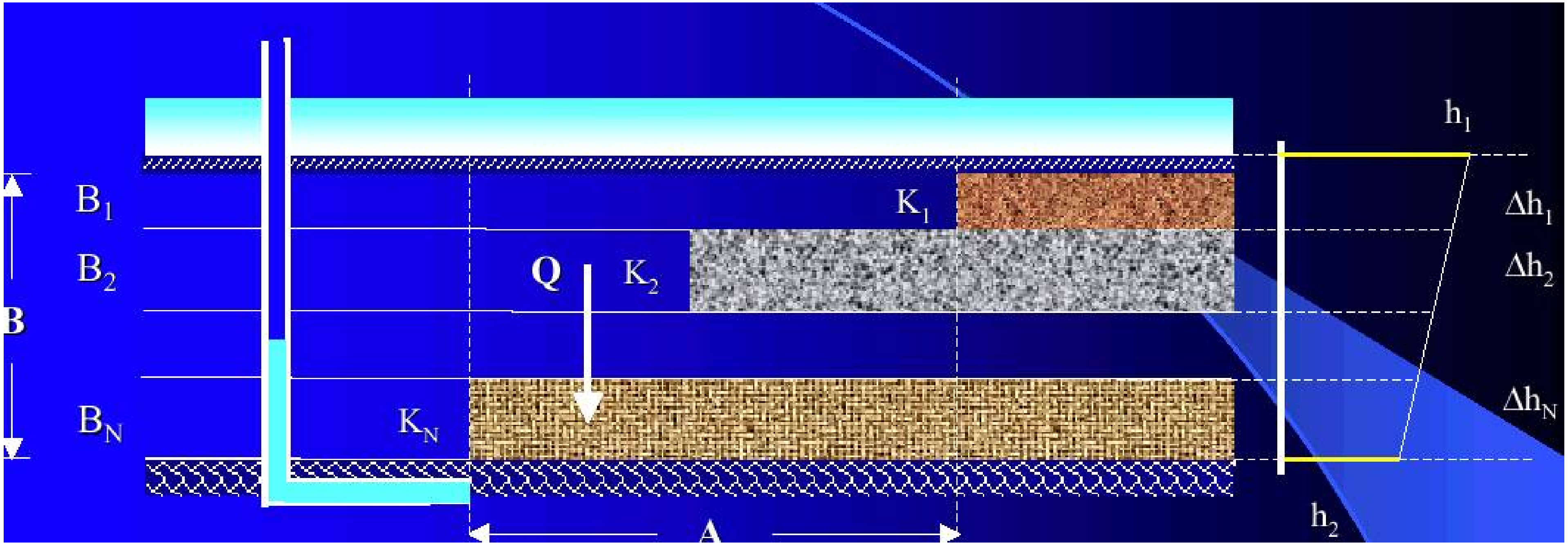
Example



$$\bar{K} = \frac{\sum_{i=1}^{3}(b_i K_i)}{b}$$

$$K_{h,A} = \frac{K_1 z_1 + K_2 z_2}{z_1 + z_2} = \frac{(2.3\,m/d)(15\,m) + (12.8\,m/d)(15\,m)}{(15\,m) + (15\,m)} = 7.55\,m/d$$

Layered Porous Media (Flow Perpendicular to Layers)



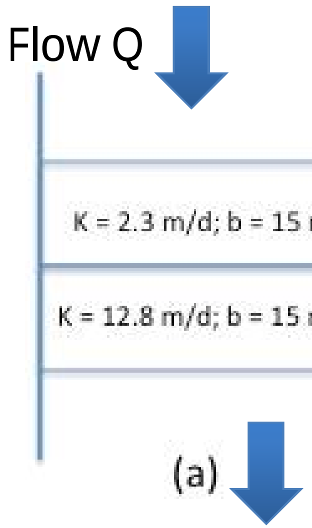
$$Q = K_1 \cdot \frac{\Delta h_1}{B_1} \cdot A = K_2 \cdot \frac{\Delta h_2}{B_2} \cdot A = \ldots = K_N \cdot \frac{\Delta h_N}{B_N} \cdot A = K_{\mathrm{eq}}^{\mathrm{N}} \cdot \frac{\Delta h}{B} \cdot A$$

$$\Delta h = \frac{Q}{A} \sum_{i=1}^{N} \frac{B_i}{K_i} = \frac{Q}{A} \cdot \frac{\mathrm{B}}{K_{\mathrm{eq}}^{\mathrm{N}}} \qquad \frac{B}{K_{\mathrm{eq}}^{\mathrm{N}}} = \sum_{i=1}^{N} \frac{B_i}{K_i}$$

K_{eq} equivalent hydraulic conductivity

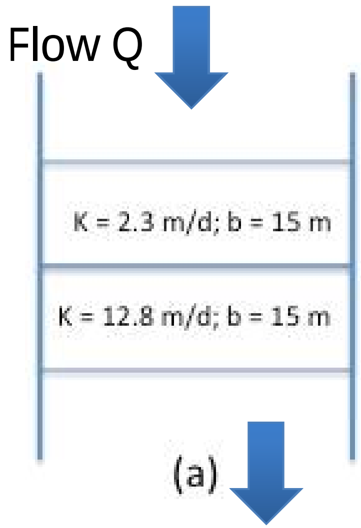
If there is a $K_i = 0 \rightarrow Q=0$

Example



- Find average K

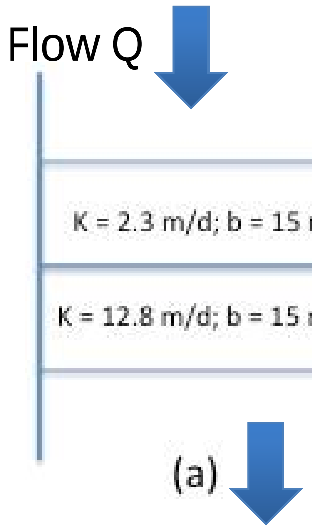
Example



$$\bar{K} = \frac{L}{\sum_{\iota=1}^{N}\left(\frac{L_\iota}{K_\iota}\right)}$$

$$K_{v,A} = \frac{z_1 + z_2}{\frac{z_1}{K_1} + \frac{z_2}{K_2}} = \frac{(15\,m) + (15\,m)}{\frac{15\,m}{2.3\,m/d} + \frac{15\,m}{12.8\,m/d}} = 3.90\,m/d$$

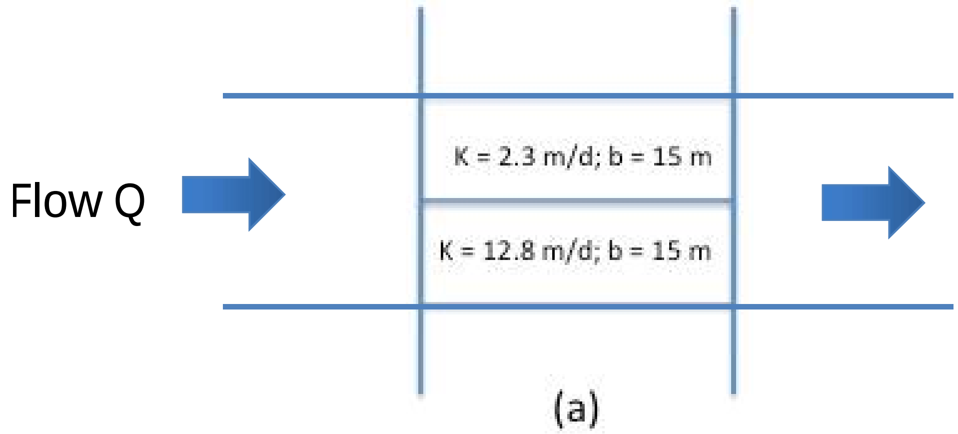
Flow Perpendicular to Layers



$$\bar{K} = \frac{L}{\sum_{i=1}^{N}\left(\frac{L_i}{K_i}\right)}$$

$$K_{v,A} = 3.90\, m/d$$

Flow Parallel to Layers



$$\bar{K} = \frac{\sum_{i=1}^{3}(b_i K_i)}{b}$$

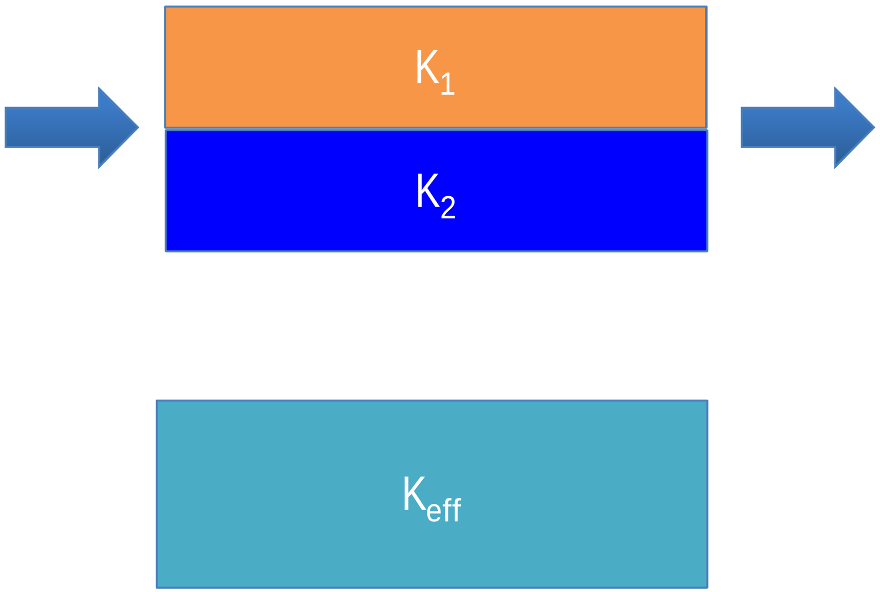
$$K_{h,A} = 7.55\, m/d$$

$$\frac{K_h}{K_v} = 1{,}9$$

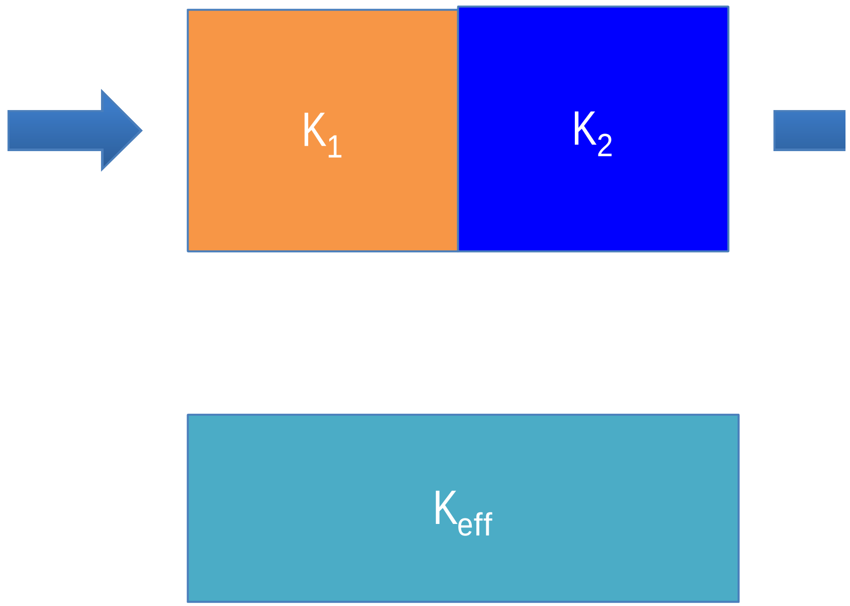
Anisotropy

Heterogeneity

- Effective Hydraulic Conductivity – We like to replace heterogeneous blocks with analogous homogeneous ones



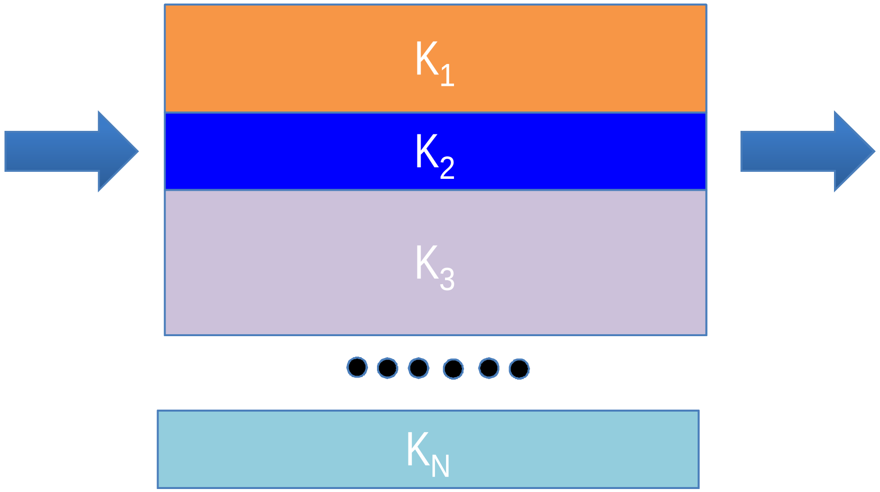
$$K_{eff} = \tfrac{1}{2}\left(K_1 + K_2\right)$$



$$\frac{1}{K_{eff}} = \frac{1}{2}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

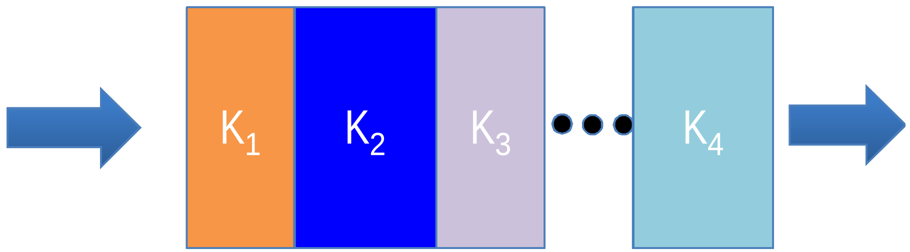
More Generally

- N parallel layers, each with conductivity K_i of thickness b_i



$$K_{eff} = \frac{\sum_{i=1}^{N} K_i b_i}{\sum_{i=1}^{N} b_i}$$

N perpendicular to flow layers, each with conductivity K_i of thickness b_i



$$K_{eff} = \frac{\sum_{i=1}^{N} b_i}{\sum_{i=1}^{N} \frac{b_i}{K_i}}$$

Esercizio

Un acquifero è composto da tre strati orizzontali: uno superficiale di spessore 1m, uno intermedio di spessore 2m e uno più profondo di spessore 3m.
Le prove in situ hanno dato i seguenti valori del coefficiente di conducibilità idraulica:

$3x10^{-1}$ (mm/s) per lo strato A
$2x10^{-1}$ (mm/s) per lo strato B
$1x10^{-1}$ (mm/s) per lo strato C

1) Determinare il rapporto di permeabilità medio nella direzione orizzontale e verticale

$$\overline{K} = \frac{L}{\sum_{i=1}^{N}\left(\frac{L_i}{K_i}\right)}$$

$$\overline{K} = \frac{\sum_{i=1}^{3}(b_i K_i)}{b}$$

Esercizio

$H_A = 1$ m $\quad k_A = 3.0 \times 10^{-1}$ (mm/s) $= 3.0 \times 10^{-4}$ (m/s) $\quad$ H=6m

$H_B = 2$ m $\quad k_B = 2.0 \times 10^{-1}$ (mm/s) $= 2.0 \times 10^{-4}$ (m/s)

$H_C = 3$ m $\quad k_C = 1.0 \times 10^{-1}$ (mm/s) $= 1.0 \times 10^{-4}$ (m/s)

Si calcola la permeabilità nella direzione parallela agli strati:

$$k_H = (H_A K_A + H_B k_B + H_C k_C)/H = 1.67 \times 10^{-4} \text{ (m/s)}$$

Si calcola la permeabilità nella direzione perpendicolare agli strati:

$$k_V = H/\ (H_A/K_A + H_B/k_B + H_C/k_C) = 1.4 \times 10^{-4} \text{ (m/s)}$$

Si calcola quindi il rapporto tra la permeabilità orizzontale e verticale:

$$k_H/k_V = 1.20$$

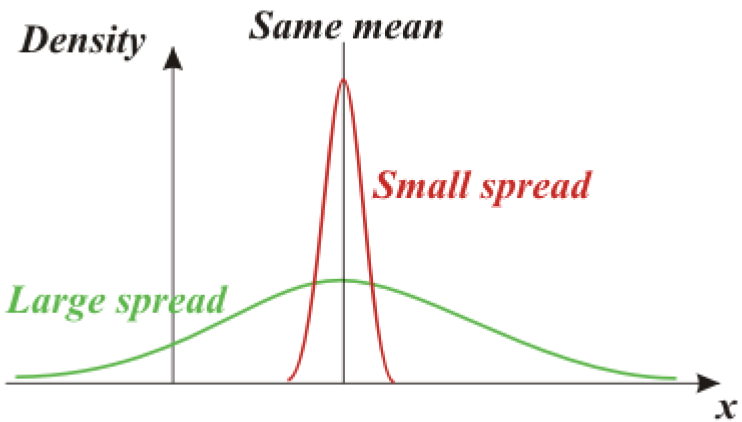
I MOMENT

MEAN

estimates the value around which central clustering occurs

The expectation of Z, $E[Z] = \mu$ tells us where the **central tendency** of the variable distribution is located, but it tells us nothing about the extent of the dispersion, or spread of this distribution around its mean.

In the illustration below, the red and the green distributions have the same mean, but very different spreads.



II MOMENT

Having characterized a distribution's central value, one conventionally next characterizes its "width" or "variability" around that value.

A natural idea for quantifying the spread of the possible values of Z is to measure how far, on the average, Z is from its mean.

The expectation of the squared distance from Z to its mean μ is the variance Var(Z) of a random variable Z :

$$Var(Z) = E[(Z - \mu)^2]$$

$$S^2 = 1/N \sum_{i=1}^{N} (z_i - \mu)^2$$

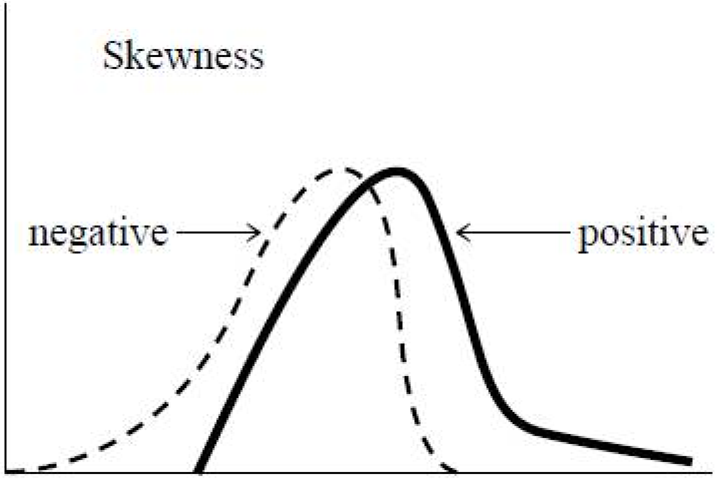
VARIANCE

Characterizes the dispersion of values that can be assumed by the variablee

Its square root is called STANDARD DEVIATION

Sometimes preferred because it is in the same measurement unit of the observations

III MOMENT



SKEWNESS

Characterizes the degree of asymmetry of the distribution

= 0 ⟶ symmetric distribution

> 0 ⟶ tail to the right

< 0 ⟶ tail to the left

The skewness is important in the analysis of

Solute breakthrough curves