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THE VISUAL APPEARANCE OF *RAPIDLY MOVING OBJECTS*

By ***V. F. Weisskopf***

I WOULD like to draw the attention of physicists to a recent paper by James Terrell [1] in which he does away with an old prejudice held by practically all of us. We all believed that, according to special relativity, an object in motion appears to be contracted in the direction of motion by a factor $[1-(v/c)^2]^{1/2}$. A passenger in a fast space ship, looking out of the window, so it seemed to us, would see spherical objects contracted to ellipsoids. This is definitely not so according to Terrell's considerations, which for the special case of a sphere were also carried out by R. Penrose.[2] The reason is quite simple. When we see or photograph an object, we record light quanta emitted by the object when they arrive simultaneously at the retina or at the photographic film. This implies that these light quanta have *not* been emitted simultaneously by all points of the object. The points further away from the observer have emitted their part of the picture earlier than the closer points. Hence, if the object is in motion, the eye or the photograph gets a "distorted" picture of the object, since the object has been at different locations when different parts of it have emitted the light seen in the picture.

In special relativity, this distortion has the remarkable effect of canceling the Lorentz contraction so that objects appear undistorted but only rotated. This is exactly true only for objects which subtend a small solid angle.

In order to understand the situation thoroughly let us consider the distortion of the picture we see of a moving object under nonrelativistic conditions, where light moves with light velocity c only in the stationary frame of reference of the observer, and a moving object does not suffer a Lorentz contraction. In the frame of the object moving with the velocity v the light velocity would be $c-v$ in the direction of motion and $c+v$ in the opposite direction.

We first consider the case of a cube of dimension l moving parallel to an edge and observed from a direction perpendicular to the motion. The observation is made at great distance in order to keep the subtended angle small (see Fig. 1). The square $ABCD$ facing the observer will be seen undistorted since all points have the same distance from the observer. The square $ABEF$ facing in the opposite direction of the motion (the rear side in regard to the motion, not in regard to the observer's position) is invisible when the cube is not in motion. However when it moves it becomes visible since the light from E and F is emitted l/c seconds earlier when the points E and F were $(v/c)l$ further behind at E' and F'. Hence the face $ABEF$ will be seen as a rectangle with a height l and a width $(v/c)l$. The picture of the cube, therefore, is a distorted one. In an undistorted picture of a rotated

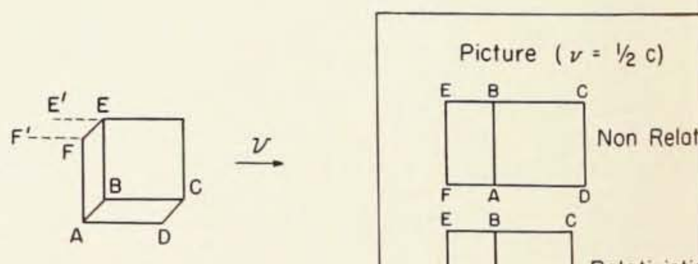

Fig. 1. A cube moving with velocity v seen by an observer at an angle of 90°.

[1] J. Terrell, Phys. Rev. **116**, 1041 (1959).
[2] R. Penrose, Proc. Cambridge Phil. Soc. **55**, 137 (1959); see also H. Salecker and E. Wigner, Phys. Rev. **109**, 571 (1958).

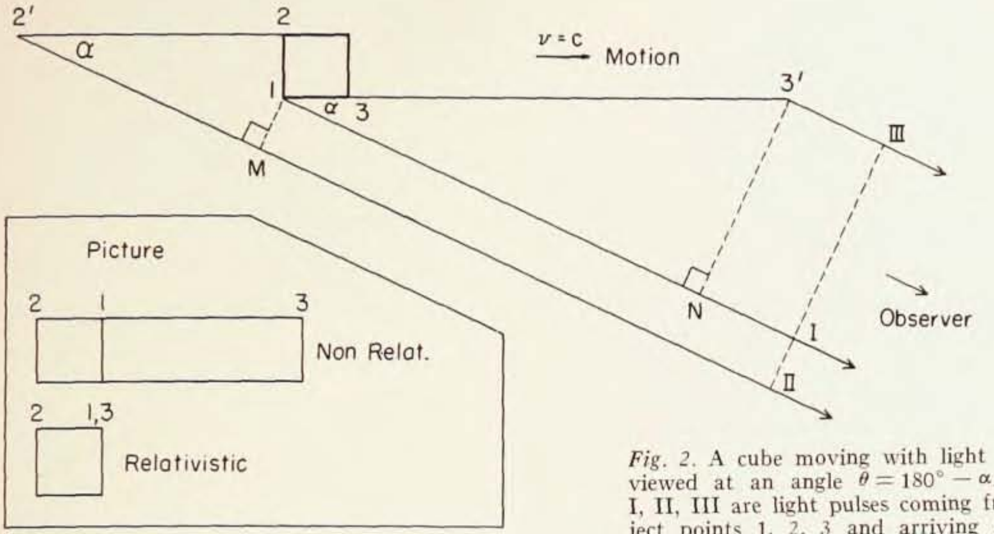

Fig. 2. A cube moving with light velocity viewed at an angle $\theta = 180° - \alpha$. Points I, II, III are light pulses coming from object points 1, 2, 3 and arriving simultaneously at the observer. (Nonrelativistic.)

cube both faces should be foreshortened; if the face *ABEF* is shortened by the factor v/c, the other face *ABCD* should be foreshortened by $(1 - v^2/c^2)^{1/2}$, whereas here *ABCD* appears as a square. Hence the picture of the cube appears dilated in the direction of motion. A similar consideration for a moving sphere shows that it would appear as an ellipse elongated in the direction of motion by a factor $(1 + v^2/c^2)^{1/2}$.

We get even more paradoxical results by considering the picture of a moving cube in a nonrelativistic world, seen not at 90° to the direction of motion but at $180 - \alpha$ degrees where α is a small angle. We now look at the object to the left when it is coming towards us from the left. We will assume now that $v/c = 1$ in order to simplify our considerations. What is the picture then? Fig. 2 illustrates the situation. The edges *AB*, *CD*, *EF* are denoted by the numbers 1, 2, 3. We assume that the edge 1 emits its light quanta at the time $t = 0$. Where must the edges 2 and 3 emit their light such that it travels in a common front with the light from 1, in order to arrive simultaneously at the observer? It is easily seen that 2 must emit its light much earlier; in fact it must happen when it is at the point marked 2′ which is determined by the equality of the distances (2′2) and (2′*M*). The interval (2′2) is the distance which the edge 2 travels between the emission of light by 2 and 1. The length (2′*M*) is the distance which the light travels from 2′ in order to be "in line" with the light emitted by 1. Both light and edge travel with the speed *c*. We can see that the distance (1*M*) is equal to (1 2) which is the size *l* of the cube. The light seen from edge 3 is emitted much later, when the edge is at 3′. The point 3′ is determined by the equality of the distances (3 3′) and (1*N*). A simple calculation shows that $(3'N) = l \sin \alpha \, (1 - \cos \alpha)^{-1}$.

What then is the picture we see of the cube? It is indicated in the figure by the points I, II, III which represent the positions of the light quanta coming from the object and form the picture. We will see a strongly deformed cube with the edge 1 in the middle, the edge 2 on the left of 1 as if we were looking from behind (from the left to the right) and the edge 3 quite far to the right of 1. Again we see a picture elongated in the direction of flight. The face between the edges 1 and 2 appears as a true square.

We now will show that relativity theory simplifies the situation. It removes the distortion of the picture and what remains is an undistorted but rotated aspect of the object. We can see this directly with the examples quoted. Consider the cube when looked at perpendicular to its motion; the Lorentz contraction reduces the distance between the edges *AB* and *CD* by the factor $(1 - v^2/c^2)^{1/2}$ and leaves the distance between *AB* and *EF* unchanged. Therefore the picture of the face *ABCD* is foreshortened precisely by the amount necessary to represent an undistorted view of a cube turned by an angle whose sine is v/c. In the case of the cube moving with light velocity towards us the Lorentz contraction reduces the distance between the edges 1 and 3 to zero. The picture one sees then is a regular square corresponding to the rear face and nothing else, since edge 3 coincides with edge 1. Hence we see an undistorted picture directly from behind. The object is undistorted but turned by an angle of $(180 - \alpha)$ degrees.

We can show by means of the following consideration that this result is quite generally true for any object. Let us consider an assembly of light pulses originating from *N* points of the object, traveling all

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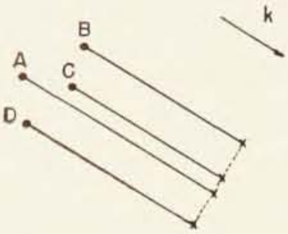

Fig. 3. A "picture". A, B, C, D are points of the object. The four crosses are the light pulses making up the "picture".

in the same direction given by a vector **k**, and such that the light pulses are all in one plane perpendicular to **k** (see Fig. 3). Then they will arrive simultaneously at the eye of the observer and produce the shape which is seen. We will call such an assembly of light pulses a "picture" of the object. Under nonrelativistic conditions a "picture" does not remain a picture when seen from a moving frame of reference. The reason is that, in the moving frame, the plane of the light pulses is no longer perpendicular to the direction of the propagation. In a relativistic world a "picture" remains a "picture" in any frame of reference. The light pulses would arrive simultaneously at a camera in every system of reference.

This fact can be proven immediately in the following way: The light pulses form a wave front or can be imagined as moving embedded in an electromagnetic wave exactly where this wave has a crest. It is known that electromagnetic waves are transverse in all frames of reference. That means that a wave front or the plane of the wave crest is perpendicular to the direction of propagation in *any* system. We can also show that the distance between the light pulses is an invariant magnitude. Here we only need to introduce a coordinate system where the x-axis is parallel to the propagation. Then for two light pulses of the picture the invariant $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - c^2(t_1 - t_2)^2$ is equal to the square of the distance d between the two pulses, since $d^2 = (y_1 - y_2)^2 + (z_1 - z_2)^2$ and $x_1 = x_2$ when $t_1 = t_2$. The latter relation expresses the fact that the pulses are in a plane perpendicular to the propagation.

The only thing that is not invariant is the direction of propagation, the vector **k**. The transformation of this direction is given by the well-known aberration formula. A light beam whose direction includes the angle θ with the x-axis is seen including an angle θ' with the x-axis in a system moving with the velocity v along the x-axis: *

$$\sin\theta' = \frac{(1 - v^2/c^2)^{\frac{1}{2}} \sin\theta}{1 + (v/c)\cos\theta}.$$

We can conclude the following result from the invariance of the "picture": The picture seen from a moving object observed at the angle θ is the *same* as one would see in the system where the object is at rest, but observed at the angle θ'. Hence we see an undistorted picture of a moving object, but a picture in which the object is seemingly rotated by the angle $\theta' - \theta$. A spherical object still appears as a sphere.

This must not by any means be interpreted as indicating that there is no Lorentz contraction. Of course, there is Lorentz contraction, but it just compensates for the elongation of the picture caused by the finite propagation of light.

It is instructive to plot the angle θ' as a function θ. Fig. 4 shows this relation for $v = 0$, for a small

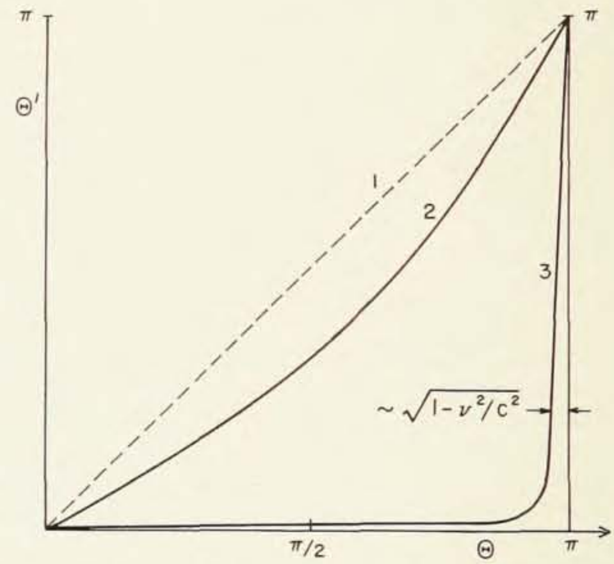

Fig. 4. The angle of observation θ' of a light beam relative to the direction of v, seen by an observer in the moving system, versus the same angle θ as seen in the rest system. Curve 1 is for $v = 0$, curve 2 is for $v = c/2$, curve 3 is for $v = c$.

value of v/c and also for the case $v/c \approx 1$. We see that the apparent rotation is always negative, which means that the object is turned such that it reveals more of its trailing side to the observer. In the extreme case of $v \approx c$, θ' is extremely small for all values of θ except when $180 - \theta$ is of the order $[1 - (v/c)^2]^{1/2}$. Since θ goes from 180° to 0° when an object moves by, we find for the case $v \approx c$ that we see the front side of the object only at the very beginning; it turns around facing its trailing side at us quite early when we still see it coming at us and remains doing so until it leaves us and naturally is seen from behind. This paradoxical situation is perhaps not so surprising when one is reminded of the fact that the aberration angle is almost 180° when $v \approx c$. Hence the light which we see coming from the object when it is moving towards us, has left the object backwards when observed from the object itself.

The situation becomes clearer when we look closer at the distribution of the emitted light as seen from

* The angles refer to the direction in which the light beam is seen; that means a direction opposite to the motion of the light pulses.

the observer. Let us assume that the moving object emits radiation which is isotropic in its own rest system, i.e., its intensity is independent of the emission angle θ'. This radiation does not at all appear isotropic in the nonmoving system; there it seems concentrated in the forward direction. If $v \approx c$, most of the light appears to be emitted such that it includes a very small angle θ with v. This is a well-known effect which causes an isotropic emission to look as if almost all radiation is emitted in the form of a focused headlight beam. One example of this effect is the radiation of electrons running along a circle with a velocity near c as one finds it in synchrotrons. In this case the radiation in the rest system is not completely isotropic; it is essentially a dipole radiation. Still it appears as an emission sharply peaked in the direction of flight.

The apparent angular distribution $I(\theta)$ of the radiated intensity in the system at rest is connected with the angular distribution $I_0(\theta')$ in the system of the moving object by an expression which is related to the aberration formula:

$$\frac{I(\theta)}{I_0(\theta')} = \frac{\sin\theta' d\theta'}{\sin\theta d\theta} = K(\theta)$$

$$K(\theta) = \frac{1-(v/c)^2}{[1+(v/c)\cos\theta]^2}$$

where θ is the angle of observation, which means that the forward direction is near $\theta = \pi$. $K(\theta)$ is plotted in Fig. 5 as a function of θ and we see that the width of the "headlight" beam is of the order $[1-(v/c)^2]^{1/2}$.

The factor $K(\theta)$ also determines the Doppler shift of the light. If the emitted light has the frequency ω_0 in the system moving with the object, the observer at rest sees a frequency $\omega = K^{1/2}\omega_0$. The frequency is increased or decreased by the square root of the factor by which the intensity is enhanced or reduced. We note that, for $\theta = 90°$ there is a reduction of frequency due to the relativistic Doppler shift.

We now describe what is seen when an object is moving by with a velocity near that of light. First, when the angle of vision is still near 180°, we see the front face of the object, strongly Doppler-shifted to very high frequencies and with high intensity. We are looking into the "headlight" beam of the radiation. When the angle of vision becomes of the order $\pi - [1-(v/c)^2]^{1/2}$ the color shifts towards lower frequencies, the intensity drops, and the object seems to turn. When $\theta \approx \pi - 2^{1/2}[1-(v/c)^2]^{1/4}$, still an angle close to 180°, the intensity becomes very low, we are out of the "headlight" beam, the color is now of much lower frequencies than it would be in the system moving with the object; the object has turned all around and we are looking at its trailing face. The front is invisible because the beams emitted forward in the moving system are concentrated into the small angle of the "headlight". The picture seen at angles smaller than $\pi - 2^{1/2}[1-(v/c)^2]^{1/4}$ remains essentially the same until the object disappears. It is the picture expected when the object is receding. However it appears already when the object is moving toward us.

We would like to emphasize that all these considerations are exact only for objects which subtend a very small solid angle. Only then the picture consists essentially of parallel light pulses. If the angle subtended is finite we must expect different rotation angles for different parts of the picture and this would lead to some distortions. It has been shown by Penrose,[2] however, that the picture of a sphere retains a circular circumference even for large angles of vision.

It is most remarkable that these simple and important facts of the relativistic appearance of objects have not been noticed for 55 years until J. Terrell discovered and fully recognized them in his recent publication.

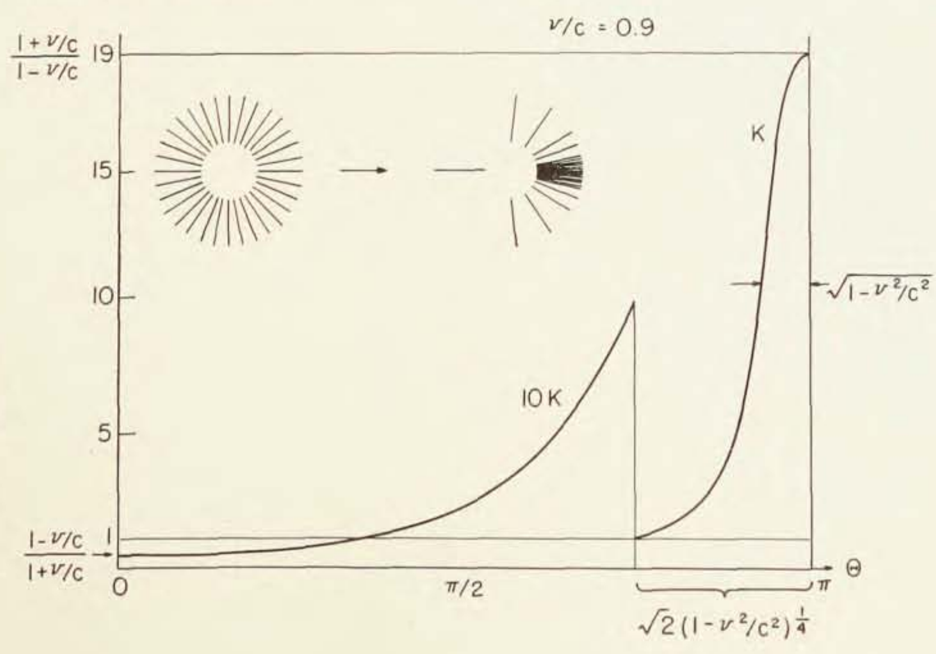

Fig. 5. The ratio K of the emitted intensity per solid angle measured in the observer's system to the intensity per solid angle measured in the system moving with the object, as function of the angle θ of observation. K also determines the Doppler shift. The observed frequency ω is related to the emitted frequency by $\omega = K^{\frac{1}{2}}\omega_0$.