Esercizio C ultimo foglio

$\underline{2x^2 + 5y^2 + 5z^2 + 4xy - 4xz - 8yz} + 7x = 0$

parte quadratica.

1. Matrice:
$$\begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix} = A$$

forma matriciale: $x^T A x + [7 \quad 0 \quad 0]\, x = 0$

2. Autovalori:
$$\begin{vmatrix} 2-\lambda & 2 & -2 \\ 2 & 5-\lambda & -4 \\ -2 & -4 & 5-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 5-\lambda & -4 \\ -4 & 5-\lambda \end{vmatrix} - 2\begin{vmatrix} 2 & -4 \\ -2 & 5-\lambda \end{vmatrix} - 2\begin{vmatrix} 2 & 5-\lambda \\ -2 & -4 \end{vmatrix}$$

$= (2-\lambda)[(5-\lambda)^2 - 16] - 2(10-2\lambda-8) - 2(-8+10-2\lambda)$

$= (2-\lambda)(25+\lambda^2-10\lambda-16) + 4\lambda - 4 - 4 + 4\lambda$

$= 50 - 25\lambda + 2\lambda^2 - \lambda^3 - 20\lambda + 10\lambda^2 - 32 + 16\lambda + 8\lambda - 8$

$= -\lambda^3 + 12\lambda^2 - 21\lambda - 40 = \lambda^3 - 12\lambda^2 + 21\lambda + 40$

$= (2-\lambda)((5-\lambda)^2-16) - 2(-2\lambda+2) - 2(-2\lambda+2)$

$= (2-\lambda)[(5-\lambda)^2-16] + (-2\lambda+2)(-4)$

$\Rightarrow \lambda = 1 \qquad \mu(1) = 2$

$\lambda = 10 \qquad \mu(10) = 1$

Autospazi: $\lambda = 1$
$$\begin{bmatrix} 1 & 2 & -2 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow x + 2y - 2z = 0 \quad \text{piano}$$

x	y	z
2	0	1
-2	1	0

$f_1 = (2, 0, 1)$

trovare secondo ortogonale a f_1: $\langle (2z-2y, y, z)(2,0,1)\rangle = 0$

$4z - 4y + z = 0$

$5z = 4y \longrightarrow z = \frac{4}{5}y$

$f_2 = \left(2\frac{4}{5}y - 2y, y, \frac{4}{5}y\right) = \left(\frac{8-10}{5}y, y, \frac{4}{5}y\right) = \left(-\frac{2}{5}, 1, \frac{4}{5}\right)$

Normalizzando: $\|f_1\| = \sqrt{4+1} = \sqrt{5} \longrightarrow f_1 = \left(\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right)$

$\|f_2\| = \sqrt{\frac{4}{25} + 1 + \frac{16}{25}} = \sqrt{\frac{45}{25}} = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$

$$f_2 = \left(-\frac{2}{5}\cdot\frac{\sqrt{5}}{3},\ \frac{\sqrt{5}}{3},\ \frac{4}{5}\cdot\frac{\sqrt{5}}{3}\right) \text{ oppure } f_2 = (-2, 5, 4) \qquad \|f_2\| = \sqrt{45} = 3\sqrt{5}$$

$$f_2 = \left(-\frac{2\sqrt{5}}{15},\ \frac{\sqrt{5}}{3},\ \frac{4\sqrt{5}}{15}\right) \quad \text{oppure} \quad f_2 = \left(-\frac{2}{3\sqrt{5}},\ \frac{5}{3\sqrt{5}},\ \frac{4}{3\sqrt{5}}\right)$$

$$\lambda = 10: \quad \begin{bmatrix} -8 & 2 & -2 \\ 2 & -5 & -4 \\ -2 & -4 & -5 \end{bmatrix} \sim \begin{bmatrix} -8 & 2 & -2 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -8 & 0 & -4 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} 2x + z = 0 \\ y + z = 0 \end{cases} \longrightarrow \begin{cases} x = -\frac{z}{2} \\ y = -z \end{cases} \qquad f_3 = \left(-\frac{1}{2}, -1, 1\right)$$

$$\|f_3\| = \sqrt{\frac{1}{4} + 1 + 1} = \sqrt{\frac{1+8}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$f_3 = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$$

$$B_{\perp n} = \left\{ \left(\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right), \left(-\frac{2}{\sqrt{5}}, \frac{5}{3\sqrt{5}}, \frac{4}{3\sqrt{5}}\right), \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right) \right\}$$

$$S = \begin{bmatrix} 2/\sqrt{5} & -2/\sqrt{5} & -1/3 \\ 0 & 5/3\sqrt{5} & -2/3 \\ 1/\sqrt{5} & 4/3\sqrt{5} & 2/3 \end{bmatrix} \qquad S^T = \begin{bmatrix} 2/\sqrt{5} & 0 & 1/\sqrt{5} \\ -2/\sqrt{5} & 5/3\sqrt{5} & 4/3\sqrt{5} \\ -1/3 & -2/3 & 2/3 \end{bmatrix}$$

$$S \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{cases} x = \frac{2}{\sqrt{5}} u - \frac{2}{\sqrt{5}} v - \frac{1}{3} w \\ y = 0\, u + \frac{5}{3\sqrt{5}} v - \frac{2}{3} w \\ z = \frac{1}{5} u + \frac{4}{3\sqrt{5}} v + \frac{2}{3} w \end{cases}$$

Riscrivere l'eq.ne: $u^2 + v^2 + 10w^2 + 7\left(\frac{2}{\sqrt{5}} u - \frac{2}{\sqrt{5}} v - \frac{1}{3} w\right) = 0$

$$\underbrace{u^2 + \frac{14}{\sqrt{5}} u}_{1} + \underbrace{v^2 - \frac{14}{\sqrt{5}} v}_{2} + \underbrace{10\left(w^2 - \frac{7}{30} w\right)}_{3} = 0$$

$$1)\ \left(u + \frac{7}{\sqrt{5}}u\right)^2 - \frac{49}{5}; \quad 2)\ \left(v - \frac{7}{3\sqrt{5}} v\right)^2 - \frac{49}{45}; \quad 3)\ 10\left(w - \frac{7}{60}\right)^2 - \frac{49}{360}$$

II Cambiamento variabile:
$$\begin{cases} t = u + \frac{7}{\sqrt{5}} \\ p = v - \frac{7}{3\sqrt{5}} \\ r = w - \frac{7}{60} \end{cases}$$

riscrivo eq.ne: $t^2 - \frac{49}{5} + p^2 - \frac{49}{45} + 10r^2 - \frac{49}{360} = 0$

$$t^2 + p^2 + 10r^2 - \frac{3528 + 392 + 49}{360} = 0$$

$$t^2 + p^2 + 10r^2 - \frac{\cancel{3969}}{\cancel{360}} \frac{441}{40} = 0$$

E' un ellissoide!