10-12-2018

$$\varphi'(x) = \left(x - \sqrt{x^2-1}\right)' = 1 - \frac{x}{\sqrt{x^2-1}}$$

$$\frac{\sqrt{x^2-1} - x}{\sqrt{x^2-1}}$$

$$\varphi(1) = 1$$
$$\varphi(-1) = -1$$

$$\phi'(x) = \frac{\sqrt{x^2-1} - x}{\sqrt{x^2-1}} = 1 - \frac{x}{\sqrt{x^2-1}}$$

$$\phi'(1) = \frac{0-1}{0} = -\infty$$

$$\phi'(-1) = \frac{0+1}{0} = +\infty$$

$$\phi''(x) = -\frac{1 \cdot \sqrt{x^2-1} - x \cdot \frac{2x}{2\sqrt{x^2-1}}}{x^2-1} = -\frac{\sqrt{x^2-1} - \frac{x^2}{\sqrt{x^2-1}}}{x^2-1}$$

$$= \frac{+1}{\sqrt{x^2-1}(x^2-1)}$$

$$\cup \quad -1 \quad + \quad 1 \quad \cup$$

$$\left(\frac{2}{\sqrt{-\Delta}}\operatorname{arctg}\frac{2ax+b}{\sqrt{-\Delta}}\right)' = \frac{2}{\sqrt{-\Delta}}\cdot\frac{1}{1+\left(\frac{2ax+b}{\sqrt{-\Delta}}\right)^2}\cdot\frac{2a}{\sqrt{-\Delta}} =$$

$\Delta < 0$

$$= \frac{4a}{-\Delta}\cdot\frac{1}{1+\frac{(2ax+b)^2}{-\Delta}} = \frac{4a}{-\Delta}\cdot\frac{1}{\frac{-\Delta+(2ax+b)^2}{-\Delta}} =$$

$$= \frac{4a}{-b+4ac+4a^2x^2+b^2+4abx} = \frac{4a}{4a(ax^2+bx+c)} = \frac{1}{ax^2+bx+c}$$

$$\int \frac{2x+3}{3x^2+4x+5}\,dx$$

$\Delta = 16 - 60 = -44$ (265)

$$\int \frac{2x}{3x^2+4x+5}\,dx + \int \frac{3}{3x^2+4x+5}\,dx = \frac{2}{6}\int \frac{6x+4-4}{3x^2+4x+5}\,dx + 3\int \frac{1}{3x^2+4x+5}\,dx$$

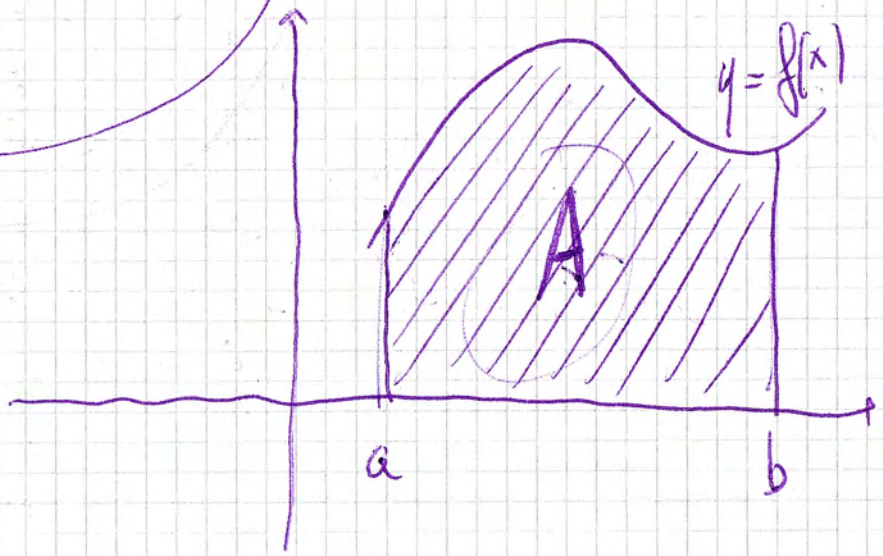
$$= \frac{1}{3}\int \frac{6x+4}{3x^2+4x+5}\,dx - \frac{4}{3}\int \frac{1}{3x^2+4x+5}\,dx + 3\int \frac{1}{3x^2+4x+5}\,dx$$

$$= \frac{1}{3}\ln\left(3x^2+4x+5\right) + \frac{5}{3}\cdot\frac{2}{\sqrt{44}}\operatorname{arctg}\frac{6x+4}{\sqrt{44}} + C$$

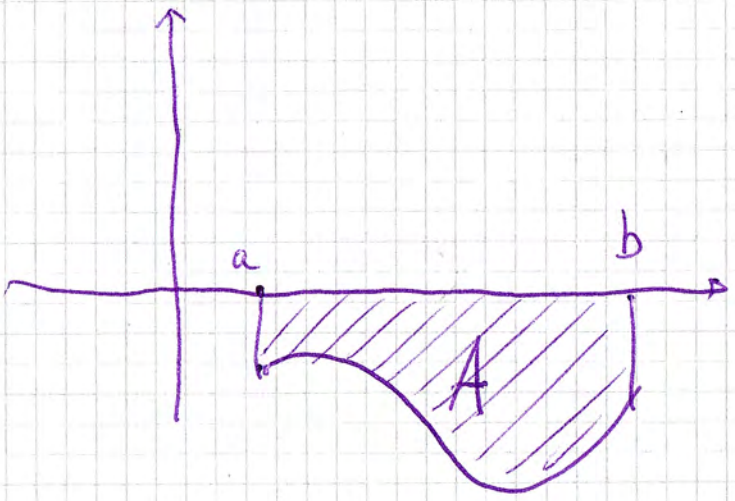
$\sqrt{44} = 2\sqrt{11}$

$\int_a^b f(x)\,dx$

$f(x)$ continue in $[a,b]$



$$\int_a^b f(x)\,dx \overset{\text{def.}}{=} A$$



$$\int_a^b f(x)\,dx = -A$$

(26.2

$$\int_a^b f(x)\,dx = 5 - 4 + 3$$

$$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$

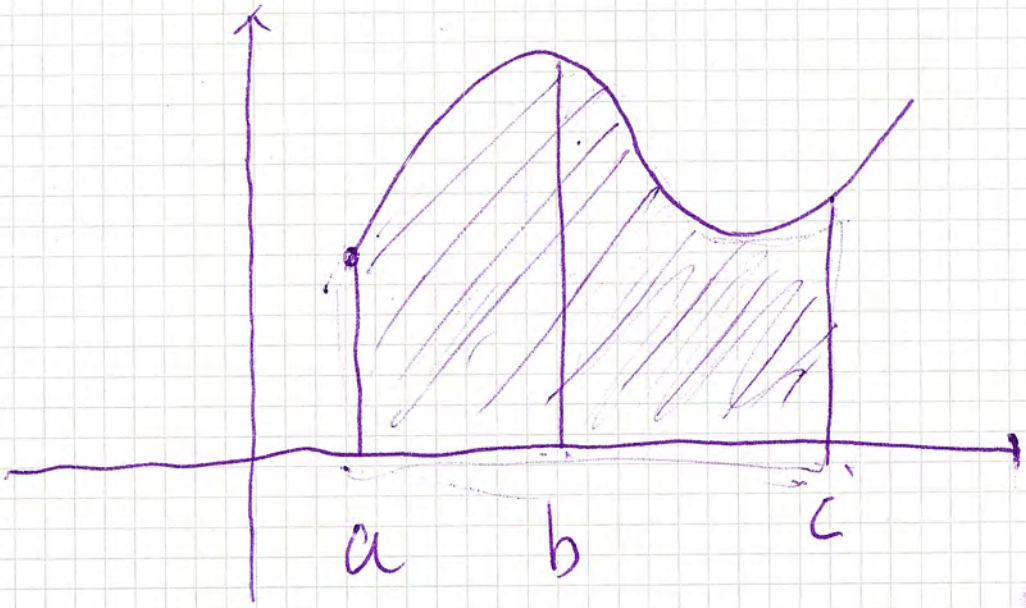
$a<b$

$$\int_a^a f(x)\,dx \overset{\text{def.}}{=} 0$$

$$\int_b^a f(x)\,dx \overset{\text{def.}}{=} -\int_a^b f(x)\,dx$$

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

(26)



$$\int \frac{7x+4}{x^2+2x+5}\,dx = \int \frac{7x}{x^2+2x+5}\,dx + \int \frac{4}{x^2+2x+5}\,dx =$$

$\Delta = 4-20 = -16 < 0$

$$= \frac{7}{2}\int \frac{2x+2-2}{x^2+2x+5}\,dx + 4\int \frac{1}{x^2+2x+5}\,dx = \frac{7}{2}\int \frac{2x+2}{x^2+2x+5}\,dx$$

$$-7\int \frac{1}{x^2+2x+5}\,dx + 4\int \frac{1}{x^2+2x+5}\,dx = \frac{7}{2}\ln\left(x^2+2x+5\right) - 3\cdot\frac{2}{\sqrt{16}}\arctg\frac{\frac{x+1}{2}}{\frac{\sqrt{16}}{4}}$$

$$\int tg^3 x\,dx = \int tg^2 x \cdot tg x = \int (tg^2 x + 1 - 1)\, tg x\,dx$$

$$= \int [(tg^2 x + 1)\, tg x - tg x]\,dx = \int (tg^2 x + 1)\, tg x\,dx + \int \underbrace{tg x\,dx}_{\frac{-\sin x}{\cos x}}$$

$$= \frac{tg^2 x}{2} + \ln|\cos x| + C$$

$$\cos x = t \longrightarrow -\sin x\,dx = dt \quad \sin x\,dx = -dt$$

$$\int \frac{\sin^3 x}{\cos^3 x}\,dx$$

$$\int \frac{\sin^2 x}{\cos^3 x} \cdot \sin x\,dx = \int \frac{1-t^2}{t^3} \cdot (-dt) = \int \frac{t^2-1}{t^3}\,dt$$

$$= \int \frac{1}{t}\,dt - \int \frac{1}{t^3}\,dt = \ln|t| - \frac{t^{-2}}{-2} + C = \ln|t| + \frac{1}{2t^2} + C$$

($\frac{1}{t^3} = t^{-3}$)

$$\int \frac{1}{(1+x^2)\operatorname{arctg} x}\,dx = \int \frac{\frac{1}{1+x^2}}{\operatorname{arctg} x}\,dx = \ln|\operatorname{arctg} x| + C$$ (272

$$\frac{1}{1+x^2} \cdot \frac{1}{\operatorname{arctg} x}$$

$\operatorname{arctg} x = t \qquad x = \operatorname{tg} t$

$\operatorname{arctg} 1 = \frac{\pi}{4}$

$\operatorname{tg} \frac{\pi}{4} = 1$

$$\int \frac{1}{(1+\operatorname{tg}^2 t)\,t}\,(1+\operatorname{tg}^2 t)\,dt = \int \frac{1}{t}\,dt = \ln|t| + C$$

$$= \ln|\operatorname{arctg} x| + C$$

$$\int x\,e^{3x+4}\,dx = \underset{g(x)}{x}\cdot\underbrace{\frac{e^{3x+4}}{3}}_{g'(x)} - \int 1\cdot\frac{e^{3x+4}}{3}\,dx$$

$$= x\,\frac{e^{3x+4}}{3} - \frac{e^{3x+4}}{9} + C$$

$$\int \frac{e^{3x}+1}{e^x}\,dx = \int\left(\frac{e^{3x}}{e^x}+\frac{1}{e^x}\right)dx = \int e^{2x}\,dx + \int e^{-x}\,dx = \frac{e^{2x}}{2} + \frac{e^{-x}}{-1} + C$$

$e^x = t \quad x = \lg t$

$$\int \frac{t^3+1}{t}\cdot\frac{1}{t}\,dt = \int \frac{t^3+1}{t^2}\,dt = \int \frac{t^3}{t^2}\,dt + \int \underbrace{\frac{1}{t^2}}_{t^{-2}}\,dt$$

$$= \frac{t^2}{2} - \frac{1}{t} + C$$