3-12-2018

INTEGRAZIONE DELLE FUNZIONI RAZIONALI FRATTE

$$\int \frac{P(x)}{H(x)}\,dx$$

Supponiamo grado $P \geq$ grado H

Divido $P(x)$ per $H(x)$ e sia $Q(x)$ il quoziente, $R(x)$ il resto dove grado $R <$ grado H

$$P(x) = H(x)Q(x) + R(x)$$

$$\frac{P(x)}{H(x)} = \frac{H(x)Q(x)+R(x)}{H(x)} = \frac{H(x)Q(x)}{H(x)} + \frac{R(x)}{H(x)}$$

$$\int Q(x)\,dx + \int \frac{R(x)}{H(x)}\,dx$$

$Q(x)$ facile

① grado $H(x)=1 \Rightarrow$ grado $R(x)=0 \Rightarrow R(x)=C$

$$\int \frac{C}{ax+b}\,dx = \frac{C}{a}\int \frac{a}{ax+b}\,dx = \frac{C}{a}\log|ax+b|+C$$

$$\int \frac{3}{2x+1}\,dx = \frac{3}{2}\int \frac{2}{2x+1}\,dx = \frac{3}{2}\log|2x+1|+C$$

$$\int \frac{3x+2}{4x-3}\,dx =$$

$$\begin{array}{r|l} 3x+2 & 4x-3 \\ -3x+\frac{9}{4} & \frac{3}{4}=Q(x) \\ \hline \frac{17}{4}=R(x) & \end{array}$$

$$= \int \frac{3}{4}dx + \int \frac{17/4}{4x-3}\,dx$$

$$\frac{3}{4}x + \frac{17}{4\cdot 4}\int \frac{4}{4x-3}\,dx$$

$$= \frac{3}{4}x + \frac{17}{16}\log|4x-3|+C$$

(2) grado $H(x)=2$

$$ax^2+bx+c=a(x-x_1)(x-x_2)$$ (250

(2a) $\Delta>0$

$$\int \frac{3x+2}{x^2+3x-4}\,dx \qquad \frac{3x+2}{(x-1)(x+4)}$$

$x^2+3x-4=0$

$\Delta=9+16=25>0$

$x=\frac{-3\pm 5}{2}$ → 1, -4

$$\frac{3x+2}{(x-1)(x+4)}=\frac{A}{x-1}+\frac{B}{x+4}=\frac{A(x+4)+B(x-1)}{(x-1)(x+4)}$$

$Ax+4A+Bx-B=(A+B)x+4A-B$

$$\begin{cases} A+B=3 \\ 4A-B=2 \end{cases}$$

$5A=5$ $A=1$

$B=2$

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$A(x+4) + B(x-1) = 3x+2$

$x=1$ $\quad 5A = 5 \quad A = 1$

$x=-4$ $\quad -5B = -10 \quad B = 2$

$$\int \frac{3x+2}{(x-1)(x+4)}\,dx = \int \left(\frac{1}{x-1} + \frac{2}{x+4}\right) dx = \int \frac{1}{x-1}\,dx + 2\int \frac{1}{x+4}\,dx$$

$$= \ln|x-1| + 2\ln|x+4| + C$$

2b $\Delta = 0$

$$a(x-x_1)(x-x_2) = a(x-x_1)^2$$

$$\int \frac{2x+3}{(3x+1)^2}\,dx$$

$$\frac{2x+3}{(3x+1)^2} = \frac{A}{3x+1} + \frac{B}{(3x+1)^2} = \frac{A(3x+1)+B}{(3x+1)^2}$$

$x = -\frac{1}{3}$ $\quad B = -\frac{2}{3} + 3 = \frac{7}{3}$ $\qquad x = 0$ $\quad A + B = 3 \rightarrow A = 3 - \frac{7}{3} = \frac{2}{3}$

$$\int \frac{2/3}{3x+1}\,dx + \int \frac{7/3}{(3x+1)^2}\,dx = \frac{2}{9}\ln|3x+1| + \frac{7}{3}\int (3x+1)^{-2}\,dx$$

$$\frac{2}{9}\ln|3x+1| + \frac{7}{3\cdot 3}\int 3|3x+1|^{-2}\,dx =$$

$$\frac{2}{9}\ln|3x+1| + \frac{7}{9}\frac{(3x+1)^{-1}}{-1} + C$$

$$= \frac{2}{9}\ln|3x+1| - \frac{7}{9(3x+1)} + C$$

$$\int f(x)^{\alpha} f'(x)\,dx$$

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$$\frac{f(x)^{\alpha+1}}{\alpha+1} + C$$

$$\int \frac{1}{x^2+x+1}\,dx = \qquad \Delta = 1-4 = -3$$

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$$x^2+x+1 = \underbrace{x^2+x+\frac{1}{4}}+\frac{3}{4} = \left(x+\frac{1}{2}\right)^2+\frac{3}{4} = \frac{3}{4}\left[\frac{4}{3}\left(x+\frac{1}{2}\right)^2+1\right] =$$

$$= \frac{3}{4}\left[\left(\frac{2}{\sqrt{3}}\right)^2\left(x+\frac{1}{2}\right)^2+1\right] = \frac{3}{4}\left[\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{2x+1}{2}\right)^2+1\right] =$$

$$= \frac{3}{4}\left[\left(\frac{2x+1}{\sqrt{3}}\right)^2+1\right]$$

$$a^n b^n = (ab)^n$$

$$\int \frac{1}{x^2+x+1}\,dx = \int \frac{1}{\frac{3}{4}\left[\left(\frac{2x+1}{\sqrt{3}}\right)^2+1\right]}\,dx = \frac{4}{3}\int \frac{\frac{2}{\sqrt{3}}}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\,dx$$

$$= \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}} + C$$

$$\left(\frac{2x+1}{\sqrt{3}}\right)' = \frac{2}{\sqrt{3}}$$

$$\int \frac{1}{ax^2+bx+c}\,dx = \frac{2}{\sqrt{-\Delta}}\operatorname{arctg}\frac{2ax+b}{\sqrt{-\Delta}} + C$$

$\Delta < 0$

$$= \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}} + C$$

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Integrazione per sostituzione

$$\int f(x)\,dx$$

$$x = g(t)$$

$$\int f(g(t))\,g'(t)\,dt$$

$$\int \frac{x+5}{\sqrt{x-3}}\,dx$$

$$\sqrt{x-3} = t \qquad x-3 = t^2$$

$$x = t^2+3 = g(t)$$

$$\int \frac{t^2+3+5}{t}\cdot 2t\,dt = 2\int (t^2+8)\,dt = 2\left(\frac{t^3}{3}+8t\right)+C$$

$$\int \frac{e^{2x}}{1+e^x}\,dx \qquad e^x = t$$

$$x = \lg t$$

$$\int \frac{t^2}{1+t}\cdot\frac{1}{t}\,dt = \int \frac{1+t-1}{1+t}\,dt = \int\left(\frac{1+t}{1+t} - \frac{1}{1+t}\right)dt$$

$$= \int 1\,dt - \int \frac{1}{1+t}\,dt = t - \lg|1+t| + C =$$

$$= e^x - \lg(1+e^x) + C$$

$$\int \frac{\ln^2 x + 1}{x \ln x}\,dx$$

$\ln x = t$

$x = e^t$

$$\int \frac{t^2+1}{e^t \cdot t} \cdot e^t\,dt = \int \frac{t^2+1}{t}\,dt = \int \left(t + \frac{1}{t}\right) dt =$$

$$= \int t\,dt + \int \frac{1}{t}\,dt = \frac{t^2}{2} + \ln|t| + C = \frac{\ln^2 x}{2} + \ln|\ln x| + C$$

$$\int \frac{\ln^2 x}{x \ln x}\,dx + \int \frac{1}{x \ln x}\,dx = \int \frac{1}{x}\ln x\,dx + \int \frac{\frac{1}{x}}{\ln x}\,dx = \frac{\ln^2 x}{2} + \ln|\ln x| + C$$

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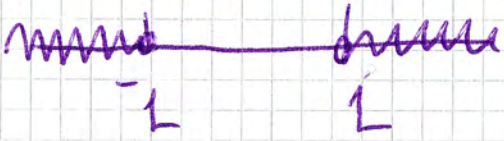
$$f(x) = x - \sqrt{x^2-1}$$

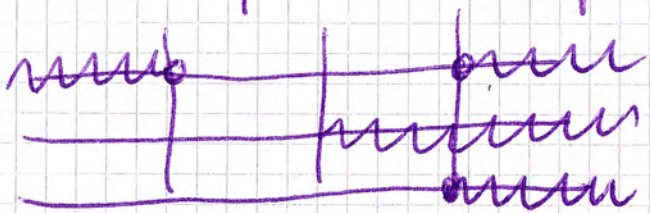
Dominio $x^2-1 \geq 0$

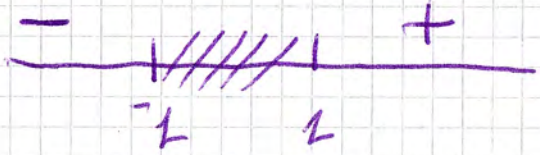


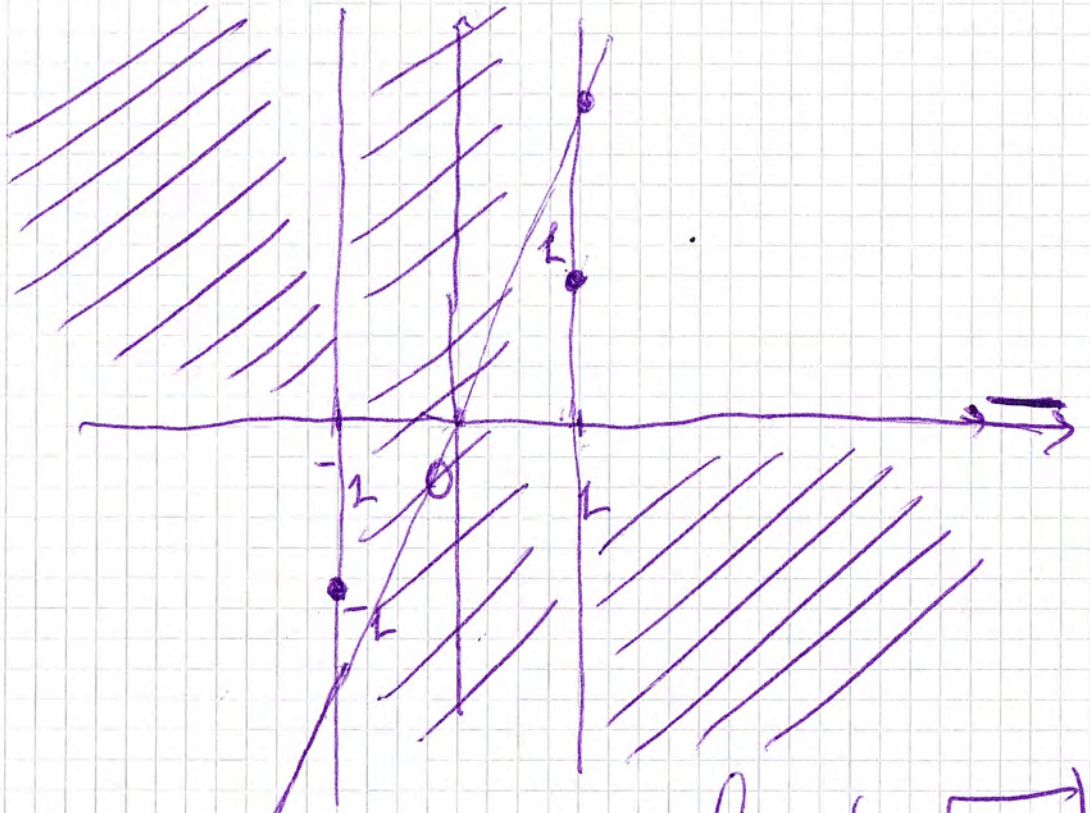
Int. ani $\begin{cases} y=0 \\ x = \sqrt{x^2-1} \end{cases} \rightarrow x^2 = x-1 \quad 0 = -1$ imp.

Segno $x - \sqrt{x^2-1} > 0 \quad x > \sqrt{x^2-1} \quad \sqrt{x^2-1} < x$

$$\begin{cases} x^2-1 \geq 0 \\ x > 0 \\ x^2-1 < x^2 \text{ A.V.} \end{cases}$$







$f(1) = 1$

$f(-1) = -1$

$$\lim_{x \to \pm\infty} \left(x - \sqrt{x^2-1}\right) = -\infty \qquad \lim_{x \to +\infty} \left(x - \sqrt{x^2-1}\right) = \lim_{x \to +\infty} \frac{\left(x - \sqrt{x^2-1}\right)\left(x + \sqrt{x^2-1}\right)}{x + \sqrt{x^2-1}} =$$

$$= \lim_{x \to +\infty} \frac{x^2 - x^2 + 1}{x + \sqrt{x^2-1}} = \frac{1}{+\infty} = 0$$

$y = 0$ as. oriz. destra

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$$\lim_{x\to-\infty}\frac{x-\sqrt{x^2-1}}{x}=\lim_{x\to-\infty}\left(1-\frac{\sqrt{x^2-1}}{x}\right)=2=m$$

$$\frac{\sqrt{x^2}}{x}=\frac{|x|}{x}=\frac{-x}{x}=-1$$

$$q=\lim_{x\to-\infty}\left(x-\sqrt{x^2-1}-2x\right)=\lim_{x\to-\infty}\left(-x-\sqrt{x^2-1}\right)=$$

$$=\lim_{x\to-\infty}\frac{\left(-x-\sqrt{x^2-1}\right)\left(-x+\sqrt{x^2-1}\right)}{-x+\sqrt{x^2-1}}=\lim_{x\to-\infty}\frac{x^2-x^2+1}{-x+\sqrt{x^2-1}}=\frac{1}{+\infty}=0$$

$q=0$

$y=2x$ asintoto obliquo sinistro