28-11-2018

$$f(x)=\frac{\sqrt{x^2+1}}{x}$$

Dominio

$$\begin{cases} x^2+1 \geq 0 \quad \forall x \\ x \neq 0 \end{cases} \qquad (x \neq 0)$$

Int. assi

~~$x=0$~~

$$\begin{cases} y=0 \\ \dfrac{\sqrt{x^2+1}}{x}=0 \end{cases} \qquad \sqrt{x^2+1}=0 \rightarrow x^2+1=0 \text{ impossibile}$$

Segno

$N. = \sqrt{x^2+1} \geq 0 \quad \forall x \in D_f$

$D. > 0 \quad x>0$

		0	
N.	+	|	+
D.	−	|	+
	−	0	+

$$f(-x)=\frac{\sqrt{(-x)^2+1}}{-x}=\frac{\sqrt{x^2+1}}{-x}=-\frac{\sqrt{x^2+1}}{x}=-f(x)$$ dispari

$$\lim_{x\to 0^{\pm}} \frac{\sqrt{x^2+1}}{x} = \frac{1}{0^{\pm}} = \pm\infty \qquad x=0 \text{ as. vert.}$$

$$\lim_{x\to\pm\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\pm\infty} \frac{\sqrt{x^2}}{x} = \lim_{x\to\pm\infty} \frac{|x|}{x}$$

$$\lim_{x\to+\infty} \frac{x}{x} = 1 \qquad \lim_{x\to-\infty} \frac{-x}{x} = -1$$

$y=1$ as. oriz. destro $\qquad y=-1$ as. oriz. sinistro

$$f'(x) = \frac{\left(\sqrt{x^2+1}\right)' x - \sqrt{x^2+1}\cdot(x)'}{x^2} = \frac{\frac{1}{2\sqrt{x^2+1}}\cdot 2x\cdot x - \sqrt{x^2+1}\cdot 1}{x^2} =$$

$$= \frac{\frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1}}{x^2} = \frac{\frac{x^2 - x^2 - 1}{\sqrt{x^2+1}}}{x^2} = -\frac{1}{x^2\sqrt{x^2+1}}$$

x^2 : — — — 0 — — —

f' : — — — 0 — — —

1236

$$f'(x) = -\frac{1}{x^2\sqrt{x^2+1}}$$

$$f''(x) = -\frac{0\cdot x^2\sqrt{x^2+1} - 1\cdot\left(x^2\sqrt{x^2+1}\right)'}{\left(x^2\sqrt{x^2+1}\right)^2}$$

$$= +\frac{2x\sqrt{x^2+1} + x^2\cdot\frac{2x}{2\sqrt{x^2+1}}}{x^4\left(x^2+1\right)} = \frac{2x\left(x^2+1\right)+x^3}{\sqrt{x^2+1}\,x^4\left(x^2+1\right)}$$

$$= \frac{2x^3+2x+x^3}{x^4\left(x^2+1\right)\sqrt{x^2+1}} = \frac{3x^3+2x}{x^4\left(x^2+1\right)\sqrt{x^2+1}} = x\left(3x^2+2\right)$$

$$+ \quad - \quad +$$
$$\cap \quad 0 \quad \cup$$

INTEGRALE INDEFINITO

DEFINIZIONE. $F(x)$ è una primitiva di $f(x)$ su $[a,b]$ se
1) $F(x)$ è derivabile
2) $F'(x) = f(x)$

1 è una primitiva di 0

$x+2$ " " " 1

$\sin x$ " " " $\cos x$

$-\cos x$ " " " $\sin x$

DEF. $\int f(x)\,dx = \{\text{totalità delle primitive di } f(x)\}$

si legge "integrale indefinito di $f(x)$ in dx"

Sia $F(x)$ una primitive di $f(x)$ allora $F(x)+k$ è ancora una primitive di $f(x)$

Sia $G(x)$ un'altra primitive di $f(x)$ (cioè $G(x)$ è derivabile e $G'(x)=f(x)$)

$G(x)$, $F(x)$ hanno la stessa derivata ($f(x)$) quindi

per il 2° Corollario di L.

$$G(x)-F(x)=k \rightarrow G(x)=F(x)+k$$

Quindi $F(x)+k$ mi dà, al variare di k, tutte le primitive di $f(x)$

$$\int f(x)\,dx = \{ F(x)+k \,:\, k\in\mathbb{R} \} = F(x)+k$$

dove $F(x)$ è una "qualunque" primitive di $f(x)$

① $$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

② $$\int k f(x)\,dx = k\int f(x)\,dx$$

INTEGRALI IMMEDIATI

1) $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + k \quad \alpha \neq -1$

2) $\int \frac{1}{x} dx = \log|x| + k$

3) $\int f(x)^\alpha f'(x) dx = \frac{f(x)^{\alpha+1}}{\alpha+1} + k \quad \alpha \neq -1$

4) $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + k$

$$\int \left(2x^3 - \frac{4}{5}x^2 + 5x - 13\right) dx = \int 2x^3 dx + \int -\frac{4}{5}x^2 dx + \int 5x dx + \int -13 dx \quad (242$$

$$= 2\int x^3 dx - \frac{4}{5}\int x^2 dx + 5\int x\, dx - 13\int 1\, dx = 2 \cdot \frac{x^4}{4} - \frac{4}{5}\cdot\frac{x^3}{3} + 5\cdot\frac{x^2}{2} - 13x + K$$

$$= \frac{1}{2}x^4 - \frac{4}{15}x^3 + \frac{5}{2}x^2 - 13x + K$$

$$\int \frac{x+1}{x^2+1} dx = \int \left(\frac{x}{x^2+1} + \frac{1}{x^2+1}\right) dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$= \frac{1}{2}\ln(x^2+1) + \operatorname{arctg} x + C$$

243

$$\int x\sqrt{5x^2+1}\,dx = \int x\left(5x^2+1\right)^{1/2} dx$$

$$= \frac{1}{10}\int 10x\left(5x^2+1\right)^{1/2} dx = \frac{1}{10}\cdot\frac{\left(5x^2+1\right)^{3/2}}{3/2} + C$$

$$= \frac{1}{15}\sqrt{\left(5x^2+1\right)^3} + C$$

$$\int \operatorname{sen}x\cos x\,dx = \int (\operatorname{sen}x)'\cos x\,dx = \frac{\operatorname{sen}^2 x}{2} + C = \frac{1-\cos^2 x}{2} + C = \frac{1}{2} - \frac{\cos^2 x}{2} + C$$

$$-\int(-\operatorname{sen}x)\cos x\,dx = -\frac{\cos^2 x}{2} + C$$

$$\int \sin x \cos x \, dx = \int \frac{\sin 2x}{2} \, dx$$

$$\sin 2x = 2 \sin x \cos x$$

$$= \frac{1}{2 \cdot 2} \int 2 \sin 2x \, dx = \frac{-\cos 2x}{4} + C$$

$$\frac{-\cos 2x}{4} + C \qquad \text{или} \quad \frac{\sin^2 x}{2} \quad -\frac{\cos^2 x}{2}$$

$$-\frac{(2\cos^2 x - 1)}{4} \qquad -\frac{1 - 2\sin^2 x}{4}$$

$$-\frac{1}{2}\cos^2 x + \frac{1}{4} \qquad -\frac{1}{4} + \frac{1}{2}\sin^2 x$$

INTEGRAZIONE PER PARTI

$$\int f'(x)\,dx = f(x) + C$$

$$\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x)$$

$$\int \left(f(x)g(x)\right)' dx = \int \left(f'(x)g(x) + f(x)g'(x)\right) dx = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$f(x)g(x) = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

regola di integrazione per parti:

$$\int \underset{\substack{\downarrow \\ f}}{x} \underset{\substack{\downarrow \\ g}}{\lg x}\, dx = \frac{x^2}{2}\lg x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx =$$

$$= \frac{x^2}{2}\lg x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\lg x - \frac{x^2}{4} + C$$

$$\int \operatorname{arctg} x\,dx = \int \underset{\substack{\downarrow \\ f}}{1}\cdot \underset{\substack{\downarrow \\ g}}{\operatorname{arctg} x}\,dx = x\operatorname{arctg} x - \int x\cdot\frac{1}{1+x^2}\,dx$$

$$= x\operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = x\operatorname{arctg} x - \frac{1}{2}\lg(1+x^2)+$$

$$\int \sin^2 x\,dx = \int \underset{\downarrow f'}{\sin x} \cdot \underset{\downarrow g}{\sin x}\,dx = (-\cos x)\cdot \sin x - \int -\cos x \cdot \cos x\,dx$$

$$= -\sin x\cos x + \int \cos^2 x\,dx = -\sin x\cos x + \int (1-\sin^2 x)\,dx$$

$$\int \sin^2 x\,dx = -\sin x\cos x + x - \int \sin^2 x\,dx$$

$$2\int \sin^2 x\,dx = x - \sin x\cos x$$

$$\boxed{\int \sin^2 x\,dx = \frac{x-\sin x\cos x}{2} + C}$$

$$\boxed{\int \cos^2 x\,dx = \frac{x+\sin x\cos x}{2} + C}$$