21-11-2018 $\left| f(x) = \frac{1}{6}\cdot\frac{x-2}{(x^2-x+1)^2} + \frac{1}{6}\cdot\frac{2x-1}{x^2-x+1} + \frac{2}{3\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} \right| 2|3$

$$\frac{1}{6}\cdot\frac{x-2+(2x-1)(x^2-x+1)}{(x^2-x+1)^2} + \frac{2}{3\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} = \frac{1}{6}\cdot\frac{x-2+2x^3-2x^2+2x-x^2+x-1}{(x^2-x+1)^2} +$$

$$+\frac{2}{3\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} = \frac{1}{6}\cdot\frac{2x^3-3x^2+4x-3}{(x^2-x+1)^2} + \frac{2}{3\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} =$$

$$f'(x) = \frac{1}{6}\cdot\frac{(6x^2-6x+4)(x^2-x+1)^2 - (2x^3-3x^2+4x-3)\,2(x^2-x+1)(2x-1)}{(x^2-x+1)^4} + \frac{2}{3\sqrt{3}}\cdot\frac{1}{1+\left(\frac{2x-1}{\sqrt{3}}\right)^2}\cdot\frac{2}{\sqrt{3}}$$

$$= \frac{1}{6}\cdot\frac{(x^2-x+1)\left(6x^4-6x^3+6x^2-6x^3+6x^2-6x+4x^2-4x+4-8x^4+4x^3+12x^3-6x^2-6x^2+8x+12x-6\right)}{(x^2-x+1)^4}$$

$$+\frac{4}{9}\cdot\frac{1}{1+\frac{4x^2-4x+1}{3}}$$

$$= \frac{1}{6} \cdot \frac{(x^2-x+1)(-2x^4+4x^3-6x^2+10x-2)}{(x^2-x+1)^3} + \frac{4}{3} \cdot \frac{1}{\frac{3+4x^2-4x+1}{3}}$$

$$= \frac{1}{6} \cdot \frac{-2x^4+4x^3-6x^2+10x-2}{(x^2-x+1)^3} + \frac{1}{4(x^2-x+1)} \cdot \frac{4}{3}$$

$$= \frac{-x^4+2x^3-3x^2+5x-1}{3(x^2-x+1)^3} + \frac{1}{3(x^2-x+1)} = \frac{}{(x^2-x+1)^2}$$

$$= \frac{-x^4+2x^3-3x^2+5x-1+x^4-x^2+1-2x^3+2x^2-2x}{3(x^2-x+1)^3} = \frac{3x}{3(x^2-x+1)^3}$$

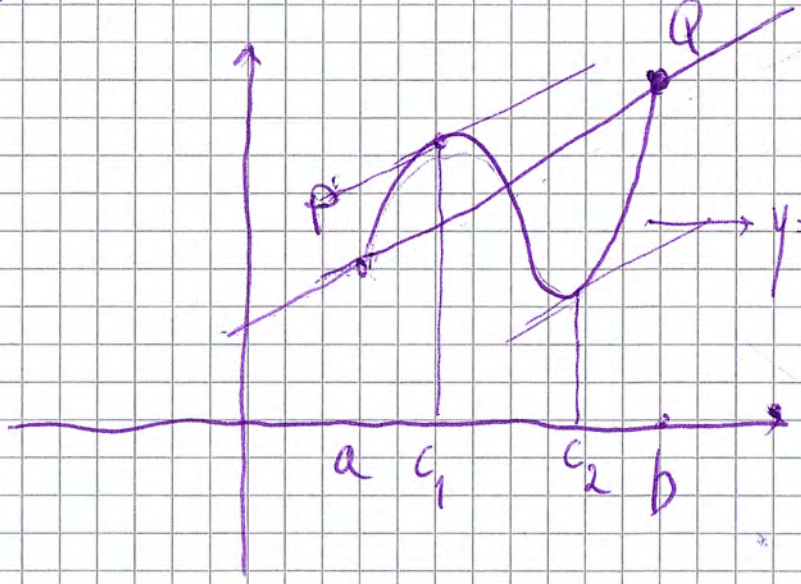
TEOREMA DI ROLLE. Se
1) $f(x)$ è continua in $[a,b]$
2) $f(x)$ è derivabile in $]a,b[$
3) $f(a) = f(b)$

$\Rightarrow \exists c \in]a,b[: f'(c) = 0$

TEOREMA DI LAGRANGE. Se

1) $f(x)$ è continua in $[a,b]$
2) $f(x)$ è derivabile in $]a,b[$

$\Rightarrow \exists c \in]a,b[: f'(c) = \dfrac{f(b)-f(a)}{b-a} = m_{PQ}$



Dim.:

$g(x) \overset{\text{def.}}{=} f(x) + kx$

Voglio determinare k in modo da poter applicare alla $g(x)$ il Th. di Rolle in $[a,b]$

1) $g(x)$ è continua in $[a,b]$? Sì perché somma di f.c. in $[a,b]$
2) $g(x)$ è derivabile in $]a,b[$? " " " " f.d. in $]a,b[$
3) $g(a) = g(b) \longrightarrow f(a) + ka = f(b) + kb \qquad k(a-b) = f(b) - f(a)$

$$k = \frac{f(b)-f(a)}{a-b} = -\frac{f(b)-f(a)}{b-a}$$

Per $k = -\frac{f(b)-f(a)}{b-a}$ posso applicare Rolle alla $g(x)$

$\exists\, c \in]a,b[: g'(c) = 0$.

$g'(x) = f'(x) + k \longrightarrow g'(c) = f'(c) + k = 0$

$\longrightarrow f'(c) = -k = \frac{f(b)-f(a)}{b-a}$.

$\frac{2}{3} = -\frac{2}{-3}$

$-\frac{2}{3} \quad -\frac{2}{3}$

$\frac{-2}{3} \quad \frac{2}{-3}$

Verificare che è possibile applicare alla funzione $f(x) = x + \ln x$ 217
nell'intervallo $[1, e]$ il th. di Lagrange e determinare i punti di cui
si parla in tale teorema.

a b

$D = \{x \in \mathbb{R} : x > 0\}$

$f(e) = e + \ln e = e + 1$
$f(1) = 1 + \ln 1 = 1$

1) $f(x)$ è continua in $[1,e]$? Sì, poiché somma di f.c. in $[1,e]$
2) $f(x)$ è derivabile in $]1,e[$? " " f.d. in $]1,e[$

$\exists\, c \in]1,e[$ t.c. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\dfrac{f(b)-f(a)}{b-a} = \dfrac{f(e)-f(1)}{e-1} = \dfrac{e+1-1}{e-1} = \dfrac{e}{e-1}$

$f'(x) = 1 + \dfrac{1}{x}$

$1 + \dfrac{1}{x} = \dfrac{e}{e-1}$

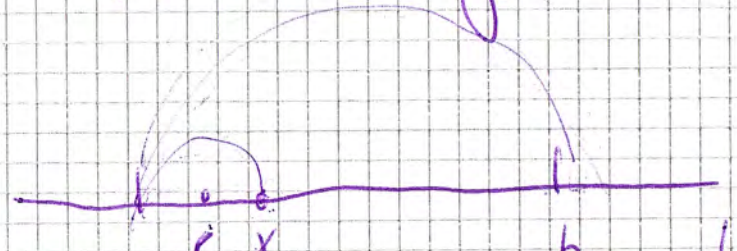
$\dfrac{1}{x} = \dfrac{e}{e-1} - 1 = \dfrac{e-e+1}{e-1} = \dfrac{1}{e-1}$

$x = e-1$

$c = e-1 \cong 1{,}71$

I° Corollario di Lagrange Sia $f(x)$ a) e b). Inoltre sia $f'(x)=0 \quad \forall x \in]a,b[$. Allora $f(x)$ è costante in $[a,b]$.

Dim.:



Applico Lagrange alle $f(x)$ in $[a,x]$ Lo posso fare!!!

$$\exists\, c \in]a,x[\text{ t.c. } f'(c) = \frac{f(x)-f(a)}{x-a} \Rightarrow f(x)-f(a)=0 \Rightarrow f(x)=f(a). \quad (A)$$

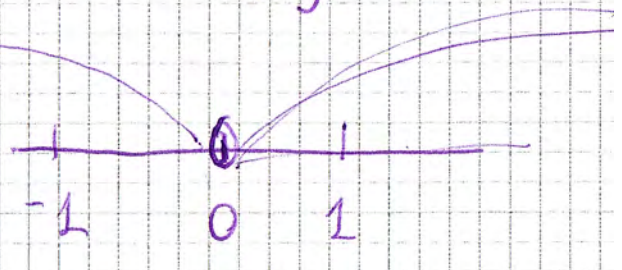
$f'(c)=0$

$\Rightarrow f(x)$ è costante.

$f(x)=\sin^2 x+\cos^2 x$ $\quad f'(x)=2\sin x\cdot\cos x+2\cos x\cdot(-\sin x)=0$

$D=\mathbb{R}$ $\quad f(0)=\sin^2 0+\cos^2 0=0+1=1$ $\quad \sin^2 x+\cos^2 x=1$

$f(x) = \operatorname{arctg} x + \operatorname{arctg} \frac{1}{x}$ $\quad f'(x) = 0$ $\quad D = \mathbb{R} \setminus \{0\}$



$f(1) = \operatorname{arctg} 1 + \operatorname{arctg} 1 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$f(-1) = \operatorname{arctg}(-1) + \operatorname{arctg}(-1) = -\frac{\pi}{4} - \frac{\pi}{4} = -\frac{\pi}{2}$

$f(\sqrt{3}) = \operatorname{arctg} \sqrt{3} + \operatorname{arctg} \frac{1}{\sqrt{3}}$

$= \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

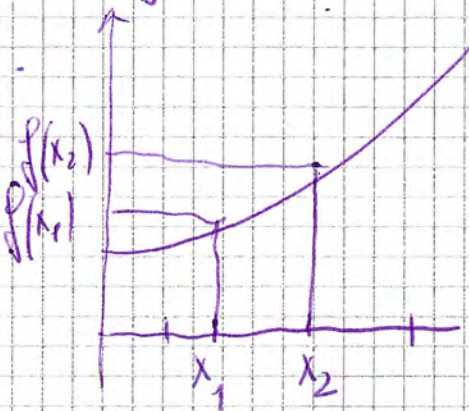
II° Corollario di Lagrange. Siano $f(x)$ e $g(x)$ a) e b).

Supponiamo poi che $f'(x)=g'(x)$ in $]a,b[$. Allora $f(x)-g(x)=k$ in $[a,b]$.

Dim: Sia $F(x)=f(x)-g(x)$. $F(x)$ verifica a) e b). $F'(x)=f'(x)-g'(x)=0$.

Quindi per il I° Corollario $F(x)=k$ in $[a,b] \Rightarrow f(x)-g(x)=k$ in $[a,b]$.

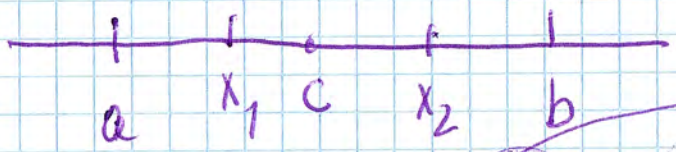
III° Corollario di Lagrange. Sia $f(x)$ a) e b). Sia $f'(x)>0$ ($f'(x)<0$) in $]a,b[$. Allora $f(x)$ è crescente in $[a,b]$ [decrescente]



$x_1<x_2 \Rightarrow f(x_1)<f(x_2)$ crescente

$x_1<x_2 \Rightarrow f(x_1)>f(x_2)$ decrescente

Dim.:



Applico Lagrange alla $f(x)$ in $[x_1, x_2]$

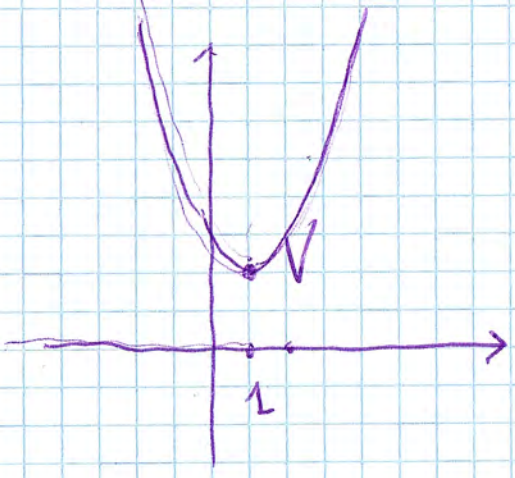
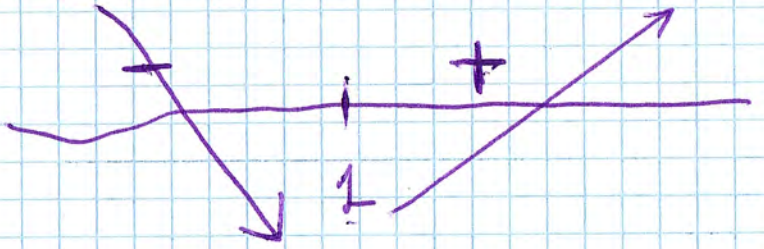
$\exists c \in]x_1, x_2[$ t.c. $f'(c) = \dfrac{f(x_2)-f(x_1)}{x_2-x_1}$ (+)

(+) $\rightarrow$ $f(x_2)-f(x_1) > 0$

$f(x_1) < f(x_2)$

$f(x) = x^2 - 2x + 3$

$V(1, 2)$



$f'(x) = 2x - 2 > 0 \quad x > 1$