14-11-2018

$$f(x) = \ln\left(\sqrt{1+e^x} - 1\right) - \ln\left(\sqrt{1+e^x} + 1\right)$$

$$f'(x) = \left(\ln\left(\sqrt{1+e^x} - 1\right)\right)' - \left(\ln\left(\sqrt{1+e^x} + 1\right)\right)' = \frac{1}{\sqrt{1+e^x} - 1}\left(\sqrt{1+e^x} - 1\right)' - \frac{1}{\sqrt{1+e^x} + 1}\cdot\left(\sqrt{1+e^x} + 1\right)'$$

$$= \frac{1}{\sqrt{1+e^x} - 1}\cdot\left(\sqrt{1+e^x}\right)' - \frac{1}{\sqrt{1+e^x} + 1}\cdot\left(\sqrt{1+e^x}\right)' = \frac{1}{\sqrt{1+e^x} - 1}\cdot\frac{1}{2\sqrt{1+e^x}}\cdot\left(1+e^x\right)' - \frac{1}{\sqrt{1+e^x} + 1}\cdot\frac{1}{2\sqrt{1+e^x}}\cdot\left(1+e^x\right)'$$

$$= \frac{1}{\sqrt{1+e^x} - 1}\cdot\frac{1}{2\sqrt{1+e^x}}\cdot e^x - \frac{1}{\sqrt{1+e^x} + 1}\cdot\frac{1}{2\sqrt{1+e^x}}\cdot e^x = \frac{e^x}{2\sqrt{1+e^x}}\left(\frac{1}{\sqrt{1+e^x} - 1} - \frac{1}{\sqrt{1+e^x} + 1}\right)$$

$$= \frac{e^x}{2\sqrt{1+e^x}}\cdot\frac{\sqrt{1+e^x} + 1 - \sqrt{1+e^x} + 1}{\left(\sqrt{1+e^x} - 1\right)\left(\sqrt{1+e^x} + 1\right)} = \frac{e^x}{2\sqrt{1+e^x}}\cdot\frac{2}{1+e^x - 1} = \frac{e^x}{2\sqrt{1+e^x}}\cdot\frac{2}{e^x} = \frac{1}{\sqrt{1+e^x}}$$

$$f(x)=\frac{1}{15}\cos^3 x\left(3\cos^2 x-5\right)$$

$$f'(x)=\left(\frac{1}{15}\cos^3 x\right)'\left(3\cos^2 x-5\right)+\frac{1}{15}\cos^3 x\cdot\left(3\cos^2 x-5\right)'$$

$$=\frac{1}{15}\left(\cos^3 x\right)'\left(3\cos^2 x-5\right)+\frac{1}{15}\cos^3 x\cdot\left(3\cos^2 x\right)'=$$

$$=\frac{1}{\cancel{15}_5}\cdot\cancel{3}\cos^2 x\left(-\sin x\right)\left(3\cos^2 x-5\right)+\frac{1}{\cancel{15}_5}\cos^3 x\cdot\cancel{3}\cdot 2\cos x\left(-\sin x\right)$$

$$=-\frac{1}{5}\cos^2 x\sin x\left(3\cos^2 x-5\right)-\frac{2}{5}\cos^4 x\sin x=-\frac{3}{5}\cos^4 x\sin x+\cos^2 x\sin x$$

$$-\frac{2}{5}\cos^4 x\sin x=-\cos^4 x\sin x+\cos^2 x\sin x=\cos^2 x\sin x\left(\underbrace{1-\cos^2 x}_{\sin^2 x}\right)$$

$$=\cos^2 x\sin^3 x$$

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$$f(x)=\frac{1}{15}\cos^3 x\left(3\cos^2 x-5\right)=\frac{1}{5}\cos^5 x-\frac{1}{3}\cos^3 x$$

$$f'(x)=\frac{1}{5}\cdot 5\cos^4 x\left(-\sin x\right)-\frac{1}{3}\cdot 3\cos^2 x\left(-\sin x\right)$$

$$=\left(-\cos^4 x\sin x+\cos^2 x\sin x=\ldots\right.$$

$$\frac{x}{2}=\frac{1}{2}x$$

$$\left(\frac{f(x)}{u}\right)'=\frac{f'(x)}{u}$$

$$f(x)=\frac{x}{2}\sqrt{2+x^2}+\ln\left(x+\sqrt{2+x^2}\right)$$

$$f'(x)=\sqrt{2+x^2}$$

$$f(x)=2\ln\left|\frac{x+3}{x+2}\right|-\frac{1}{x+2}-\frac{1}{x+3}$$

$$f'(x)=2\cdot\frac{1}{\frac{x+3}{x+2}}\cdot\left(\frac{x+3}{x+2}\right)'-\frac{1\cdot(x+2)-1\cdot(x+2)'}{(x+2)^2}-\frac{1\cdot(x+3)-1\cdot(x+3)'}{(x+3)^2}$$

$$=2\cdot\frac{x+2}{x+3}\cdot\frac{(x+3)'(x+2)-(x+3)(x+2)'}{(x+2)^2}-\frac{-1}{|x+2|^2}-\frac{-1}{|x+3|^2}$$

$$=2\cdot\frac{x+2-x-3}{(x+3)(x+2)}+\frac{1}{(x+2)^2}+\frac{1}{(x+3)^2}$$

$$\frac{-2}{(x+3)(x+2)}+\frac{1}{(x+2)^2}+\frac{1}{(x+3)^2}=\frac{-2(x+3)(x+2)+(x+3)^2+(x+2)^2}{(x+2)^2(x+3)^2}=$$

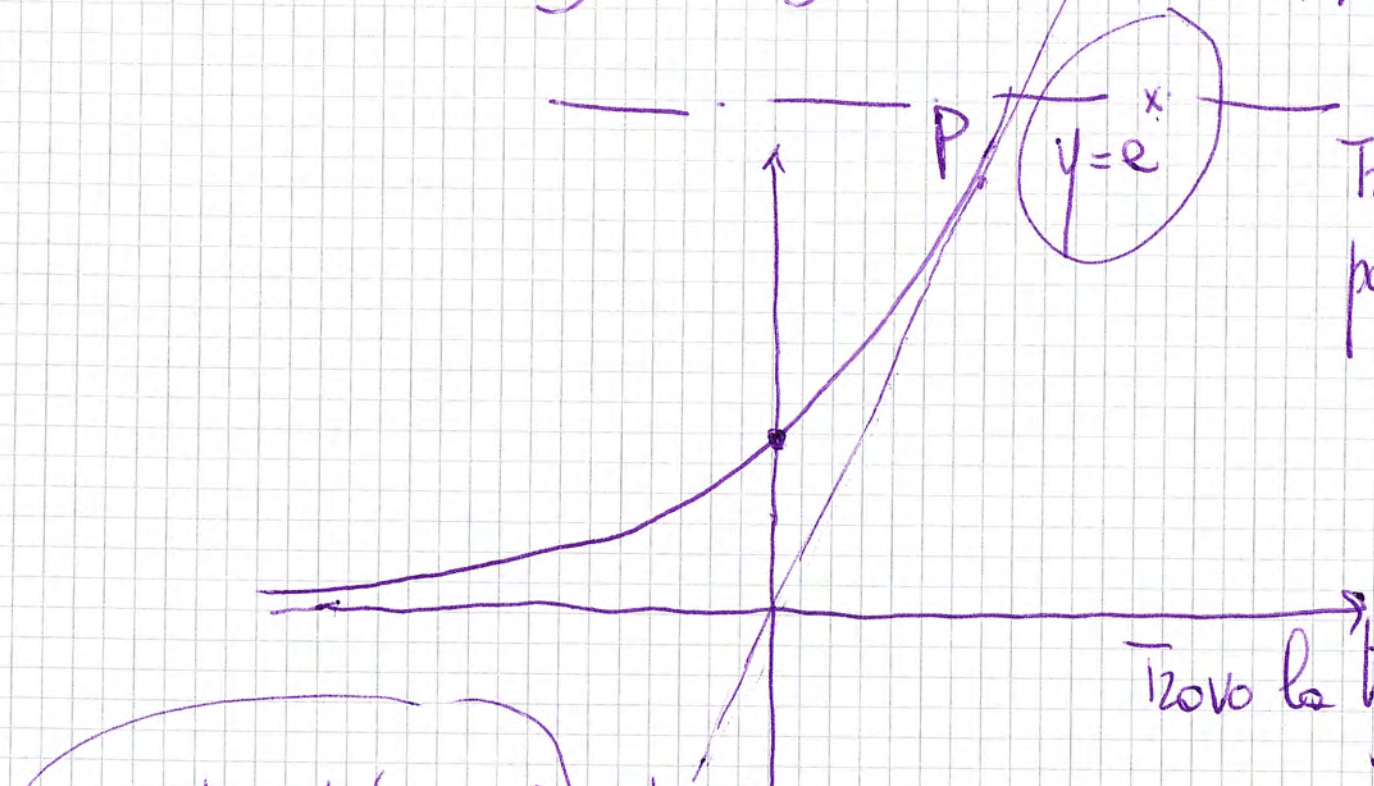
$$=\frac{-2x^2-4x-6x-12+x^2+9+6x+x^2+4+4x}{(x+2)^2(x+3)^2}=\frac{1}{(x+2)^2(x+3)^2}$$

$$\frac{1}{(x+2)^2(x+3)^2}$$

$$\left(\ln|f(x)|\right)'=\begin{cases}\ln f(x) & \text{если } f(x)>0\\ \ln(-f(x)) & \text{если } f(x)<0\end{cases}$$

$$\left(\ln|f(x)|\right)'=\begin{cases}\frac{1}{f(x)}f'(x)=\frac{f'(x)}{f(x)}\\ \frac{1}{-f(x)}(-f'(x))=\frac{f'(x)}{f(x)}\end{cases}$$

$$f(x) = \frac{tg^3 x}{3} + \frac{tg^5 x}{5} \qquad \left[\frac{sen^2 x}{cos^6 x}\right]$$



Trovare la retta tg alla curva passante per l'origine.

$P(t, e^t)$

Trovo la tg. in P $\quad y' = e^x$

$y'(t) = e^t$

$y - e^t = e^t(x - t)$ è tg alla curva in P

$0 - e^t = e^t(0 - t) \quad + e^t = + te^t \quad e^t(1 - t) = 0$

$\neq 0 \quad \neq 0$

$t = 1 \qquad P(1, e)$

REGOLA DI DERIVAZIONE DELLE FUNZIONI INVERSE

$y = f(x)$ derivabile e invertibile con inversa $x = g(y)$

$$g'(y_0) = \frac{1}{f'(x_0)} \quad \text{dove} \quad f(x_0) = y_0 \quad \left(f'(x_0) \neq 0\right)$$

$y = e^x \qquad x = \log y \qquad (\log y)' = \dfrac{1}{e^x} = \dfrac{1}{y}$

$y = \log x \qquad x = e^y \qquad (e^y)' = \dfrac{1}{\frac{1}{x}} = x = e^y$

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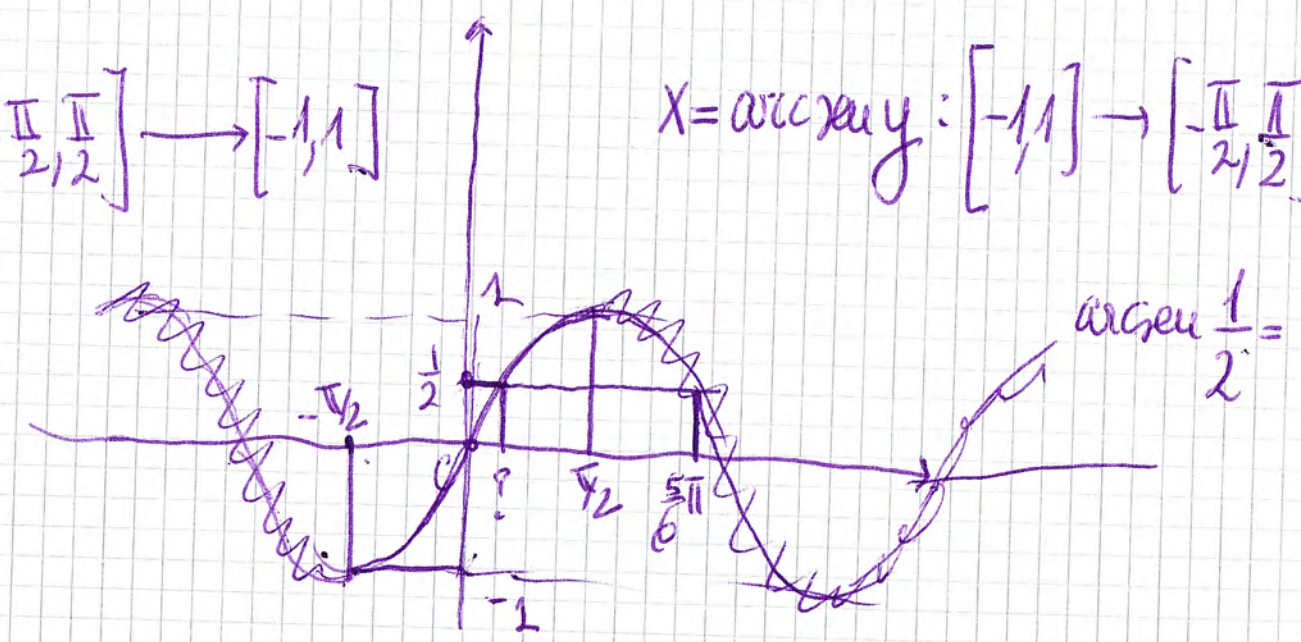
$y = \sin x : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$

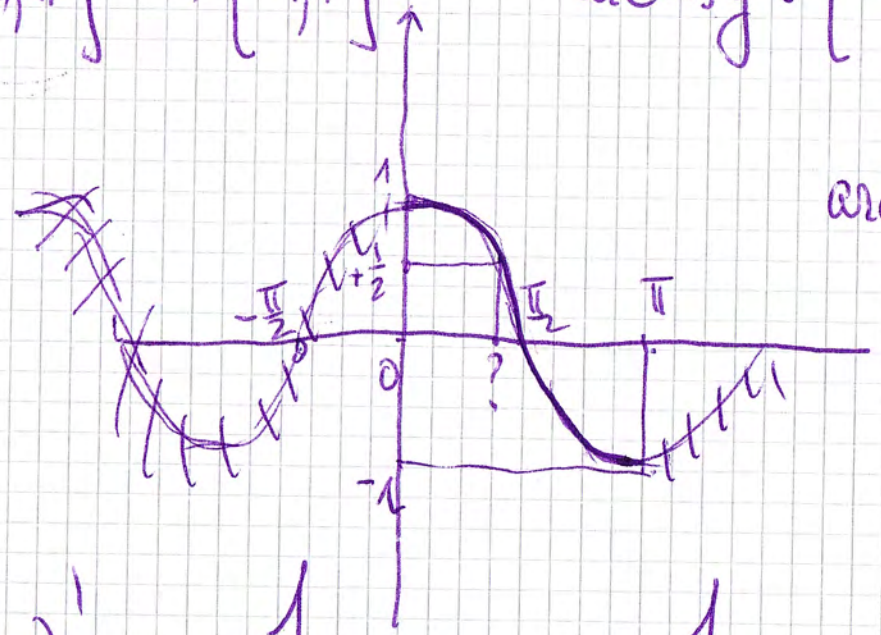
$x = \arcsin y : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$$(\arcsin y)' = \frac{1}{\cos x} = \frac{1}{\pm\sqrt{1-\sin^2 x}} = \frac{1}{\sqrt{1-y^2}}$$

$\cos x$: $+$

$y = \cos x : [0, \pi] \to [-1, 1]$ $\qquad \arccos y : [-1, 1] \to [0, \pi]$



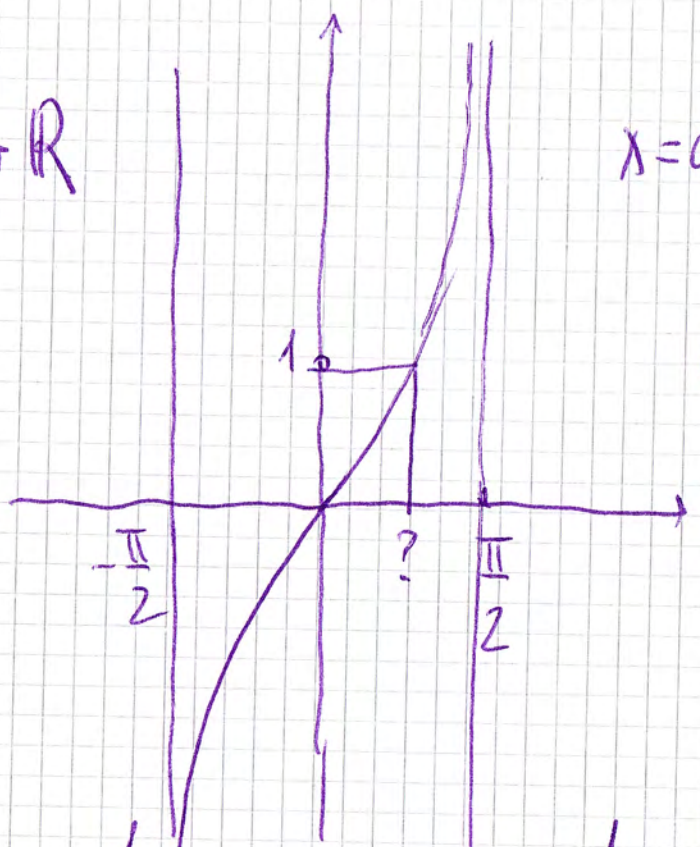
$\arccos \frac{1}{2} = \frac{\pi}{3}$

$$(\arccos y)' = \frac{1}{-\sin x} = -\frac{1}{\pm\sqrt{1-\cos^2 x}} = -\frac{1}{\sqrt{1-y^2}}$$

$y = \operatorname{tg} x : \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\longrightarrow \mathbb{R}$

$x = \operatorname{arctg} y : \mathbb{R} \longrightarrow \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

$\operatorname{arctg} 1 = \frac{\pi}{4}$



$$(\operatorname{arctg} y)' = \frac{1}{1+\operatorname{tg}^2 x} = \frac{1}{1+y^2}$$

1) $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

2) $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$

3) $(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$

$$f(x)^{g(x)} = e^{\ln f(x)^{g(x)}} = e^{g(x)\ln f(x)}$$

$$y = x^x = e^{\ln x^x} = e^{x\ln x}$$

$$y' = e^{x\ln x}\cdot\left(1\cdot\ln x + x\cdot\frac{1}{x}\right) = x^x(\ln x + 1)$$

$$y = \operatorname{arctg} x + \operatorname{arctg} \frac{1}{x}$$

$$\left(x^{-1}\right)' = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

$$y' = \left(\operatorname{arctg} x\right)' + \left(\operatorname{arctg} \frac{1}{x}\right)' = \frac{1}{1+x^2} + \frac{1}{1+\frac{1}{x^2}} \cdot \left(-\frac{1}{x^2}\right) =$$

$$= \frac{1}{1+x^2} - \frac{1}{x^2+1} = 0$$