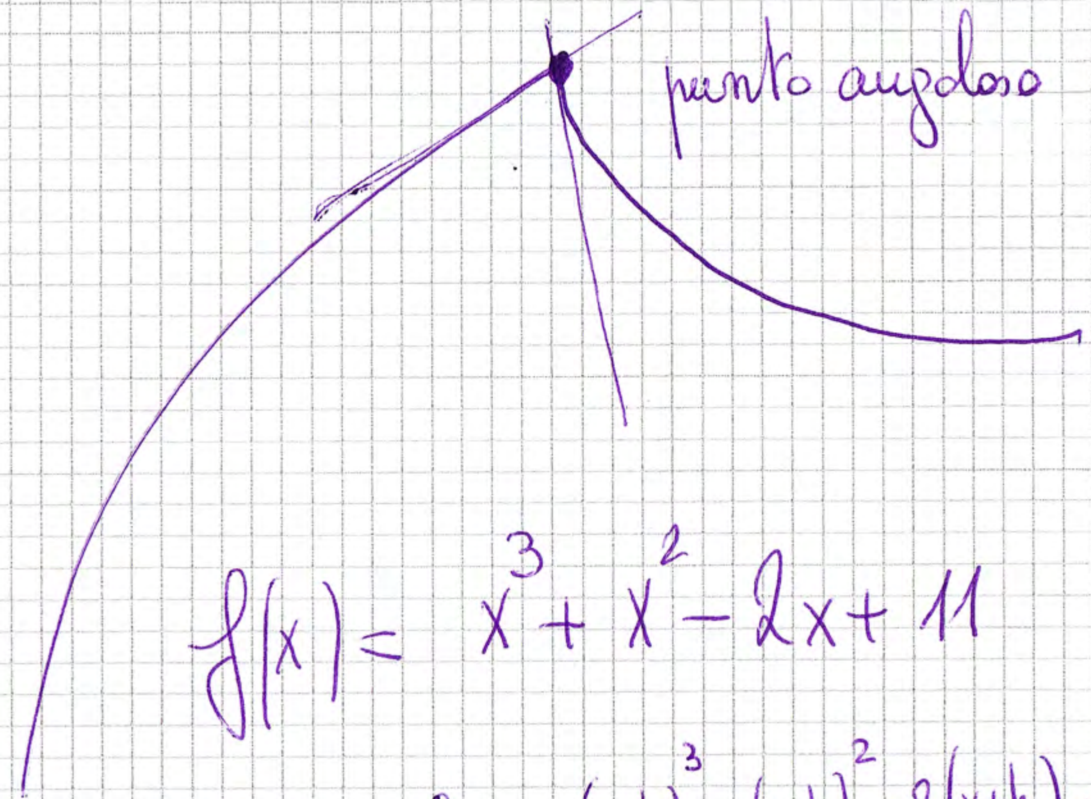

183

$$f(x) = x^3 + x^2 - 2x + 11$$

$$\lim_{h\to0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to0}\frac{(x+h)^3+(x+h)^2-2(x+h)+\cancel{11}-x^3-x^2+2x-\cancel{11}}{h}$$

$$= \lim_{h\to0}\left(\frac{(x+h)^3-x^3}{h}+\frac{(x+h)^2-x^2}{h}-\frac{2(x+h)-2x}{h}\right)$$

$$= \lim_{h\to0}\left(\frac{(x+h)^3-x^3}{h}\right)+\lim_{h\to0}\left(\frac{(x+h)^2-x^2}{h}\right)-2\lim_{h\to0}\left(\frac{(x+h)-x}{h}\right) = \underline{3x^2+2x-2}$$

12-11-2018

① Siano $f(x), g(x)$ funzioni derivabili in x. Considero

$F(x) = (f+g)(x)$

$$F(x) = (f+g)(x) \overset{def.}{=} f(x) + g(x)$$

$$\lim_{h\to 0} \frac{F(x+h)-F(x)}{h} = \lim_{h\to 0} \frac{(f+g)(x+h)-(f+g)(x)}{h} = \lim_{h\to 0} \frac{f(x+h)+g(x+h)-f(x)-g(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h} \right) = f'(x) + g'(x) \quad (f+g)' = f' + g'$$

② $(f-g)' = f' - g'$

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$f(x) = x \qquad f(x) = 1 \cdot x$

③ $F(x) = f(x)g(x) = (f \cdot g)(x)$

$$\lim_{h\to 0} \frac{F(x+h)-F(x)}{h} = \lim_{h\to 0} \frac{(fg)(x+h)-(fg)(x)}{h} = \lim_{h\to 0} \frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{\overbrace{f(x+h)g(x+h)-f(x)g(x+h)}+\overbrace{f(x)g(x+h)-f(x)g(x)}}{h} =$$

$$= \lim_{h\to 0} \left[\underbrace{\frac{f(x+h)-f(x)}{h}}_{f'(x)} \underbrace{g(x+h)}_{g(x)} \right] + \lim_{h\to 0} \left[\underbrace{f(x)}_{f(x)} \underbrace{\frac{g(x+h)-g(x)}{h}}_{g'(x)} \right] =$$

$$f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

(*) $g(x)$ è derivabile in x quindi è continua in x

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$(fg)' = f'g + fg'$

$(1 \cdot x)' = 1'x + 1 \cdot x'$
$= \cancel{0x} + 1 \cdot 1 = 1$

$(kf)' = \cancel{k'f} + kf' =$

$(2x^4)' = 2 \cdot 4x^3 = 8x^3$

$(fgh)' = ((fg)h)' = (fg)'h + (fg)h' = (f'g + fg')h + fgh' =$
$= f'gh + fg'h + fgh'$

$(fghk)' = f'ghk + fg'hk + fgh'k + fghk'$

$$\left[f(x)^{n}\right]' = \Big[\underbrace{f(x)f(x)\cdots\cdots f(x)}_{n\ \text{fattori}}\Big]' = f'(x)f(x)^{n-1} + f'(x)f(x)^{n-1} + \cdots + f'(x)f(x)^{n-1} = n f'(x)f(x)^{n-1}$$

$$\boxed{\left[f(x)^{n}\right]' = n f(x)^{n-1} f'(x)}$$

$$\left(\operatorname{sen}^3 x\right)' = \left((\operatorname{sen} x)^3\right)' = 3(\operatorname{sen} x)^2 \cos x$$

$$\left(\operatorname{sen}^2 x + \cos^2 x\right)' = \left(\operatorname{sen}^2 x\right)' + \left(\cos^2 x\right)' = 2\operatorname{sen} x \cdot \cos x + 2\cos x \cdot (-\operatorname{sen} x)$$
$$= 2\operatorname{sen} x \cos x - 2\operatorname{sen} x \cos x = 0$$

④ $F(x) = \left(\frac{f}{g}\right)(x) \overset{\text{def.}}{=} \frac{f(x)}{g(x)}$ $\qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

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$$(\text{tg}\,x)' = \left(\frac{\text{sen}\,x}{\cos x}\right)' = \frac{(\text{sen}\,x)'\cos x - \text{sen}\,x\cdot\boxed{(\cos x)'}}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \text{sen}^2 x}{\cos^2 x}$$

$$\boxed{\frac{1}{\cos^2 x}}$$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\text{sen}^2 x}{\cos^2 x} = \boxed{1 + \text{tg}^2 x}$$

$$(\text{cotg}\,x)' =$$

$$-\frac{1}{\text{sen}^2 x}$$

$$-(1 + \text{cotg}^2 x)$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x^2+1)' = 2x$$ 189

$$f(x) = \sqrt{x^2+1}$$

$$\lim_{h\to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h} = \lim_{h\to 0} \frac{\sqrt{(x+h)^2+1} - \sqrt{x^2+1}}{h} \cdot \frac{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}{\sqrt{(x+h)^2+1} + \sqrt{x^2+1}}$$

$$= \lim_{h\to 0} \frac{(x+h)^2+1-(x^2+1)}{h\left[\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right]} = \lim_{h\to 0} \frac{\cancel{x^2}+h^2+2hx+\cancel{1}-\cancel{x^2}-\cancel{1}}{h\left[\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right]} =$$

$$\lim_{h\to 0} \frac{\cancel{h}(h+2x)}{\cancel{h}\left[\sqrt{(x+h)^2+1} + \sqrt{x^2+1}\right]} = \frac{2x}{\sqrt{x^2+1} + \sqrt{x^2+1}} = \boxed{\frac{2x}{2\sqrt{x^2+1}}} = \frac{x}{\sqrt{x^2+1}}$$

$$\frac{1}{2\sqrt{x^2+1}} \cdot 2x$$

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$y = \lg \sin x \qquad y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \operatorname{cotg} x$

$y = \sin \sqrt{\lg \cos x} \qquad y' = \cos \sqrt{\lg \cos x} \cdot \frac{1}{2\sqrt{\lg \cos x}} \cdot \frac{1}{\cos x} (-\sin x)$

$y = \lg \left(x + \sqrt{x^2+1}\right) \qquad y' = \frac{1}{x+\sqrt{x^2+1}} \left(x + \sqrt{x^2+1}\right)' =$

$= \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \left(\sqrt{x^2+1}\right)'\right) = \frac{1}{x+\sqrt{x^2+1}} \left(1 + \frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x\right)$

$= \frac{1}{x+\sqrt{x^2+1}} \left(1 + \frac{x}{\sqrt{x^2+1}}\right) = \frac{1}{\cancel{x+\sqrt{x^2+1}}} \cdot \frac{\cancel{\sqrt{x^2+1} + x}}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$

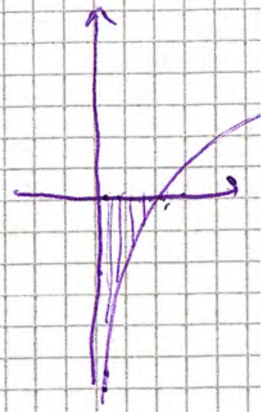
$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, +\infty - \infty$ $1^{\infty}, 0^{0}, \infty^{0}$

(19)

$x \longrightarrow a^{x} \longrightarrow \log_a a^{x} = x$

$x > 0 \longrightarrow \log_a x \longrightarrow a^{\log_a x} = x$

$$\boxed{a^{\log_a f(x)} = f(x)}$$



$f(x)^{g(x)} = a^{\log_a f(x)^{g(x)}} = a^{g(x)\log_a f(x)}$

$\left(1+\frac{1}{x}\right)^{x}$

$\overset{\infty}{g(x)} \log_e \overset{\to 1}{f(x)} = \infty \cdot 0$

$\overset{0}{g(x)} \log \overset{\to \infty}{f(x)} \to +\infty$ $0 \cdot \infty$

$\overset{0}{g(x)} \log \overset{\to 0^{+}}{f(x)} \to -\infty = 0 \cdot \infty$

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$$(x^{\sqrt{2}})' = \sqrt{2}\, x^{\sqrt{2}-1}$$

$$y = x^{m} \qquad y' = m x^{m-1} \qquad m \in \mathbb{N}$$

$$y = \sqrt[n]{x} \qquad y' = \frac{1}{n\sqrt[n]{x^{n-1}}} = \frac{1}{n} x^{\frac{1}{n}-1}$$

$$y = x^{\frac{1}{n}}$$

$$y = x^{3/2} \qquad y' = \frac{3}{2} x^{\frac{3}{2}-1} = \frac{3}{2} x^{\frac{1}{2}}$$

$$y = x^{3/2} = \sqrt{x^{3}}$$

$$y' = \frac{1}{\boxed{2}\sqrt{x^{3}}} \cdot \boxed{3} x^{2} = \frac{3}{2} \cdot \frac{x^{2}}{x^{3/2}} = \frac{3}{2} x^{1/2}$$

$$y = x^{\alpha} = e^{\lg x^{\boxed{\alpha}}} = \boxed{e^{\alpha \lg x}}$$

$$y' = \boxed{e^{\alpha \lg x}} \cdot \alpha \cdot \frac{1}{x} = \boxed{x^{\alpha}} \cdot \alpha \cdot \frac{1}{\boxed{x}} = \alpha x^{\alpha-1}$$

$$(x^{\alpha})' = \alpha x^{\alpha-1}$$