7-11-2018

FUNZIONE DERIVATA

La funzione derivata di $f(x)$ (indicata con $f'(x)$) è quella funzione il cui valore in ogni punto è dato dalla derivata di $f(x)$ calcolata in tale punto.

$$f(x) = x^2 \longrightarrow f'(x) = 2x$$

1) $f(x) = K \longrightarrow f'(x) = 0$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{K - K}{h} = \lim_{h\to 0} \frac{0}{h}$$

$$= \lim_{h\to 0} 0 = 0$$

2) $f(x) = x \longrightarrow f'(x) = 1$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{x+h-x}{h} = \lim_{h\to 0} \frac{h}{h} =$$

$$= \lim_{h\to 0} 1 = 1$$

3) $f(x) = x^2 \longrightarrow f'(x) = 2x$

4) $f(x) = x^3 \longrightarrow f'(x) = 3x^2$

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{x^3+h^3+3x^2h+3xh^2-x^3}{h}$$

$$= \lim_{h\to 0} \frac{h(h^2+3x^2+3xh)}{h} = 3x^2$$

5) $f(x) = x^n \qquad n \in \mathbb{N}$

$$\lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$P(a) = a^n - b^n$ è divisibile per $a-b$

$P(b) = b^n - b^n = 0$

	1	0	0	...	0	$-b^n$
b		b	b^2		b^{n-1}	b^n
	1	b	b^2		b^{n-1}	0

$$Q(a) = a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1}$$

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \dots + b^{n-1})$$

$$\frac{(x+h)^n - x^n}{h} = \frac{(x+h-x)\left[(x+h)^{n-1} + (x+h)^{n-2}x + (x+h)^{n-3}x^2 + \dots + x^{n-1}\right]}{h}$$

$$n-2+1$$

$$\lim_{h\to 0}\left[(x+h)^{n-1} + (x+h)^{n-2}x + \dots + (x+h)x^{n-2} + x^{n-1}\right] =$$

$$= x^{n-1} + x^{n-2}\cdot x^{1} + \dots + x^{1}\cdot x^{n-2} + x^{n-1} = n\,x^{n-1}$$

$$f(x) = x^n \longrightarrow f'(x) = n\,x^{n-1} \qquad n \in \mathbb{N}$$

6) $f(x) = \log_a x$

$$\lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h} = \lim_{h \to 0} \frac{\log_a \frac{x+h}{x}}{h} =$$

$$= \lim_{h \to 0} \frac{\log_a\left(1 + \frac{h}{x}\right)}{\frac{h}{x} \cdot x} = \frac{1}{x}\log_a e$$

$$\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \log_a e$$

$\log_a e$

$$f(x) = \log_a x \longrightarrow f'(x) = \frac{1}{x}\log_a e$$

$$f(x) = \log_e x = \ln x \longrightarrow f'(x) = \frac{1}{x}$$

7) $f(x) = a^x$

$$\lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x (a^h - 1)}{h}$$

$$= a^x \lg a$$

$$f(x) = a^x \longrightarrow f'(x) = a^x \lg a$$

$$f(x) = e^x \longrightarrow f'(x) = e^x$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \lg a$$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

8) $f(x)=\sin x$

$$\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=$$

$$\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}=\lim_{h\to 0}\frac{\sin x(\cos h-1)+\cos x\sin h}{h}$$

$$\lim_{h\to 0}\left(\sin x\cdot\frac{\cos h-1}{h^2}\cdot h+\cos x\cdot\frac{\sin h}{h}\right)=\cos x \qquad \lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$\sin x\cdot\frac{\cos h-1}{h^2}\cdot h$: $\sin x\to 0$, $\frac{\cos h-1}{h^2}\to -\frac{1}{2}$, $h\to 0$

$\cos x\cdot\frac{\sin h}{h}$: $\cos x\to 1$, $\frac{\sin h}{h}\to 1$

$0+\cos x$

$f'(x)=\cos x$

9) $f(x)=\cos x$

$f(x)=-\sin x$

10

$$\phi(x) = \sqrt[n]{x}$$

$$\phi'(x) = \frac{1}{n \sqrt[n]{x^{n-1}}}$$

$$\phi(x) = \sqrt{x} \rightarrow \phi'(x) = \frac{1}{2\sqrt{x}}$$

$$\phi(x) = \sqrt[3]{x^2} \rightarrow \phi'(x) = \frac{1}{3\sqrt[3]{x}}$$

$$\phi(x) = \sqrt[n]{x^m} = x^{\frac{m}{n}}$$

$$\phi'(x) = \frac{m}{n} x^{\frac{m}{n}-1} = \frac{m}{n} x^{\frac{m-n}{n}} = \frac{m}{n \sqrt[n]{x^{m-n}}} = \frac{m}{n \sqrt[n]{x^m} \cdot \sqrt[n]{x^{-n}}} = \frac{m \sqrt[n]{x^m}}{n x}$$

TEOREMA $f(x)$ derivabile in $x_0 \Rightarrow f(x)$ continua in x_0.

Dim.: **Hp** $\lim_{h\to 0} \dfrac{f(x_0+h)-f(x_0)}{h} = f'(x_0)$ (esiste ed è finito)

Tesi $\lim_{x\to x_0} f(x) = f(x_0)$

$h = x - x_0$

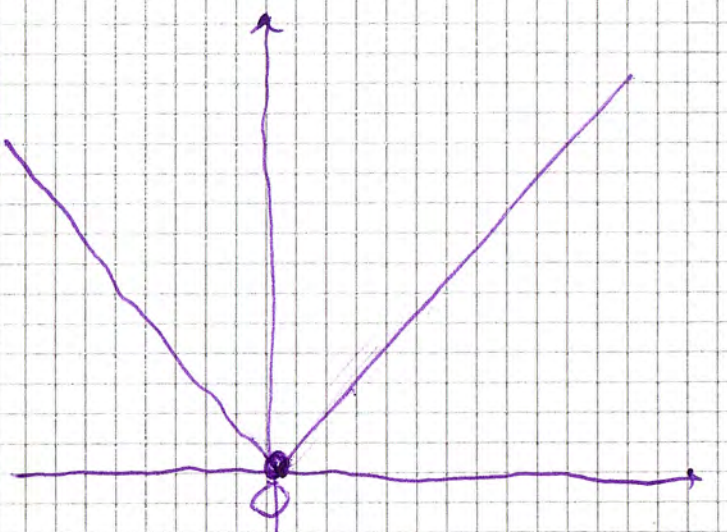
$\lim_{h\to 0} f(x_0+h) = f(x_0)$

$$f(x_0+h) = \frac{f(x_0+h)-f(x_0)}{h}\cdot h + f(x_0)$$

$h\to 0$ ↓ $\quad$ $h\to 0$ ↓ $\quad$ $h\to 0$ ↓

$$f(x_0) = f'(x_0)\cdot 0 + f(x_0)$$

$f(x) = |x|$



① continua in 0

$$\lim_{x \to x_0} f(x) = f(x_0) \to \lim_{x \to 0} |x| = 0 \qquad \left| |x| - 0 \right| < \varepsilon$$

$$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} \to \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|h| - 0}{h} = \lim_{h \to 0} \frac{|h|}{h} =$$

non esiste

$|x| < \varepsilon \quad (-\varepsilon < x < \varepsilon)$

$$\lim_{h \to 0^+} \frac{h}{h} = \lim_{h \to 0^+} 1 = 1$$

$$\lim_{h \to 0^-} \frac{-h}{h} = \lim_{h \to 0^-} (-1) = -1$$