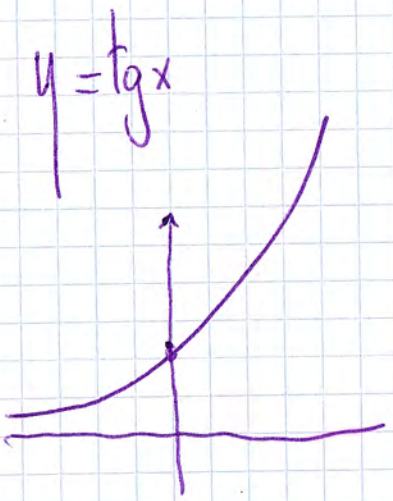
5-10-2018

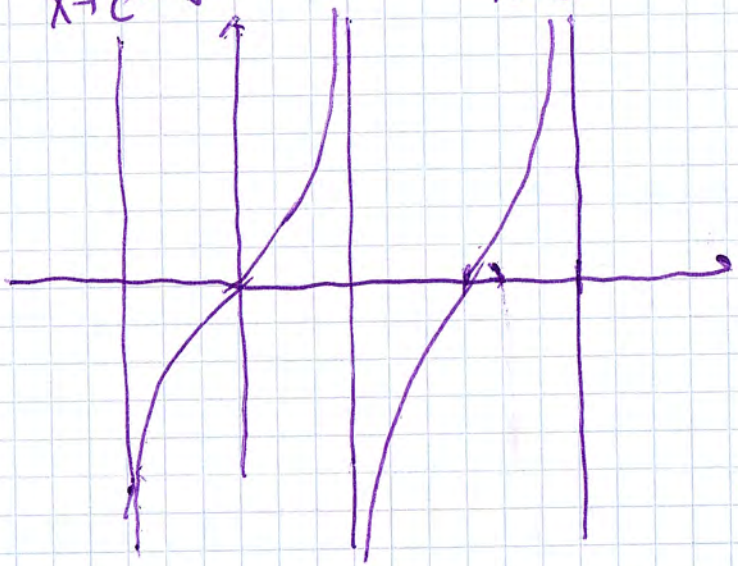
ASINTOTI

1) VERTICALI

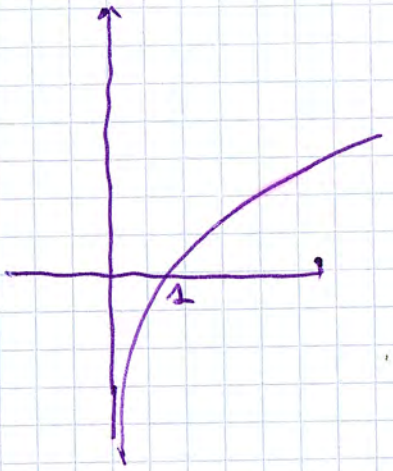
$x = c$ è asintoto verticale per $y = f(x)$ se

$\lim_{x \to c^+} f(x)$ v $\lim_{x \to c^-} f(x)$ = uno infiniti





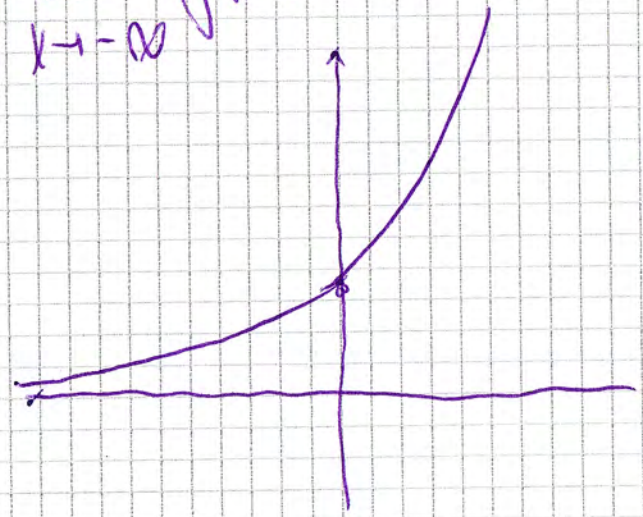
$x = \frac{\pi}{2} + k\pi$ as. vert.



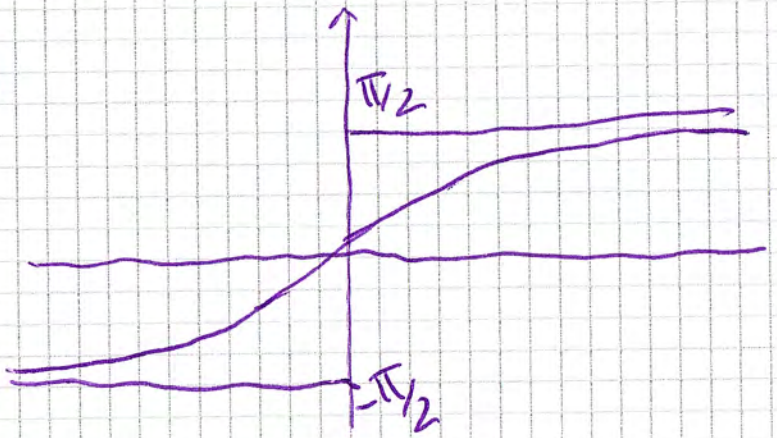
2) ORIZZONTALI

se $\lim_{x \to +\infty} f(x) = l$ allora $y = l$ asintoto orizzontale

se $\lim_{x \to -\infty} f(x) = m$ " $y = m$ " "



$\lim_{x \to -\infty} a^x = 0$ $y = 0$ as. oriz. sinistro



$\lim_{x \to +\infty} \operatorname{arctg} x = \frac{\pi}{2}$

$\lim_{x \to -\infty} \operatorname{arctg} x = -\frac{\pi}{2}$

3) OBLIQUI $\quad y = mx + q \quad m \neq 0$

$$m = \lim_{x \to \pm\infty} \frac{f(x)}{x} \qquad q = \lim_{x \to \pm\infty} \left(f(x) - mx \right)$$

$$y = \frac{P(x)}{H(x)} \qquad D = \mathbb{R} \setminus \{\text{zeri di } H\} \qquad c \text{ zero di } H \quad H(c) = 0$$

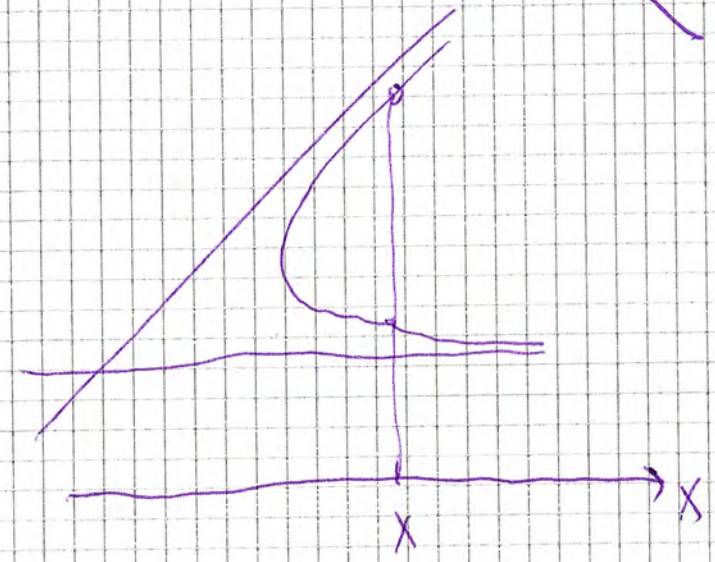
$$\lim_{x \to c} \frac{P(x)}{H(x)} = \frac{P(c) \neq 0}{H(c)} = \frac{l \neq 0}{0} = \infty$$

$$\lim_{x \to \pm\infty} \frac{P(x)}{H(x)} = \begin{cases} \infty \\ 0 \\ \frac{a}{b} \neq 0 \end{cases}$$

grado P > grado H

grado P < grado H $y = 0$ as. oriz.

grado P = grado H $y = \frac{a}{b}$ as. oriz.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\lim_{x\to\pm\infty} \frac{\frac{P(x)}{H(x)}}{x} = \lim_{x\to\pm\infty} \frac{P(x)}{x\,H(x)}$$

$y = \frac{P(x)}{H(x)}$ he un asintoto obliquo solo se $\text{grado } P(x) = \text{grado } H + 1$

$$y = \frac{x^3+2x+1}{2x^2-x-2}$$

$$m = \lim_{x\to\pm\infty} \frac{\frac{x^3+2x+1}{2x^2-x-2}}{x} = \lim_{x\to\pm\infty} \frac{x^3+2x+1}{2x^3-x^2-2x} = \frac{1}{2}$$

m

$$q = \lim_{x\to\pm\infty} \left(\frac{x^3+2x+1}{2x^2-x-2} - \frac{1}{2}x \right) = \lim_{x\to\pm\infty} \frac{2x^2+4x+2-2x^2+x^2+2x}{2(2x^2-x-2)}$$

$$= \lim_{x\to\pm\infty} \frac{x^2+6x+2}{4x^2-2x-4} = \frac{1}{4} = q$$

$y = \frac{1}{2}x + \frac{1}{4}$ as. obliquo

$$y = \left(\frac{x^2-1}{x^2+1}\right)^2$$

Dominio $\mathbb{R}$

Int. ass.

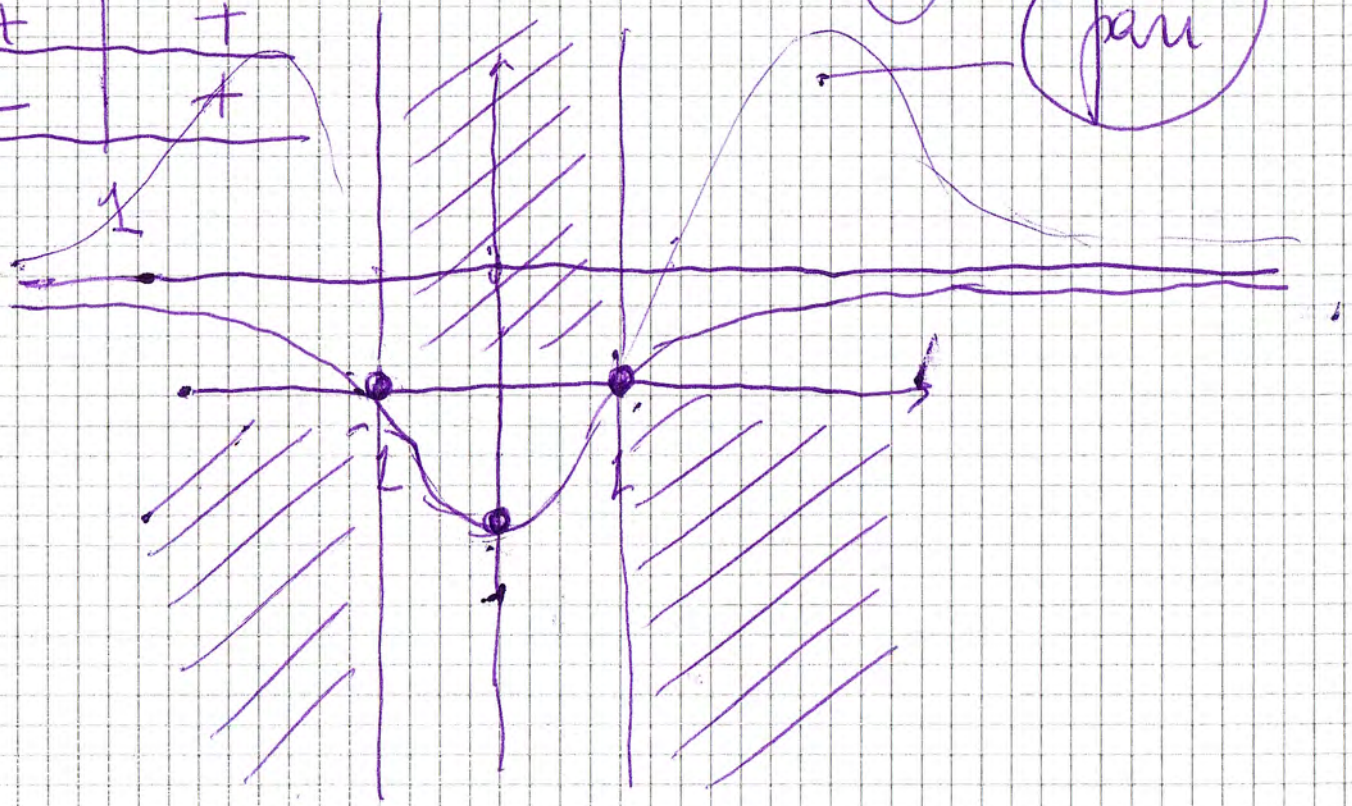
$\begin{cases} x = 0 \\ y = -1 \end{cases}$

$\begin{cases} y = 0 \\ \frac{x^2-1}{x^2+1} = 0 \rightarrow x^2 - 1 = 0 \\ x^2 = 1 \rightarrow x = \pm 1 \end{cases}$

$(1,0) \quad (-1,0)$

Segno

N		+		−		+	
D		+		+		+	
		+		−		+	
			−1		1		



$\lim_{x \to \pm\infty} f(x) = 1$

$y = 1$ as. oriz.

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$$\frac{x^2-1}{x^2+1} = 1$$

$$\frac{x^2-1}{x^2+1} = \frac{x+1}{x^2+1}$$

$$x^2-1 = x+1$$

$$x^2-x-2 = 0$$

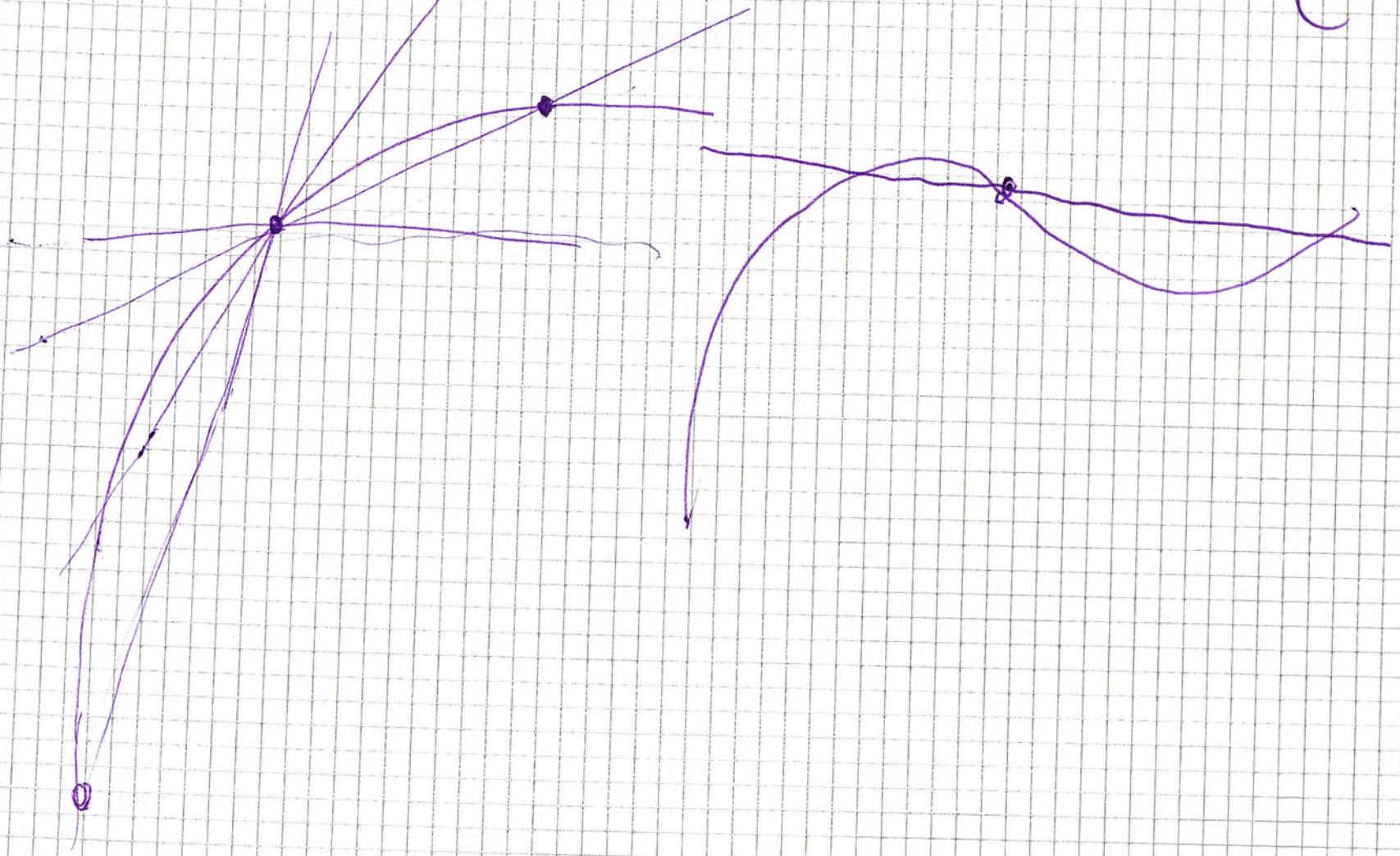
$$D = 1+8 = 9$$

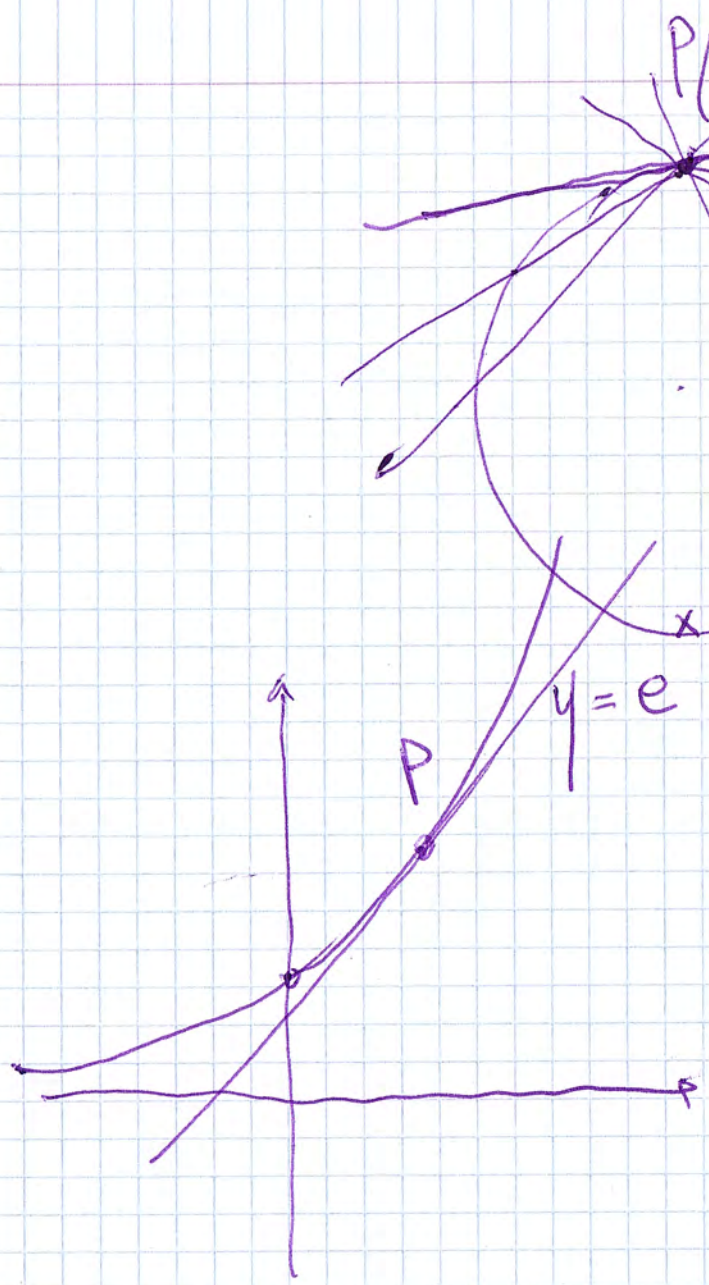
$$x_1 = \frac{1+3}{2} = 2$$

$$x_2 = \frac{1-3}{2} = -1$$

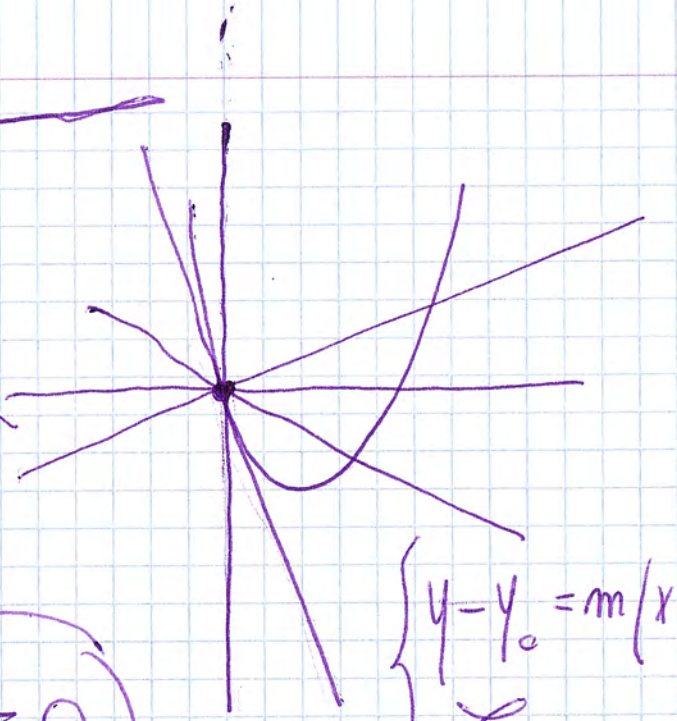
Imp.

DERIVATE





$\Delta = 0$



$$\begin{cases} y - y_0 = m(x - x_0) \\ e \end{cases}$$

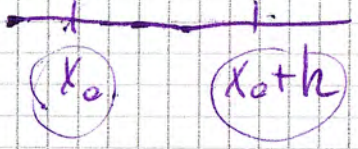
$$\begin{cases} y - y_0 = m(x - x_0) \\ y = e^x \end{cases}$$

$$e^x - y_0 = mx - mx_0$$

$y = f(x)$ definita in $]a,b[$ e sia $x_0 \in]a,b[$.

$h \in \mathbb{R}$

$$\frac{f(x_0+h) - f(x_0)}{h}$$ rapporto incrementale di $f(x)$



$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$ derivata della $f(x)$ in x_0

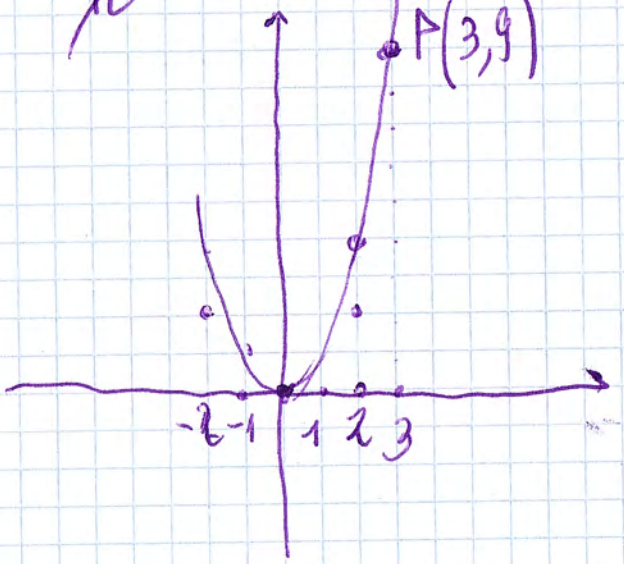
e si indica con $f'(x_0)$

se però tale limite esiste ed è finito

$y = x^2$ $\quad x_0 = 3$ $\quad f(x) = x^2$ $\quad$ 169

$$\lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{(3+h)^2 - 3^2}{h} = \lim_{h\to 0} \frac{9+h^2+6h-9}{h}$$

$$= \lim_{h\to 0} \frac{h(h+6)}{h} = 6 \qquad f'(3) = 6$$



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$P(3,9)$ $\begin{cases} y-9=m(x-3) \\ y=x^2 \end{cases}$ $x^2-9=mx-3m$

$x^2-mx+3m-9=0$ $\Delta=b^2-4ac=m^2-4(3m-9)=0$

$=m^2-12m+36=0$ $(m-6)^2=0$ $m=6$

y = f(x)
f(x0+h)
f(x0)
f(x0+h)-f(x0)
P
Q
h
α
x0
x0+h
m_PQ = (f(x0+h)-f(x0))/h = tan α
P(x0, f(x0))
Q(x0+h, f(x0+h))

$y = x^2 \qquad x_0 = 7$

$$\lim_{h\to 0} \frac{f(7+h)-f(7)}{h} = \lim_{h\to 0} \frac{(7+h)^2 - 7^2}{h} = \lim_{h\to 0} \frac{49+h^2+14h-49}{h}$$

$$= \lim_{h\to 0} \frac{h(h+14)}{h} = (14) \qquad f'(7) = 14.$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+h^2+2hx-x^2}{h}$$

$$= \lim_{h\to 0} \frac{h(h+2x)}{h} = 2x \qquad f'\left(\frac{17}{2}\right) = 2\cdot\frac{17}{2} = 17$$