31-10-2018

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x}=1$$

$$\frac{\operatorname{tg} x}{x}=\frac{\sin x}{\cos x}\cdot\frac{1}{x}=\underbrace{\frac{\sin x}{x}}_{\to 1}\cdot\underbrace{\frac{1}{\cos x}}_{\to 1}\to 1$$

$$\sin^2 x+\cos^2 x=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$$\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x}=\frac{1-\cos^2 x}{x^2(1+\cos x)}=\frac{\sin^2 x}{x^2(1+\cos x)}$$

$$=\underbrace{\frac{\sin x}{x}}_{\to 1}\cdot\underbrace{\frac{\sin x}{x}}_{\to 1}\cdot\underbrace{\frac{1}{1+\cos x}}_{\to \frac{1}{2}}\to\frac{1}{2}$$

147

$$\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}\frac{2\sin x\cos x}{x}=2$$

$$\left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot 2=2$$

$$\lim_{x\to 0}\frac{\sin 5x}{3x}=\lim_{x\to 0}\frac{\sin 5x}{5x}\cdot\frac{5}{3}=\frac{5}{3}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{\sin^2 4x}{1-\cos 2x}=\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{\sin 4x}{4x}\cdot\frac{4x\cdot 4x}{1-\cos 2x}$$

$$\lim_{x\to 0}\frac{16x^2}{1-\cos 2x}=\lim_{x\to 0}16x^2\cdot\frac{1}{\frac{1-\cos 2x}{4x^2}}\cdot\frac{1}{4x^2}=8$$

$$\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^{x} = e \cong 2{,}71\ldots$$

$g(x)$

$f(x)$

$f(1) > 0$

$1 + \frac{1}{x} > 0 \qquad \frac{x+1}{x} > 0$

N.	−	|	+	+
D.	−	|	=	+
	nutrum	|	=	+

$-1 \qquad 0$

$a \qquad b \qquad a, b \in \mathbb{R}$

$a > 0$

$(-2)^{1/2} = \not{\sqrt{-2}}$

1^{∞} , ∞^{0} , 0^{0}

$$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \log_a e$$

$\log x = \log_e x = \ln x$

$\log_2 x \quad \log_{10} x$

$$\frac{\log_a(1+x)}{x} = \frac{1}{x}\log_a(1+x) = \log_a(1+x)^{\frac{1}{x}}$$

pongo $\frac{1}{x} = t \to x = \frac{1}{t}$

se $x \to 0 \quad t \to \pm\infty$

$$\lim_{t\to\pm\infty} \log_a\left(1+\frac{1}{t}\right)^t = \log_a e$$

$\left(1+\frac{1}{t}\right)^t \to e$

$$\lim_{x\to 0} \frac{\log_e(1+x)}{x} = 1$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\frac{0}{0}$$

$$\log_a b=\frac{1}{\log_b a}$$

ISO

$a^x=t$

$\log_a a^x=\log_a t$

$x\log_a a=\log_a t$

$x=\log_a t$

$\frac{a^x-1}{x}$ $\quad a^x-1=t$ $\quad x=\log_a t$

$a^x=t+1$

$x=\log_a(t+1)$

$$\lim_{t\to 0}\frac{t}{\log_a(t+1)}=\frac{1}{\log_a e}=\log a$$

$$\boxed{\lim_{x\to 0}\frac{a^x-1}{x}=\log a}$$

$$\boxed{\lim_{x\to 0}\frac{e^x-1}{x}=1}$$

151

$$\lim_{x\to 0}\frac{e^{2x}-1}{3x}=\lim_{x\to 0}\frac{e^{2x}-1}{2x}\cdot\frac{2}{3}=\frac{2}{3}$$

$\frac{e^{2x}-1}{2x}\to 1$

$$\lim_{x\to+\infty} x\left(\log\left|\frac{x+1}{x-1}\right|\right)=\infty\cdot 0$$

$\log\left|\frac{x+1}{x-1}\right|\to 0$, $\frac{x+1}{x-1}\to 1$

$$\frac{x+1}{x-1}-1=\frac{x+1-x+1}{x-1}=\frac{2}{x-1}$$

$$\log\left|\frac{x+1}{x-1}\right|=\log\left(1+\left(\frac{x+1}{x-1}-1\right)\right)=\log\left(1+\frac{2}{x-1}\right)$$

$\frac{x+1}{x-1}-1\to 0$, $\frac{2}{x-1}\to 0$

$$x\log\left(\frac{x+1}{x-1}\right)=x\cdot\frac{\log\left(1+\frac{2}{x-1}\right)}{\frac{2}{x-1}}\cdot\frac{2}{x-1}=\frac{2x}{x-1}\xrightarrow{x\to+\infty}2$$

$\frac{\log\left(1+\frac{2}{x-1}\right)}{\frac{2}{x-1}}\to 1$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$$\lim_{x\to 0}\left(1+\frac{1}{x}\right)^x=e$$

$$\lim_{x\to 0}\frac{\log(1+x)}{x}=1$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1$$

$$\left.\begin{aligned}&\lim_{x\to 0}\frac{\sin f(x)}{f(x)}=1\\&\lim_{x\to 0}\frac{\operatorname{tg} f(x)}{f(x)}=1\\&\lim_{x\to 0}\frac{1-\cos f(x)}{f^2(x)}=\frac{1}{2}\end{aligned}\right\}\begin{aligned}&\text{se}\\&f(x)\to 0\\&\text{per } x\to 0\end{aligned}$$

$$\left.\begin{aligned}&\lim_{x\to 0}\frac{\log(1+f(x))}{f(x)}=1\\&\lim_{x\to 0}\frac{e^{f(x)}-1}{f(x)}=1\end{aligned}\right\}\begin{aligned}&f(x)\to 0\\&\text{per } x\to 0\end{aligned}$$

$$\lim_{x\to 0}\frac{(e^{3x}-1)\ln(1+5x)}{1-\cos 21x}=\frac{0}{0}$$

$$\frac{(e^{3x}-1)\ln(1+5x)}{1-\cos 21x}=\frac{\frac{e^{3x}-1}{3x}\cdot 3x\cdot\frac{\ln(1+5x)}{5x}\cdot 5x}{\frac{1-\cos 21x}{(21x)^2}\cdot(21x)^2}=$$

($\frac{e^{3x}-1}{3x}\to 1$, $\frac{\ln(1+5x)}{5x}\to 1$, $\frac{1-\cos 21x}{(21x)^2}\to\frac{1}{2}$)

$$=\frac{15x^2}{\frac{441}{2}x^2}\longrightarrow\frac{30}{441}=\frac{10}{147}$$

INFINITESIMI

Def. La funzione $f(x)$ si dice infinitesimo per $x \to c$ se $\lim_{x \to c} f(x) = 0$.

Def. Siano $f(x)$ e $g(x)$ infinitesimi per $x \to c$ $\left(\text{cioè } \lim_{x \to c} f(x) = 0 = \lim_{x \to c} g(x)\right)$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \begin{cases} 0 & f(x) \text{ si dice infinitesimo di ordine superiore a } g(x) \\ \infty & \text{" " " " inferiore " } \\ l \neq 0 & f(x) \text{ e } g(x) \text{ si dicono infinitesimi dello stesso ordine} \\ \text{se } l = 1 & f(x) \text{ e } g(x) \text{ si } \underline{\text{dicono infinitesimi equivalenti}} \\ & \text{e si scrive } f(x) \sim g(x) \end{cases}$$

Siano $f(x)$ e $g(x)$ infinitesimi equivalenti per $x \to c$, cioè

$$\lim_{x \to c} \frac{f(x)}{g(x)} = 1.$$

$$\lim_{x \to c} f(x)\left(\frac{g(x)}{f(x)}\right) = \lim_{x \to c} \left(\frac{f(x)}{g(x)}\right) g(x)\left(\frac{f(x)}{g(x)}\right) =$$

($\frac{f(x)}{g(x)} \to 1$)

$$= \lim_{x \to c} g(x)\left(\frac{f(x)}{g(x)}\right)$$

Se $f(x) \sim g(x)$ per $x \to c$ posso scambiare $f(x)$ con $g(x)$ in un limite ($x \to c$) se sono fattori

Per $x \to 0$ $\quad \sin x \sim x$

$\tg x \sim x$

$1 - \cos x \sim \frac{1}{2} x^2$

$\ln(1+x) \sim x$

$e^x - 1 \sim x$

Per $f(x) \to 0$ $\quad \sin f(x) \sim f(x)$

$\tg f(x) \sim f(x)$

$1 - \cos f(x) \sim \frac{1}{2} f^2(x)$

$\ln(1+f(x)) \sim f(x)$

$e^{f(x)} - 1 \sim f(x)$

$$\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}\frac{2x}{x}=2$$

$$\lim_{x\to 0}\frac{\sin 5x}{3x}=\lim_{x\to 0}\frac{5x}{3x}=\frac{5}{3}$$

$$\lim_{x\to 0}\frac{\sin^2 4x}{1-\cos 2x}=\lim_{x\to 0}\frac{(4x)^2}{\frac{1}{2}\cdot(2x)^2}=\lim_{x\to 0}\frac{16x^2}{\frac{1}{2}\cdot 4x^2}=8$$

$$\lim_{x\to 0}\frac{e^{2x}-1}{3x}=\lim_{x\to 0}\frac{2x}{3x}=\frac{2}{3}$$

$$\lim_{x\to+\infty} x\log\left(\frac{x+1}{x-1}\right)=\lim_{x\to+\infty} x\log\left(1+\frac{2}{x-1}\right)=\lim_{x\to+\infty} x\cdot\frac{2}{x-1}=\lim_{x\to+\infty}\frac{2x}{x-1}=2$$

$$\lim_{x \to c} f(x) = \ell$$

$$|f(x) - \ell| < \varepsilon \qquad (\varepsilon < 1)$$

tra le soluzioni $\exists H$ intorno di c

$$|f(x) - \ell| < 2$$

se $x \in H$ $\quad |f(x) - \ell| < \varepsilon < 1 < 2$