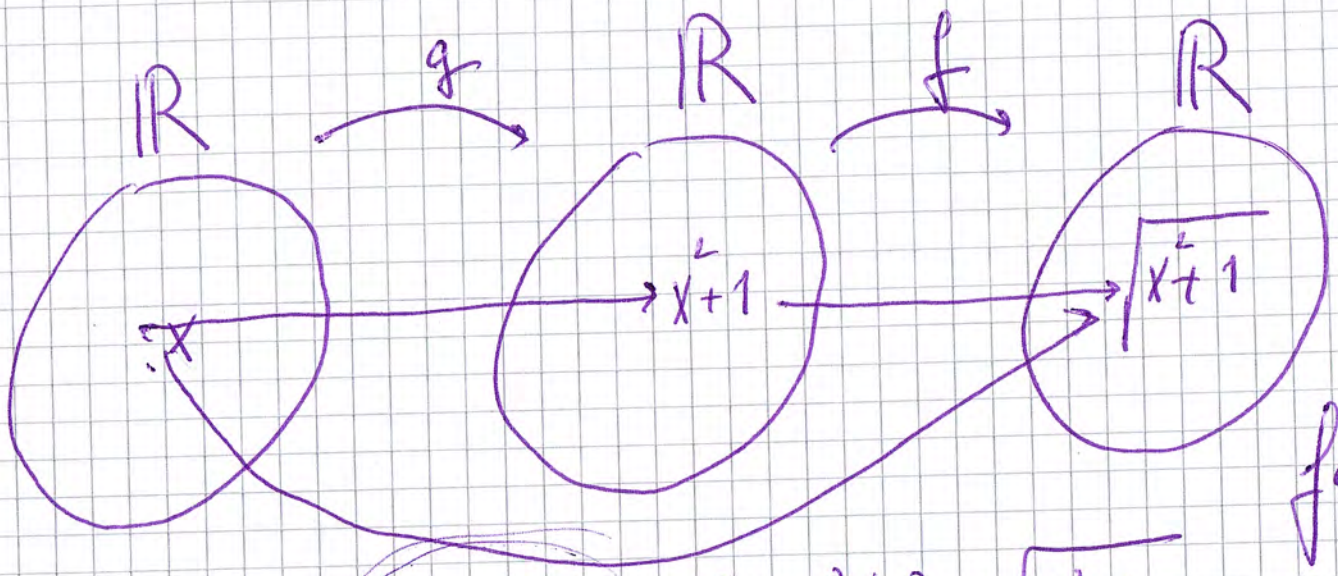
29-10-2018

$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

11.1



$f \circ g \neq g \circ f$

$g(x) = x^2 + 1$

$f(x) = \sqrt{x}$ $\quad (f \circ g)(x) = \sqrt{x^2+1}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$

$f(x) = 2^x \qquad g(x) = x^3 + x + 1$

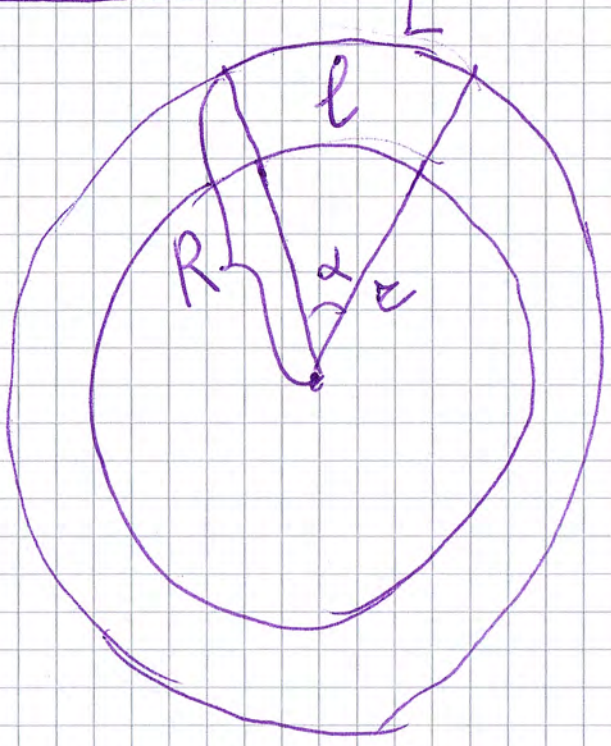
$(f \circ g)(x) \overset{def}{=} f(g(x)) = f(x^3+x+1) = 2^{x^3+x+1}$

$(g \circ f)(x) \overset{def}{=} g(f(x)) = g(2^x) = (2^x)^3 + 2^x + 1 = 2^{3x} + 2^x + 1$

$$f(x) = \log\left(\sqrt{x^2+1} + 1\right) \qquad D_f = \mathbb{R}$$

Teorema. La composizione di funzioni continue è una funzione continua.

FUNZIONI GONIOMETRICHE



$$\alpha^{rad} = \frac{l}{r} = \frac{L}{R}$$

137

$180° \leftrightarrow \pi$ rad

$30° \leftrightarrow \frac{\pi}{6}$

$60° \leftrightarrow \frac{\pi}{3}$

$90° \leftrightarrow \frac{\pi}{2}$

$45° \leftrightarrow \frac{\pi}{4}$

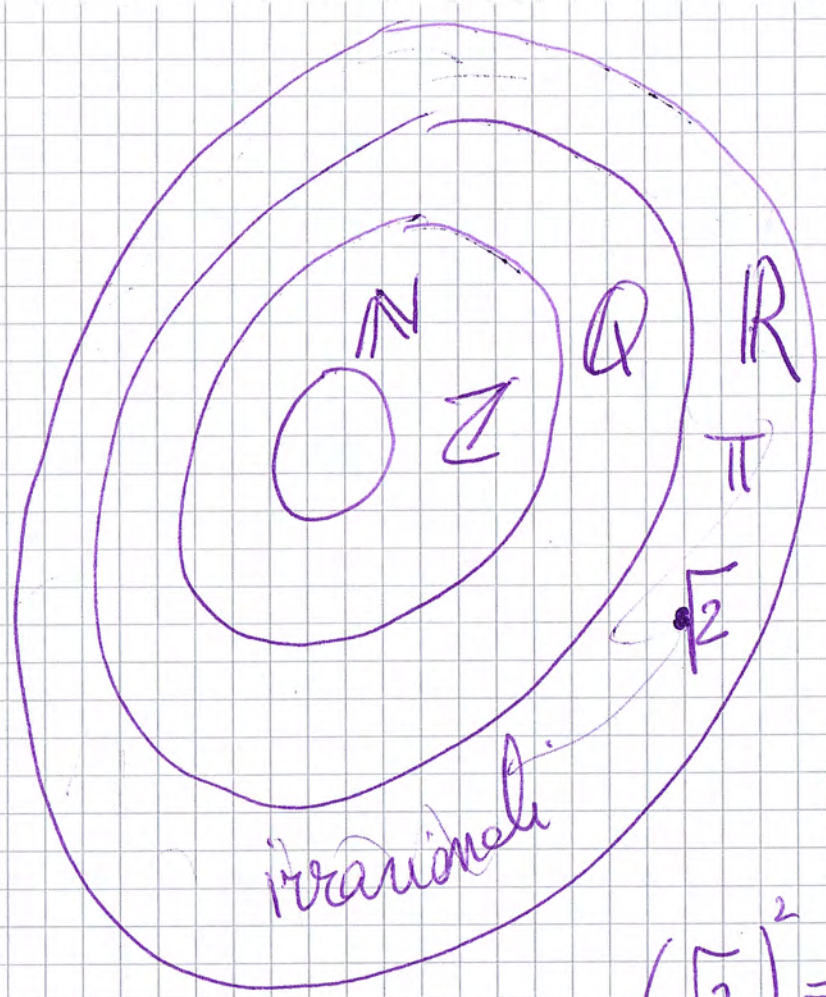
$360° \leftrightarrow 2\pi$

$180° : 180° = \alpha : \pi$

$\alpha = \frac{\pi}{10}$

$\alpha = \frac{180° \cdot \pi}{180°}$

$18° : 180° = \alpha : \pi$

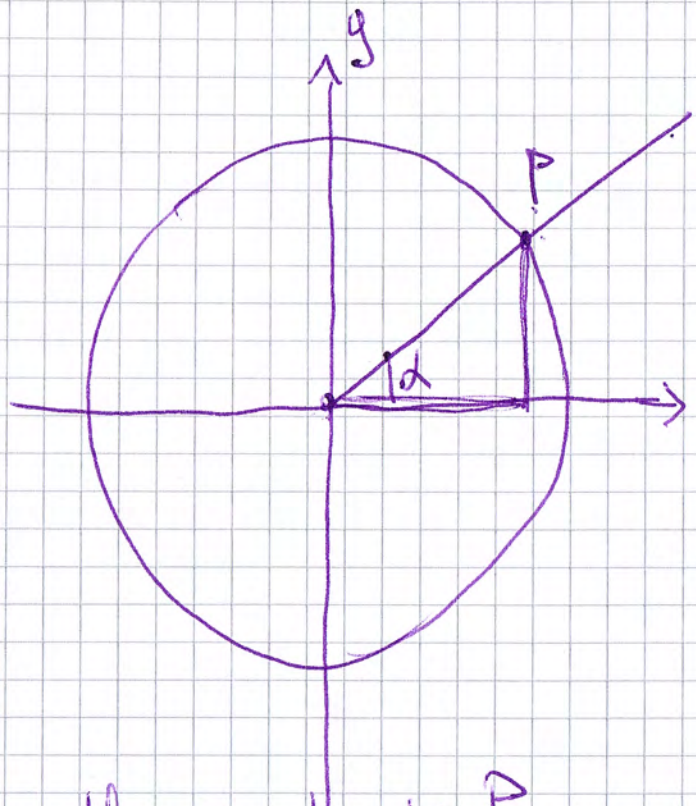


$2 : 0 = ?$ non esiste

$0 : 3 = 0$

$(\sqrt{2})^2 = 2$

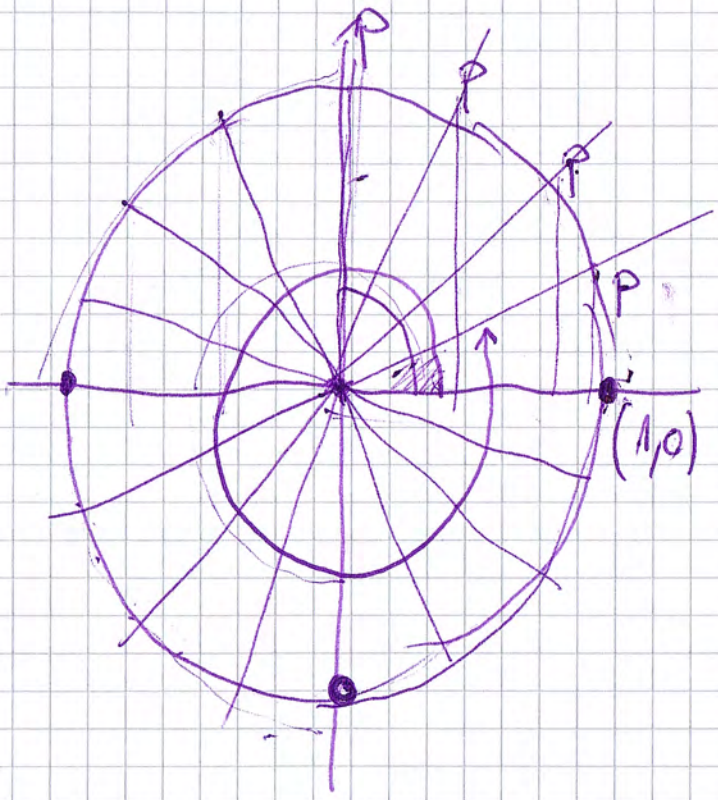
$x^2 - 2 = 0$



$$(x-x_0)^2+(y-y_0)^2=r^2$$

$$x^2+y^2=1$$

$\sin\alpha \overset{\text{def.}}{=}$ ordinata di P $\quad \cos\alpha \overset{\text{def.}}{=}$ ascissa di P



$\sin 0° = \sin 0 = 0$

$\sin 90° = \sin \frac{\pi}{2} = 1$

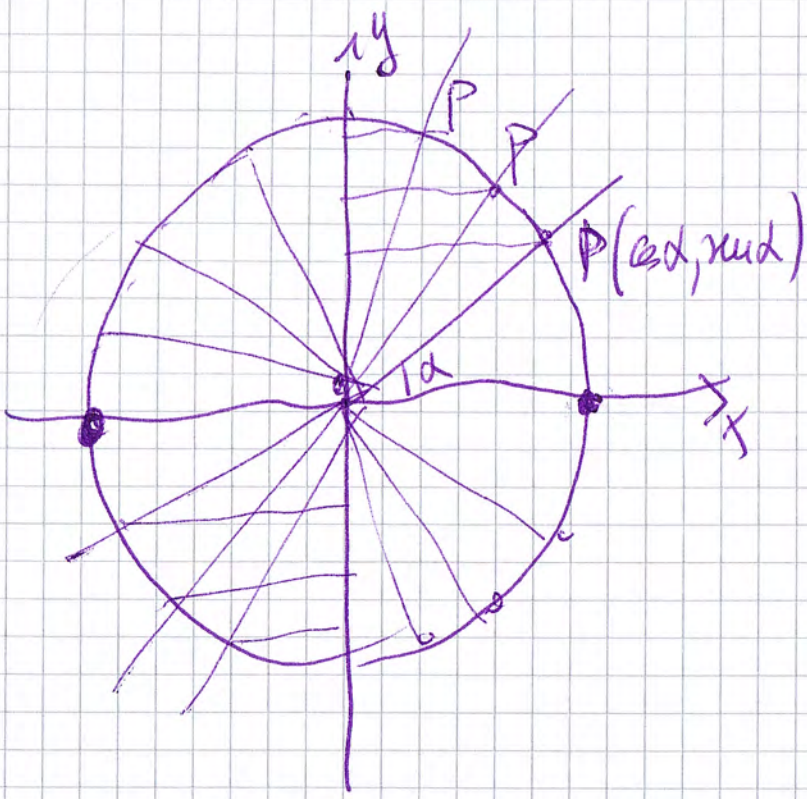
$\sin 180° = \sin \pi = 0$

$\sin 270° = \sin \frac{3}{2}\pi = -1$

$\sin 360° = \sin 2\pi = 0$

1) $\sin$ è una funzione periodica di periodo 2π

2) $-1 \leq \sin \alpha \leq 1$



$\cos 0° = \cos 0 = 1$

$\cos 90° = \cos \frac{\pi}{2} = 0$

$\cos 180° = \cos \pi = -1$

$\cos 270° = \cos \frac{3\pi}{2} = 0$

$\cos 360° = \cos 2\pi = 1$

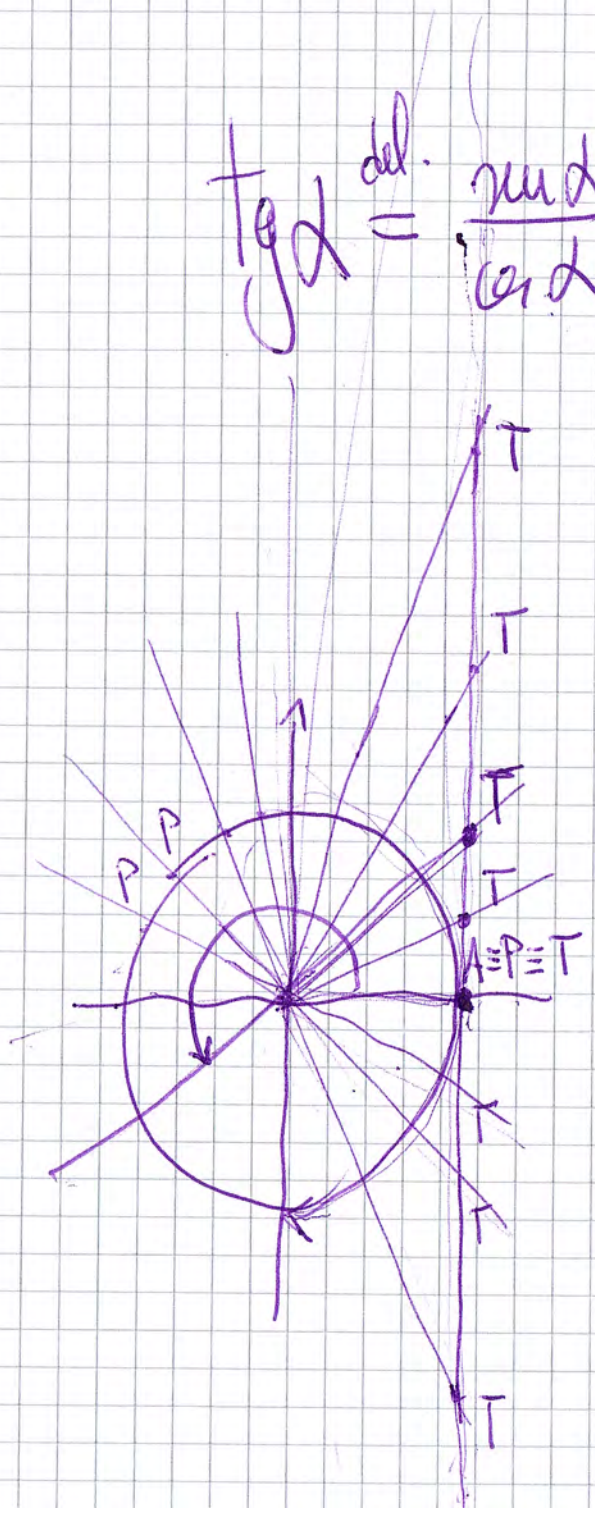
1) $\cos$ è una funzione periodica di periodo 2π

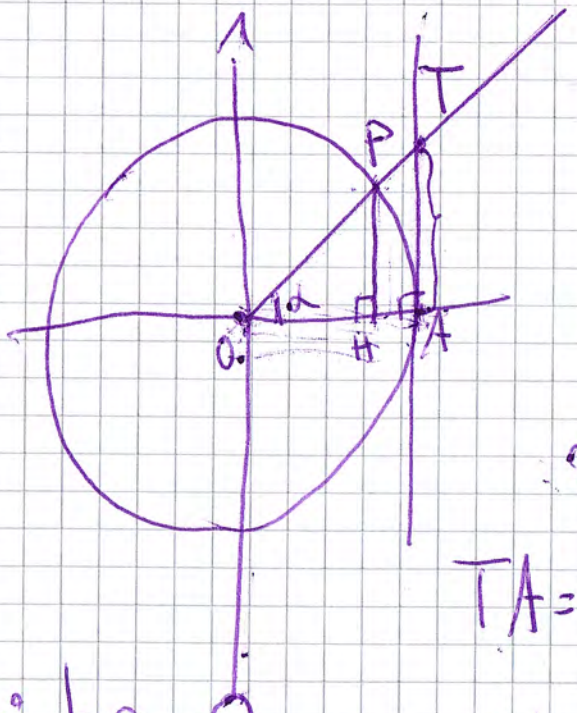
2) $-1 \leq \cos\alpha \leq 1$

$x^2 + y^2 = 1$

$$\boxed{\cos^2\alpha + \text{sen}^2\alpha = 1}$$

$$\operatorname{tg}\alpha \overset{\text{def.}}{=} \frac{\operatorname{sen}\alpha}{\cos\alpha}$$





$\hat{OPH}$ e $\hat{OTA}$ simili

$PH : OH = TA : OA$

$\operatorname{sen}\alpha : \cos\alpha = TA : 1$

$TA = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \operatorname{tg}\alpha$ = ordinata di T

$\operatorname{tg} 0° = \operatorname{tg} 0 = 0$

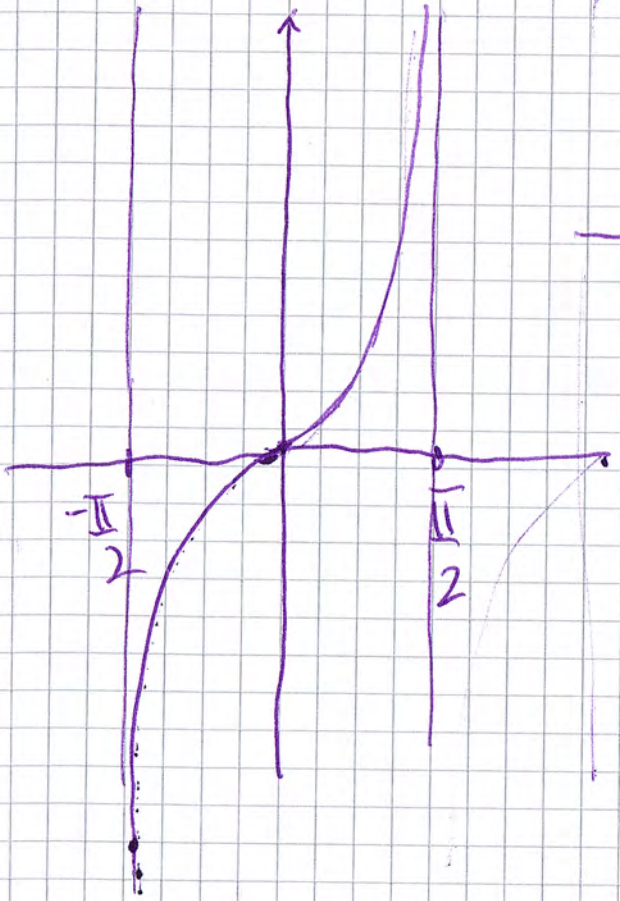
$\operatorname{tg} 90° = \operatorname{tg} \frac{\pi}{2} =$ non esiste

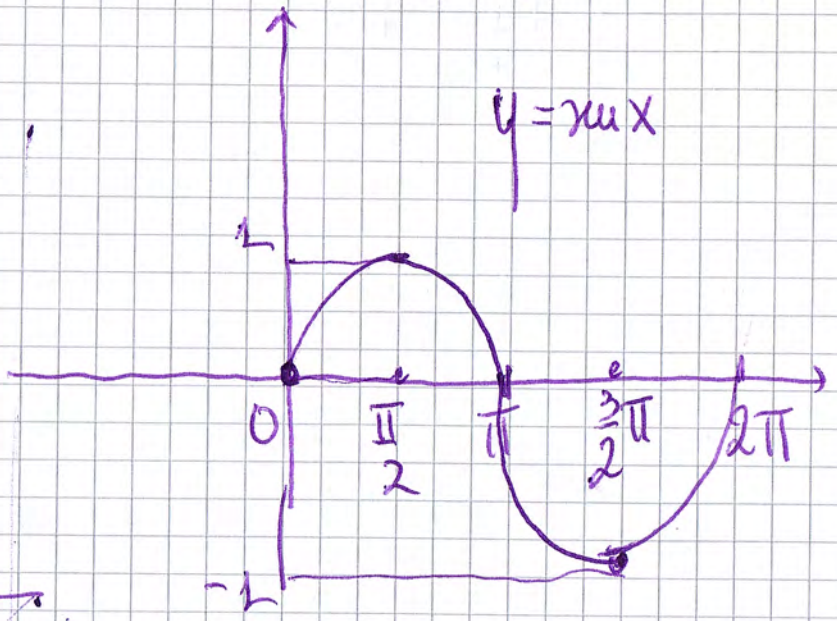
$\operatorname{tg} 180° = \operatorname{tg} \pi = 0$

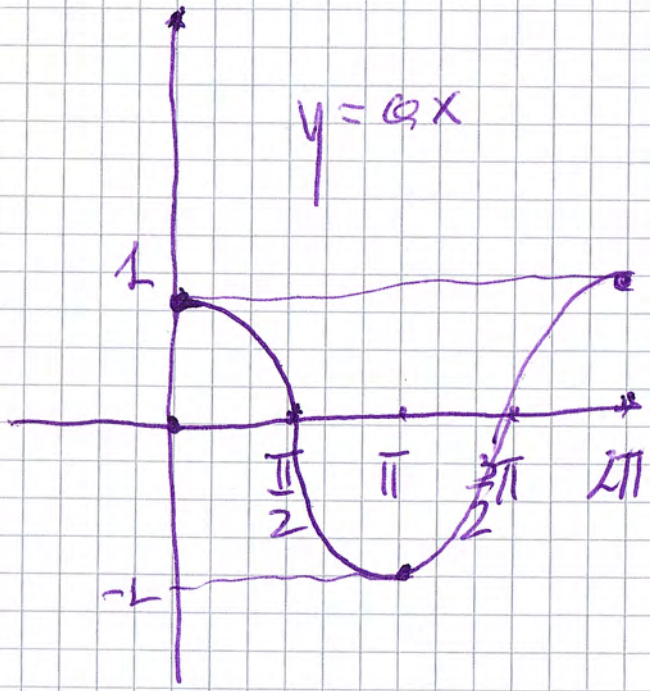
1) $\operatorname{tg}\alpha$ è una funzione periodica di periodo π

2) $-\infty < \operatorname{tg}\alpha < +\infty$

$$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\operatorname{tg}\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$$







$\sin 30° = \sin \frac{\pi}{6} = \frac{1}{2}$ $\quad \sin 60° = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \sin 45° = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ | 44

$\cos 30° = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\quad \cos 60° = \cos \frac{\pi}{3} = \frac{1}{2}$ $\quad \cos 45° = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\operatorname{tg} 30° = \operatorname{tg} \frac{\pi}{6} = \frac{\sqrt{3}}{3}$ $\quad \operatorname{tg} 60° = \operatorname{tg} \frac{\pi}{3} = \sqrt{3}$ $\quad \operatorname{tg} 45° = \operatorname{tg} \frac{\pi}{4} = 1$

$f(x) = \sin x$ $\quad f(x) = \cos x$ sono funzioni continue in $\mathbb{R}$

$f(x) = \operatorname{tg} x$ è continua in $\mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \right\}$

$\lim_{x \to 0} \sin x = \sin 0 = 0$ $\quad \lim_{x \to 0} \log \cos x = \log \cos 0 = \log 1 = 0$

I° LIMITE FONDAMENTALE

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0^+} \frac{\sin x}{x} = 1$$

$\sin x < x < \operatorname{tg} x \quad : \sin x$

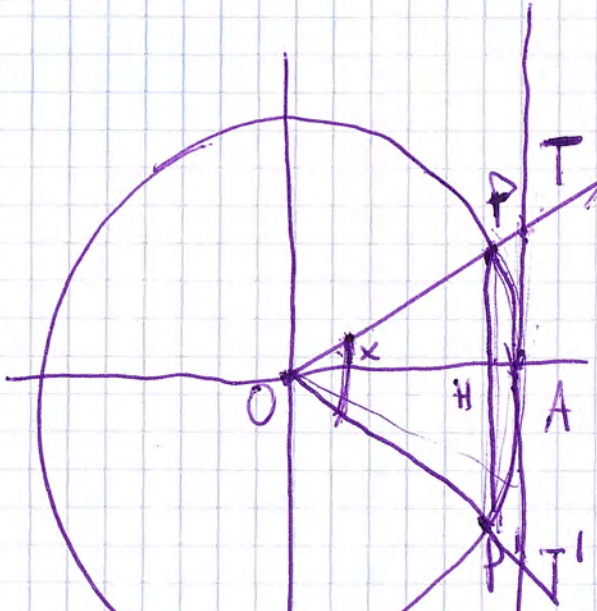
$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$

$\cos x < \frac{\sin x}{x} < 1$

$x\to 0^+$: $\cos x \to 1$, $\frac{\sin x}{x} \to 1$, $1 \to 1$

$\widetilde{PP'} < \overline{PP'} < \widetilde{TT'}$

$2\sin x < 2x < 2\operatorname{tg} x$



$2x = \frac{\widetilde{PP'}}{1}$

$\lim_{x\to 0^-} \frac{\sin x}{x} = \lim_{t\to 0^+} \frac{\sin(-t)}{-t}$

$t = -x \to x = -t$

$= \lim_{t\to 0^+} \frac{t \sin t}{-t} = 1$