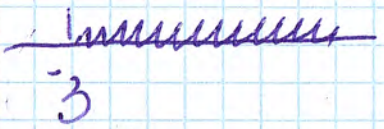
22-10-2018

108

$$\lim_{x \to -2} \log_{10}(x+3) = 0$$

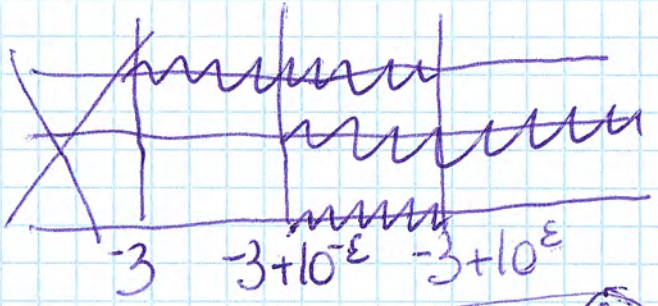
$$D = \{x+3 > 0\} = \{x > -3\}$$



$$|\log_{10}(x+3) - 0| < \varepsilon$$

$$\begin{cases} \log_{10}(x+3) < \varepsilon \cdot 1 = \varepsilon \log_{10} 10 = \log_{10} 10^{\varepsilon} \\ \log_{10}(x+3) > -\varepsilon \cdot 1 = -\varepsilon \log_{10} 10 = \log_{10} 10^{-\varepsilon} \end{cases}$$

$$\begin{cases} x+3 < 10^{\varepsilon} \\ x+3 > 10^{-\varepsilon} \end{cases} \qquad \begin{cases} x < -3+10^{\varepsilon} \\ x > -3+10^{-\varepsilon} \end{cases}$$



$$-3+10^{-\varepsilon} < -2 < -3+10^{\varepsilon}$$

$$10^{-\varepsilon} < 1 < 10^{\varepsilon}$$

$$10^{0}$$

$$\lim_{x \to +\infty} 2^{x+1} = +\infty$$

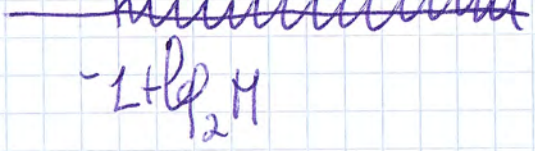
$D = \mathbb{R}$

$2^{x+1} > M$

$\log_2 2^{x+1} > \log_2 M$

$x+1 > \log_2 M$

$x > -1 + \log_2 M$



$+\infty - \infty$ $\quad 0 \cdot \infty$

$\frac{0}{0}$ $\quad \frac{\infty}{\infty}$

$$\lim_{x \to -\infty} (-3x^3) = +\infty$$

$$\lim_{x \to +\infty} (5x^5) = -\infty$$

$$= \lim_{x \to \pm\infty} a_m x^m =$$

$$= \lim_{x \to \pm\infty} \left(a_m x^m + a_{m-1} x^{m-1} + \cdots + a_1 x + a_0 \right)$$

REGOLA DEI GRADI

$$\lim_{x \to +\infty} \frac{3x^3 + 4x - 2}{5x^4 - 4x^2 + 11} = \frac{+\infty}{+\infty}$$

$$\lim_{x \to +\infty} \frac{3x^3}{5x^4} = \lim_{x \to +\infty} \frac{3}{5x} = 0$$

$$\lim_{x \to -\infty} \frac{3x^3 - 4x + 2}{x^2 + x + 11} = \lim_{x \to -\infty} \frac{3x^3}{-x^2} = \lim_{x \to -\infty} \frac{-3x}{1} = +\infty$$

$$\lim_{x\to+\infty}\frac{2x^4+x^2+x-11}{3x^4-4x^3+x+2}=\lim_{x\to+\infty}\frac{2x^4}{3x^4}$$

112

$$=\lim_{x\to+\infty}\left(\frac{2}{3}\right)=\frac{2}{3}$$

REGOLA DEI GRADI

$$\lim_{x \to \pm\infty} \frac{a_n x^n + \dots + a_1 x + a_0}{b_m x^m + \dots + b_1 x + b_0}$$

- $n < m$ → 0
- $n > m$ → ∞
- $n = m$ → $\frac{a_n}{b_m}$

$$\lim_{x\to+\infty} \frac{3x^2+4x+1}{-x+2} = -\infty \qquad \lim_{x\to+\infty} \frac{3x^2}{-x} = -\infty$$

$$\lim_{x\to+\infty} \frac{x^3+x^2+x+1}{x+2x^3+1} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{x^3+x+1}{2x^4-4x+11} = 0 \qquad \lim_{x\to-\infty} \frac{x^3}{2x^4} = \lim_{x\to-\infty} \frac{1}{2x} = 0^-$$

115

$$\lim_{x\to+\infty} \frac{3x^2 + x\sqrt{x} - 1}{x^2\sqrt{x} - x + 2} = \lim_{x\to+\infty} \frac{3x^2}{x^2\sqrt{x}} = 0$$

$3x^2$: 2 — $x\sqrt{x}$: $3/2$ — $x^2\sqrt{x}$: $5/2$ — -1: 0

$$\sqrt{x^2} \overset{?}{=} |x|$$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2}}{x} = \lim_{x\to+\infty} \frac{|x|}{x} = \lim_{x\to+\infty} \frac{x}{x} = 1$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2}}{x}$$

$x^2+1 \geq 0$ — $x \neq 0$

$$= \lim_{x\to-\infty} \frac{|x|}{x} = \lim_{x\to-\infty} \frac{-x}{x} = -1$$

(116

$$\lim_{x \to +\infty} \left(\sqrt{x^2+1} - x \right) = +\infty - \infty$$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\lim_{x \to +\infty} \frac{\sqrt{x^2+1} - x}{1} \cdot \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x} = \frac{\left(\sqrt{x^2+1}\right)^2 - x^2}{\sqrt{x^2+1} + x} =$$

$$= \frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1} + x} = \frac{1}{\sqrt{x^2+1} + x} \to \frac{1}{+\infty} = 0$$

$$\lim_{x \to 2} \left(3x^3 + 4x^2 - 4x + 2 \right) = 3 \cdot 2^3 + 4 \cdot 2^2 - 4 \cdot 2 + 2$$

$$3x^3 = 3 \cdot x \cdot x \cdot x$$

$$3 \quad 2 \quad 2 \quad 2$$

$$3 \cdot 2^3 \quad 4 \cdot 2^2 \quad -4 \cdot 2 \quad 2$$

$$\lim_{x \to c} P(x) = P(c)$$

$P(x)$ → polinomio

$$\lim_{x \to c} \frac{P(x)}{H(x)} = \frac{P(c)}{H(c)} \quad \text{se } H(c) \neq 0$$

Se $H(c)=0$ e $P(c) \neq 0$ ∞ Se $H(c)=0=P(c)$ $\frac{0}{0}$

$$\lim_{x \to 1} \frac{\overbrace{3x^4+4x^2-7}^{P(x)}}{\underbrace{x^3+x^2+x-3}_{H(x)}} = \frac{3+4-7}{1+1+1-3} = \frac{0}{0}$$

$P(1)=0$

$H(1)=0$

Teorema di Ruffini $P(x)$ è divisibile per $x-c \iff P(c)=0$

$$P(x)=(x-c)Q(x)$$

$$\lim_{x\to 1}\frac{(x-1)(3x^3+3x^2+4x+7)}{(x-1)(x^2+2x+3)}=\frac{20}{6}=\frac{10}{3}$$

	3	0	4	0	-7
1		3	3	7	7
	3	3	7	7	0

	1	1	1	-3
1		1	2	3
	1	2	3	0

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{\sqrt{x+2}-\sqrt{2}}=\frac{\sqrt{1}-1}{\sqrt{2}-\sqrt{2}}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{\sqrt{x+2}-\sqrt{2}}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}=\lim_{x\to 0}\frac{x+1-1}{\left(\sqrt{x+2}-\sqrt{2}\right)\left(\sqrt{x+1}+1\right)}$$

$$=\lim_{x\to 0}\frac{x}{\left(\sqrt{x+2}-\sqrt{2}\right)\left(\sqrt{x+1}+1\right)}\cdot\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}=$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{x+2}+\sqrt{2}\right)}{\left(x+2-2\right)\left(\sqrt{x+1}+1\right)}=$$

(12)

$$\lim_{x \to 0} \frac{\cancel{(\sqrt{x+2} + \sqrt{2})}}{\cancel{(\sqrt{x+1} + 1)}} = \frac{\sqrt{2}+\sqrt{2}}{1+1} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$