17-10-2018

$$\lim_{x\to 0} \frac{x}{x+1} = 0 \qquad D = \mathbb{R}\setminus\{-1\}$$

$$\left|\frac{x}{x+1} - 0\right| < \varepsilon$$

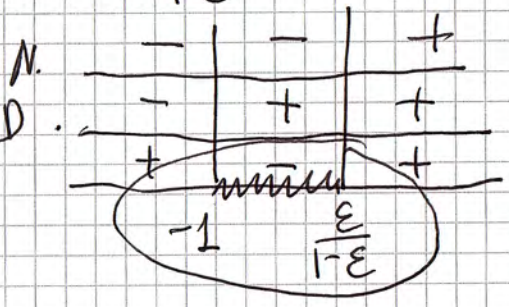
$$\begin{cases} \dfrac{x}{x+1} < \varepsilon \\ \dfrac{x}{x+1} > -\varepsilon \end{cases} \qquad \begin{cases} \dfrac{x - \varepsilon x - \varepsilon}{x+1} < 0 \\ \dfrac{x + \varepsilon x + \varepsilon}{x+1} > 0 \end{cases}$$

$1-\varepsilon > 0 \iff \varepsilon < 1$

$$\begin{cases} \dfrac{x(1-\varepsilon) - \varepsilon}{x+1} < 0 & (1) \\ \dfrac{x(1+\varepsilon) + \varepsilon}{x+1} > 0 & (2) \end{cases}$$

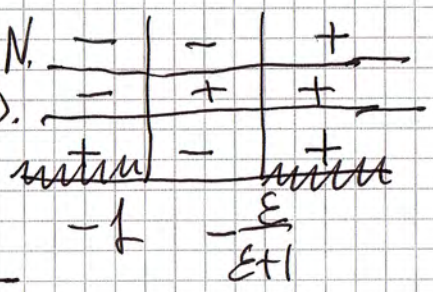
(1) $N > 0 \quad x(1-\varepsilon) > \varepsilon \quad x > \dfrac{\varepsilon}{1-\varepsilon}$

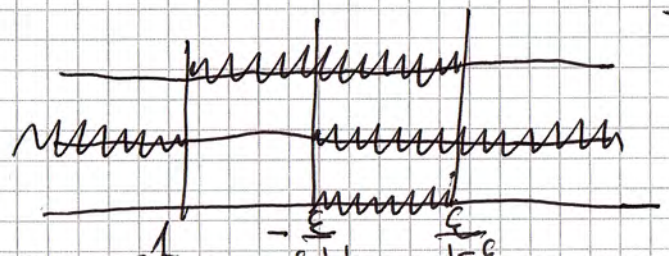
$D > 0 \quad x > -1$



(2) $N. > 0 \quad x(1+\varepsilon) > -\varepsilon \quad x > -\dfrac{\varepsilon}{1+\varepsilon}$

$D. > 0 \quad x > -1$





$$\frac{\varepsilon}{1+\varepsilon} < 1 \qquad -\frac{\varepsilon}{1+\varepsilon} > -1$$

$$-\frac{\varepsilon}{\varepsilon+1} > -1 \qquad \frac{-\varepsilon}{\varepsilon+1} > \frac{-\varepsilon-1}{\varepsilon+1} \qquad -\varepsilon > -\varepsilon-1$$

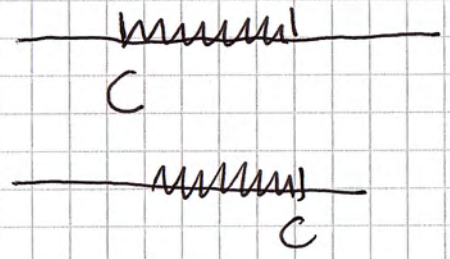
$$0 > -1$$

$$\lim_{x\to c} f(x) = l \qquad \lim_{x\to c} f(x) = \begin{cases} +\infty \\ -\infty \end{cases}$$

Def. $\lim_{x\to c^{\pm}} f(x) = l$ vuol dire che

$\forall \varepsilon > 0 \ \exists$ H intorno (sinistro / destro) di c t.c. $\forall x \in H \cap D_f \setminus \{c\}$ risulta

$$|f(x) - l| < \varepsilon$$



$\lim_{x \to c^+} f(x) = +\infty$ $\forall M > 0$ $\exists$ H intorno destro di c t.c. $\forall x \in H \cap D \setminus \{c\}$

risulti

$f(x) > M$

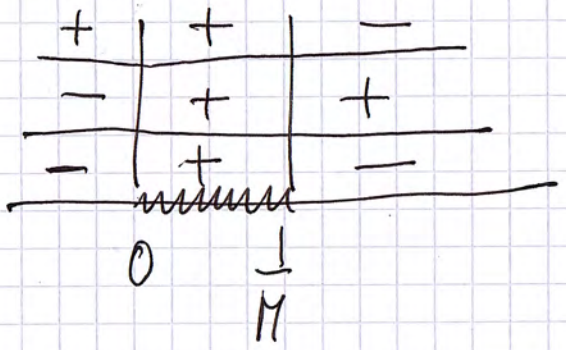
$D = \mathbb{R} \setminus \{0\}$

$\lim_{x \to 0^+} \frac{1}{x} = +\infty$

$\frac{1}{x} > M$ $\quad \frac{1 - Mx}{x} > 0$

$N. > 0 \quad 1 - Mx > 0 \quad Mx < 1 \quad x < \frac{1}{M}$

$D. > 0 \quad x > 0$



TEOREMA

$$\lim_{x\to c} f(x) = l \iff \lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = l$$

$$\lim_{\substack{x\to+\infty \\ x\to-\infty}} f(x) = l \qquad \forall \varepsilon>0 \ \exists N>0 \text{ t.c. } \forall x>N \ (x\in D_f) \text{ zwykle } (\forall x<-N)$$

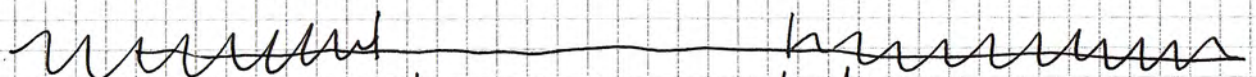
$$|f(x)-l|<\varepsilon$$

$$\lim_{x\to+\infty} \frac{x}{x+1} = 1 \qquad D = \mathbb{R}\setminus\{-1\}$$

$$\left|\frac{x}{x+1}-1\right|<\varepsilon \qquad \left|\frac{x-x-1}{x+1}\right|<\varepsilon \qquad \left|\frac{1}{x+1}\right|<\varepsilon$$

$$\left|\frac{1}{x+1}\right| < \varepsilon \qquad |x+1| > \frac{1}{\varepsilon}$$

$$x+1 > \frac{1}{\varepsilon} \quad \vee \quad x+1 < -\frac{1}{\varepsilon} \iff x > -1+\frac{1}{\varepsilon} \quad \vee \quad x < -1-\frac{1}{\varepsilon}$$



N

$\frac{1}{\varepsilon} - 1 > 0$

$1-\varepsilon > 0$

$\varepsilon < 1$

$$\lim_{x \to -\infty} \frac{x}{x+1} = 1$$

(1) $\lim_{x \to +\infty} f(x) = +\infty$ $\forall M > 0 \; \exists N > 0$ t.c. $\forall x > N$ risulta $f(x) > M$

(2) $\lim_{x \to +\infty} f(x) = -\infty$ $\forall M > 0 \; \exists N > 0$ t.c. $\forall x > N$ risulta $f(x) < -M$

(3) $\lim_{x \to -\infty} f(x) = +\infty$ $\forall M > 0 \; \exists N > 0$ t.c. $\forall x < -N$ risulta $f(x) > M$

(4) $\lim_{x \to -\infty} f(x) = -\infty$ $\forall M > 0 \; \exists N > 0$ t.c. $\forall x < -N$ risulta $f(x) < -M$

$$\lim_{x\to+\infty} \frac{x^2}{x+1} = +\infty$$

$D = \mathbb{R} \setminus \{-1\}$

$x^2 - Mx - M$

$+ \quad - \quad -$

Var. +

Per. −

$$\frac{x^2}{x+1} > M \qquad \frac{x^2 - Mx - M}{x+1} > 0$$

$N > 0$ $\quad x^2 - Mx - M > 0$ $\quad \Delta = M^2 + 4M > 0$ $\quad x = \frac{M \pm \sqrt{M^2+4M}}{2}$ V.e.

$D > 0$ $\quad x > -1$

	-1		$\frac{M-\sqrt{M^2+4M}}{2}$		$\frac{M+\sqrt{M^2+4M}}{2}$	
$+$		$+$		$-$		$+$
$-$		$+$		$+$		$+$
$-$		$+$		$-$		$+$

$$\lim_{x\to-1^+} \frac{x^2}{x+1} = +\infty$$

$$\frac{M-\sqrt{M^2+4M}}{2} > -1$$

$$M-\sqrt{M^2+4M} > -2$$

$$M+2 > \sqrt{M^2+4M}$$

$$M^2+4+4M > M^2+4M$$

$$4 > 0$$

① Teorema dell'unicità del limite

Se il limite esiste allora è unico.

② Teorema del confronto (Teorema dei due carabinieri)

Siano $f(x), g(x), h(x)$ funzioni definite in un intorno di c e tali che in tale intorno sia

$$f(x) \leq g(x) \leq h(x).$$

Sia poi $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = l$. Allora $\lim_{x \to c} g(x) = l$.

$$f(x) \leq g(x) \leq h(x)$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$l \quad l \quad l$$

3) Teorema della permanenza del segno

Sia $\lim_{x \to c} f(x) = l \neq 0$. Allora esiste un intorno di c t.c. $f(x)$ ha lo stesso segno di l in tale intorno.

Operazioni sui limiti

Supponiamo che $\lim_{x \to c} f(x) = l$ e $\lim_{x \to c} g(x) = m$. Allora

1) $\lim_{x \to c} (f(x) + g(x)) = l + m$

2) $\lim_{x \to c} (f(x) - g(x)) = l - m$

3) $\lim_{x \to c} f(x) g(x) = lm$

4) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{l}{m}$ se $m \neq 0$

$l + \infty = +\infty$ $\quad +\infty + \infty = +\infty$

$l - \infty = -\infty$ $\quad -\infty - \infty = -\infty$

$+\infty - \infty$ = FORMA INDETERMINATA

$l^{+0} \cdot (+\infty) = +\infty$ $\quad l^{+0} \cdot (-\infty) = -\infty$ $\quad l^{-0} \cdot (+\infty) = -\infty$ $\quad l^{-0} \cdot (-\infty) = +\infty$

$(+\infty)(+\infty) = +\infty$ $\quad (+\infty)(-\infty) = -\infty$ $\quad (-\infty)(-\infty) = +\infty$

$0 \cdot \infty$ = F. I.

$\frac{l}{\infty} = 0$ $\quad \frac{\infty}{l} = \infty$ $\quad \frac{\infty}{\infty}$ = F. I. $\quad \frac{0}{0}$ = F. I.

$$\lim_{x\to+\infty}\left(3x^3-4x^2+x-2\right)$$

$3x^3$: $3\cdot x\cdot x\cdot x$ → 3, $+\infty$, $+\infty$, $+\infty$ → $+\infty$

$-4x^2$ → $+\infty$ → $-\infty$

$+x$ → $+\infty$ → $+\infty$

-2 → -2

$$\lim_{x\to+\infty} 3x^3\left(1-\frac{4x^2}{3x^3}+\frac{x}{3x^3}-\frac{2}{3x^3}\right) = \lim_{x\to+\infty} 3x^3\left(1-\frac{4}{3x}+\frac{1}{3x^2}-\frac{2}{3x^3}\right)$$

$3x^3 \to +\infty$

$1 \to 1$

$-\frac{4}{3x} \to 0$

$+\frac{1}{3x^2} \to 0$

$-\frac{2}{3x^3} \to 0$

$1-0+0-0 = 1$

$+\infty \cdot 1 = +\infty$

Il limite per $x\to\infty$ di un polinomio dipende solo dalla potenza più alta

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$$\lim_{x \to -\infty} \left(-2x^5 + x^4 - x^2 + 3x + 11 \right) =$$

$$\lim_{x \to -\infty} \left(\underbrace{-2}_{-} \cdot \underbrace{x^5}_{\downarrow -\infty} \right) = +\infty$$