15-10-2018

$a < b$

$[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

$]a,b[= \{x \in \mathbb{R} : a < x < b\}$

$[a,b[= \{x \in \mathbb{R} \quad a \leq x < b\}$

$]a,b] = \{x \in \mathbb{R} : a < x \leq b\}$

$[a,+\infty[= \{x \in \mathbb{R} : x \geq a\}$

$]a,+\infty[= \{x \in \mathbb{R} : x > a\}$

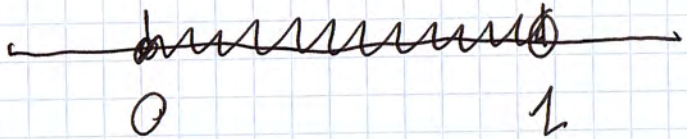
$]-\infty,a] = \{x \in \mathbb{R} : x \leq a\}$

$]-\infty,a[= \{x \in \mathbb{R} : x < a\}$

INTERVALLI

Intorno di $c \in \mathbb{R}$ è un qualunque intervallo che contenga al suo interno il punto c.

$[0,1[$

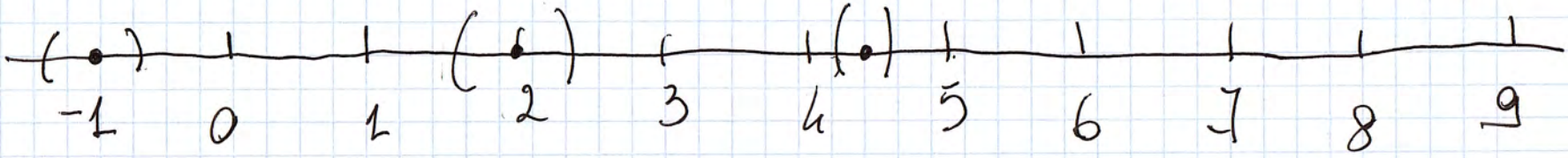


PUNTO DI ACCUMULAZIONE DI UN INSIEME

$c \in \mathbb{R}$ è punto di accumulazione per $A \subseteq \mathbb{R}$ se in ogni intorno di c cadono infiniti punti di A.

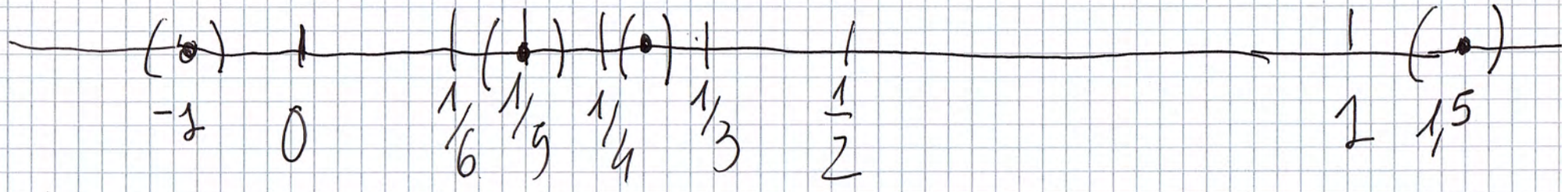
1) Se A è finito A non ha punti di accumulazione.

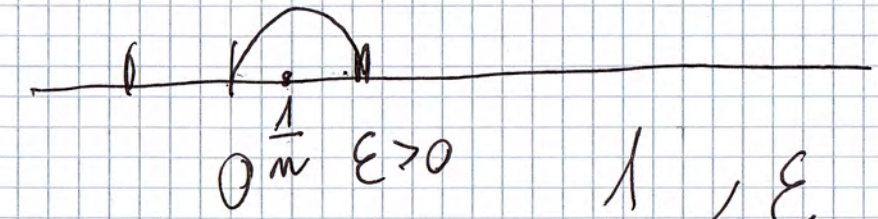
$A = \mathbb{N}$



$\mathbb{N}$ non ha punti di accumulazione

$$A = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \dots \right\}$$





$$\frac{1}{n} < \varepsilon$$

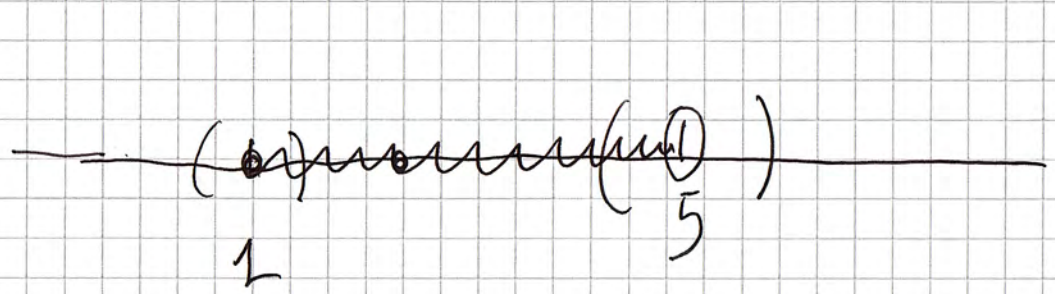
$$n > \frac{1}{\varepsilon}$$

$$\frac{1}{4} \quad \frac{1}{3}$$

$$\frac{3}{12} \quad \frac{4}{12}$$

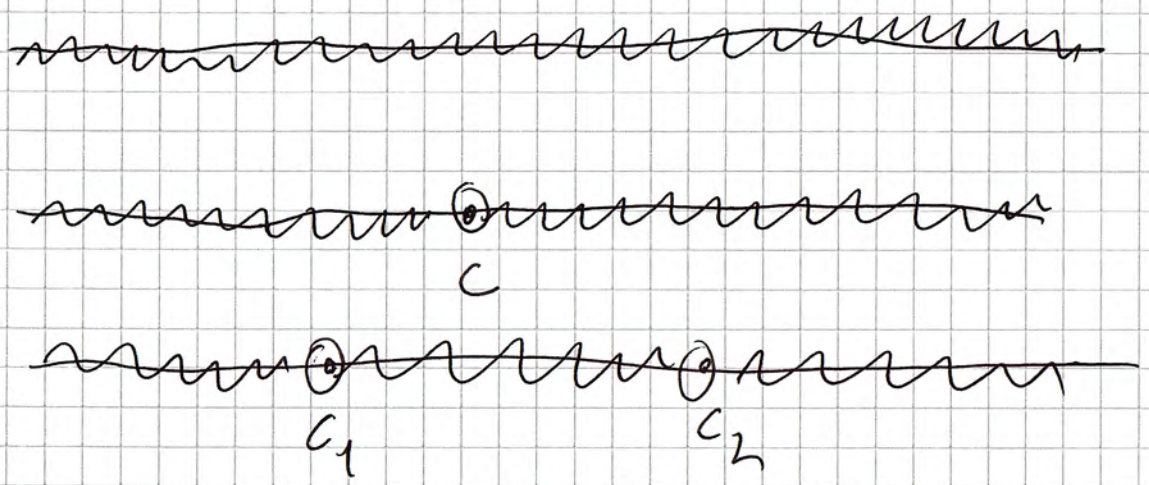
$$\frac{6}{24} \quad \frac{8}{24}$$

$A = [1, 5[$



punti di acc. A zero $[1,5]$

$\mathbb{R}$



$\mathbb{R} = \{c_1, c_2, \dots, c_n\}$ p. di acc. zero $\mathbb{R}$

Sia $y = f(x)$ funzione e sia D_f il suo dominio. Sia poi c punto di accumulazione per D_f

$\lim_{x \to c} f(x) = l$ limite per x tendente a c di $f(x)$ è uguale a l

$\Leftrightarrow$

$\forall \varepsilon > 0 \ \exists H$ intorno di c t.c. $\forall x \in H \cap D_f \setminus \{c\}$ risulta

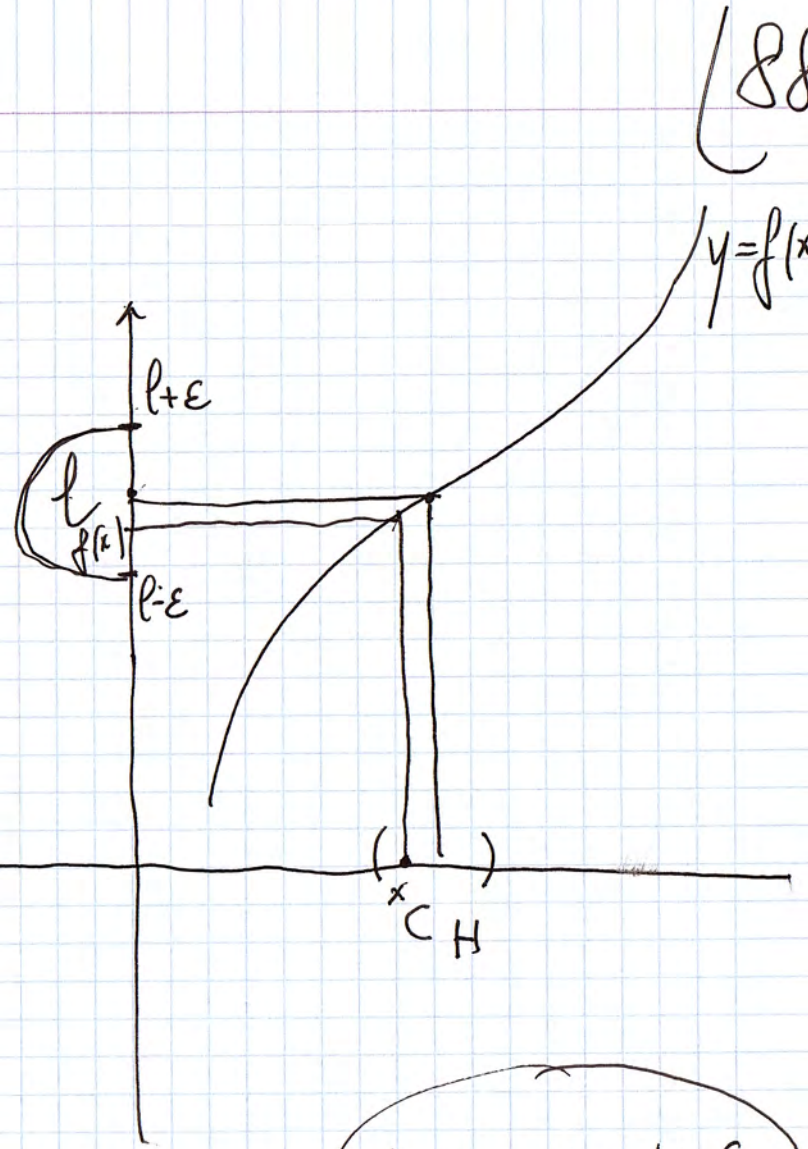
$$|f(x) - l| < \varepsilon.$$

ε piccolo a piacere

$$|f(x)-l|<\varepsilon$$

$$-\varepsilon<f(x)-l<\varepsilon$$

$$l-\varepsilon<f(x)<l+\varepsilon$$

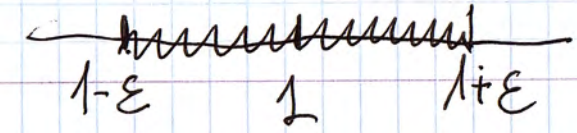


$$\lim_{x\to 1}(x+2)=3 \qquad f(x)=x+2 \qquad D=\mathbb{R}$$

$$|x+2-3|<\varepsilon \qquad |x-1|<\varepsilon$$

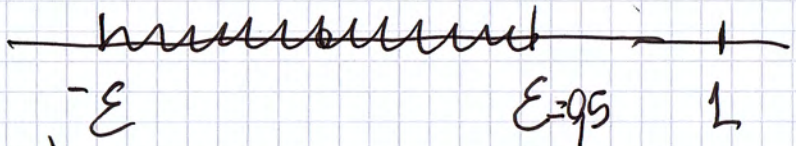
$$-\varepsilon<x-1<\varepsilon$$

$$1-\varepsilon<x<1+\varepsilon$$



$\lim_{x \to 1} (x+2) = 2$ vero

$|x+2-2| < \varepsilon \qquad |x| < \varepsilon \qquad -\varepsilon < x < \varepsilon$



$\varepsilon = \frac{1}{2}$

$\lim_{x \to 0} |x+2| = 2$

$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$

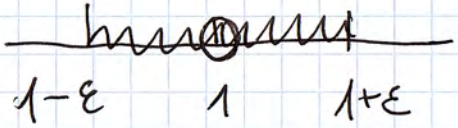
$D = R \setminus \{1\}$

$\left| \frac{x^2-1}{x-1} - 2 \right| < \varepsilon$

$$\left|\frac{(x+1)(x-1)}{x-1}-2\right|<\varepsilon \qquad |x+1-2|<\varepsilon \qquad |x-1|<\varepsilon$$

$$\left|\frac{x^2-1}{x-1}-2\right|<\varepsilon \qquad \left|\frac{x^2-1-2x+2}{x-1}\right|<\varepsilon \qquad \left|\frac{x^2-2x+1}{x-1}\right|<\varepsilon \qquad |x-1|<\varepsilon$$

$$(x-1)^2$$



$$\lim_{x\to 0}\frac{x}{x+1}=0$$

$$D_f=\mathbb{R}\setminus\{-1\}$$

$$\left|\frac{x}{x+1}-0\right|<\varepsilon$$

$$\left|\frac{x}{x+1}\right|<\varepsilon$$

$$\lim_{x \to c} f(x) = \begin{cases} +\infty & \text{(1)} \\ -\infty & \text{(2)} \end{cases}$$

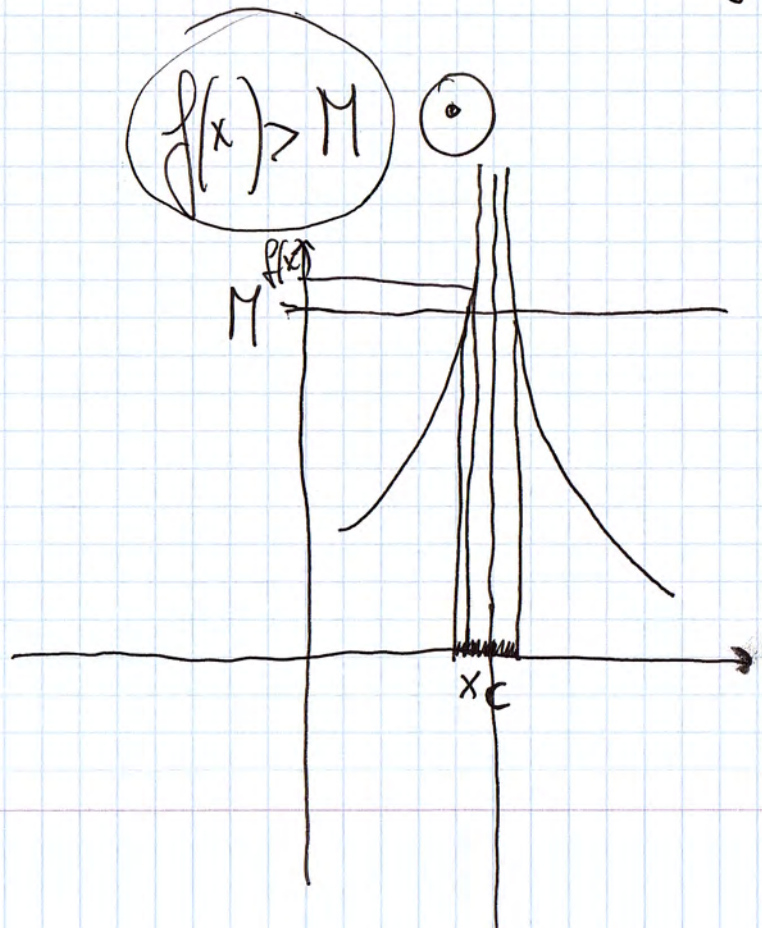
c punto di acc. per D_f

$M > 0$ grande a piacere

$\forall M > 0 \ \exists H$ intorno di c t.c. $\forall x \in H \cap D_f \setminus \{c\}$ risulta

(1) $f(x) > M$

(2) $f(x) < -M$



$$\lim_{x\to 1}\frac{1}{(x-1)^2}=+\infty \qquad D=\mathbb{R}\setminus\{1\}$$

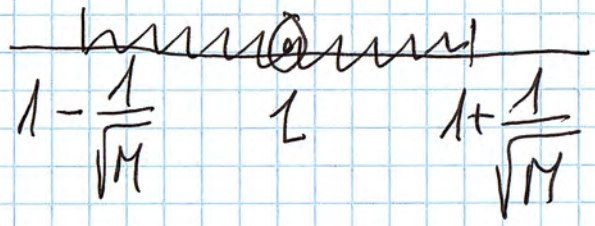
$x^2<9$

$-3<x<3$

$f(x)>M \qquad \frac{1}{(x-1)^2}>M \qquad (x-1)^2<\frac{1}{M}$

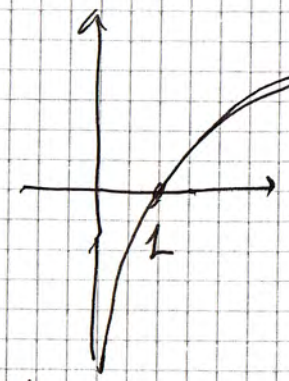
$$-\frac{1}{\sqrt{M}}<x-1<\frac{1}{\sqrt{M}}$$

$$1-\frac{1}{\sqrt{M}}<x<1+\frac{1}{\sqrt{M}}$$



$$\lim_{x\to 0} 2^{\frac{1}{x^2}} = +\infty$$

$D = \mathbb{R}\setminus\{0\}$



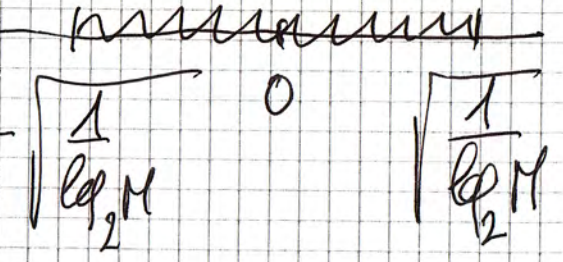
$$2^{\frac{1}{x^2}} > M \qquad \log_2 2^{\frac{1}{x^2}} > \log_2 M$$

$$\frac{1}{x^2}\log_2 2 > \log_2 M \qquad \frac{1}{x^2} > \log_2 M \qquad x^2 < \frac{1}{\log_2 M}$$

$\log_2 2 = 1$

$\frac{1}{x^2} > 0$, $\log_2 M > 0$

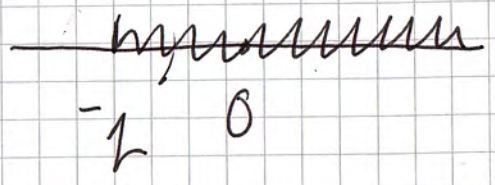
$$-\sqrt{\frac{1}{\log_2 M}} < x < \sqrt{\frac{1}{\log_2 M}}$$



$$\lim_{x\to 0} \log_{10}(x+1) = +\infty$$

$x+1>0$

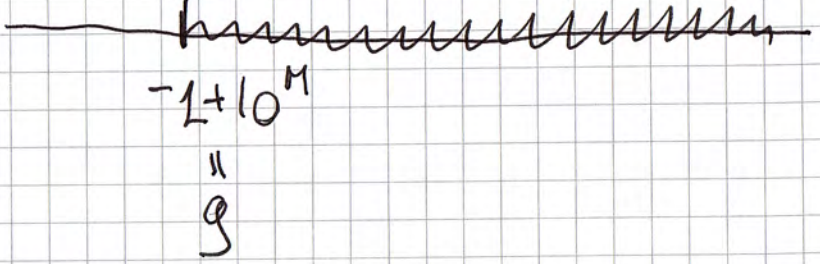
$x>-1$



$$\log_{10}(x+1) > M = M\cdot 1 = M\log_{10} 10 = \log_{10} 10^M$$

$$\log_{10}(x+1) > \log_{10} 10^M \qquad x+1 > 10^M$$

$$x > -1+10^M$$



$M=1$