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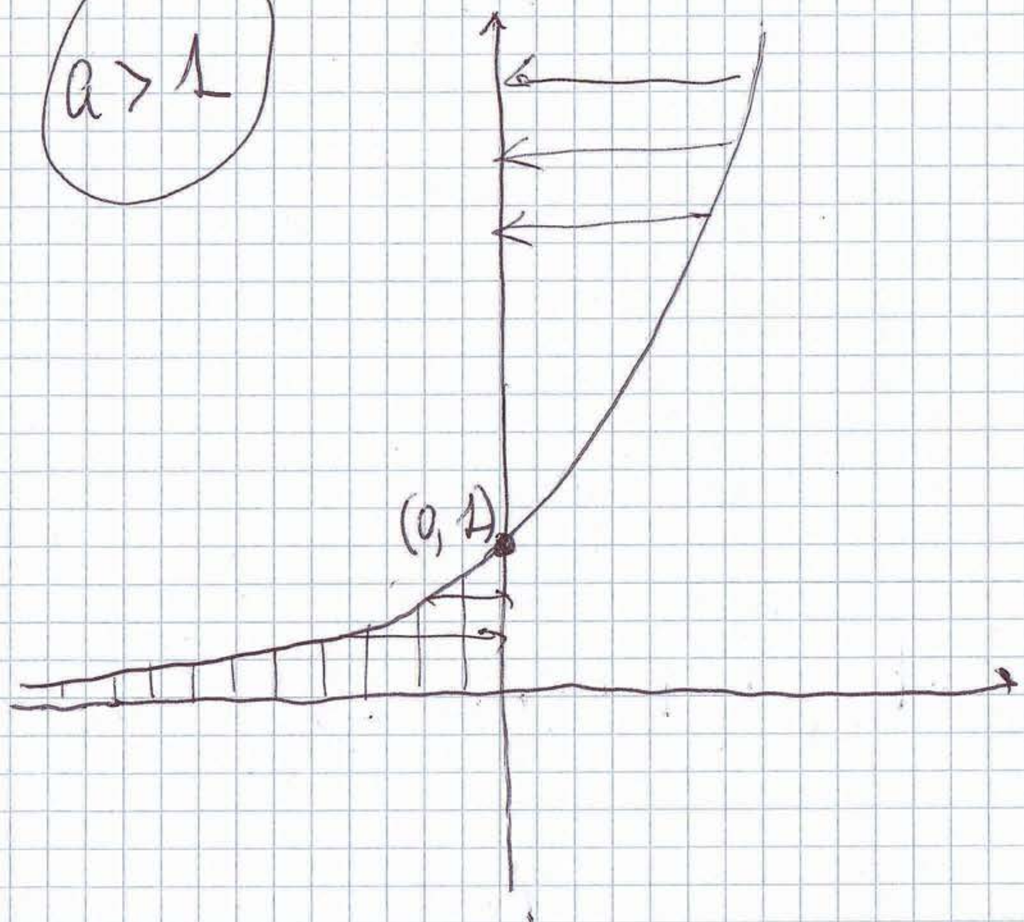
10-10-2018

$y = a^x$

$a > 0 \quad a \neq 1$

$a \in \mathbb{R}$

① $a > 1$



1) $D = \mathbb{R}$

2) $a^x > 0 \quad \forall x$

3) $a^0 = 1$

4) è crescente

5) $\lim_{x \to +\infty} a^x = +\infty$

$\lim_{x \to -\infty} a^x = 0$

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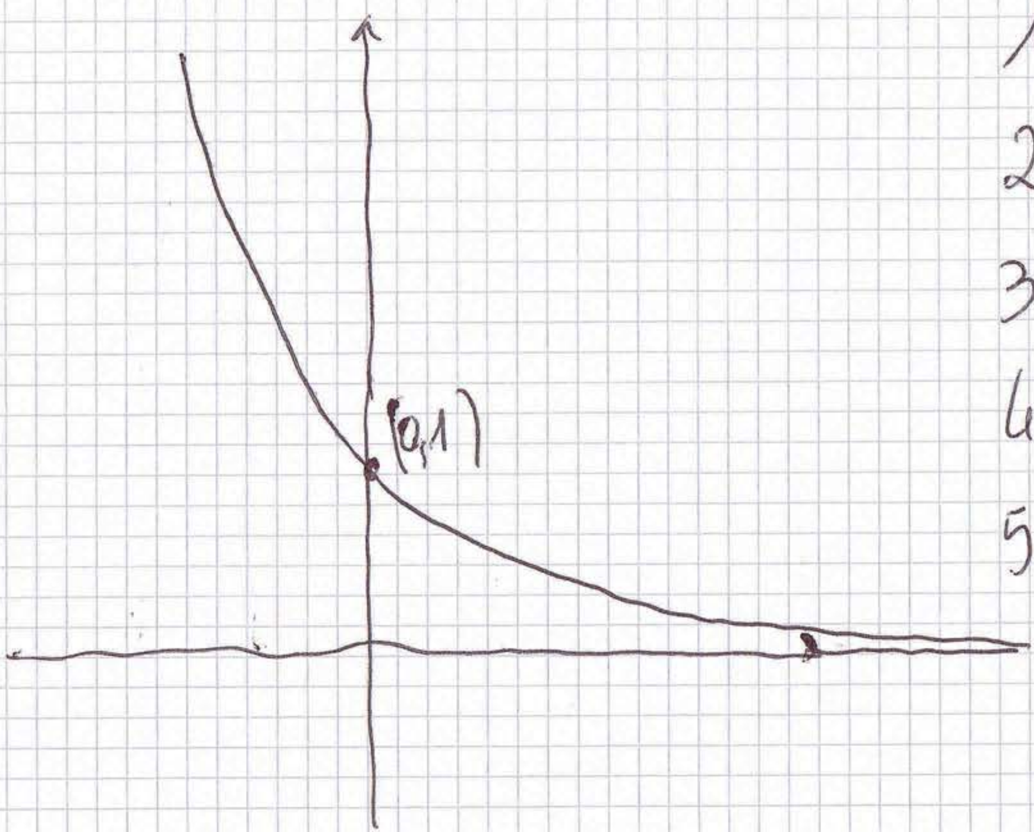
2) $y = a^x \quad 0 < a < 1$



1) $D = \mathbb{R}$
2) $a^x > 0 \ \forall x$
3) $a^0 = 1$
4) è decrescente
5) $\lim_{x \to +\infty} a^x = 0$

$\lim_{x \to -\infty} a^x = +\infty$

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$y = a^x : \mathbb{R} \longrightarrow \mathbb{R}^+$ è iniettiva
e suriettiva
è biiettiva

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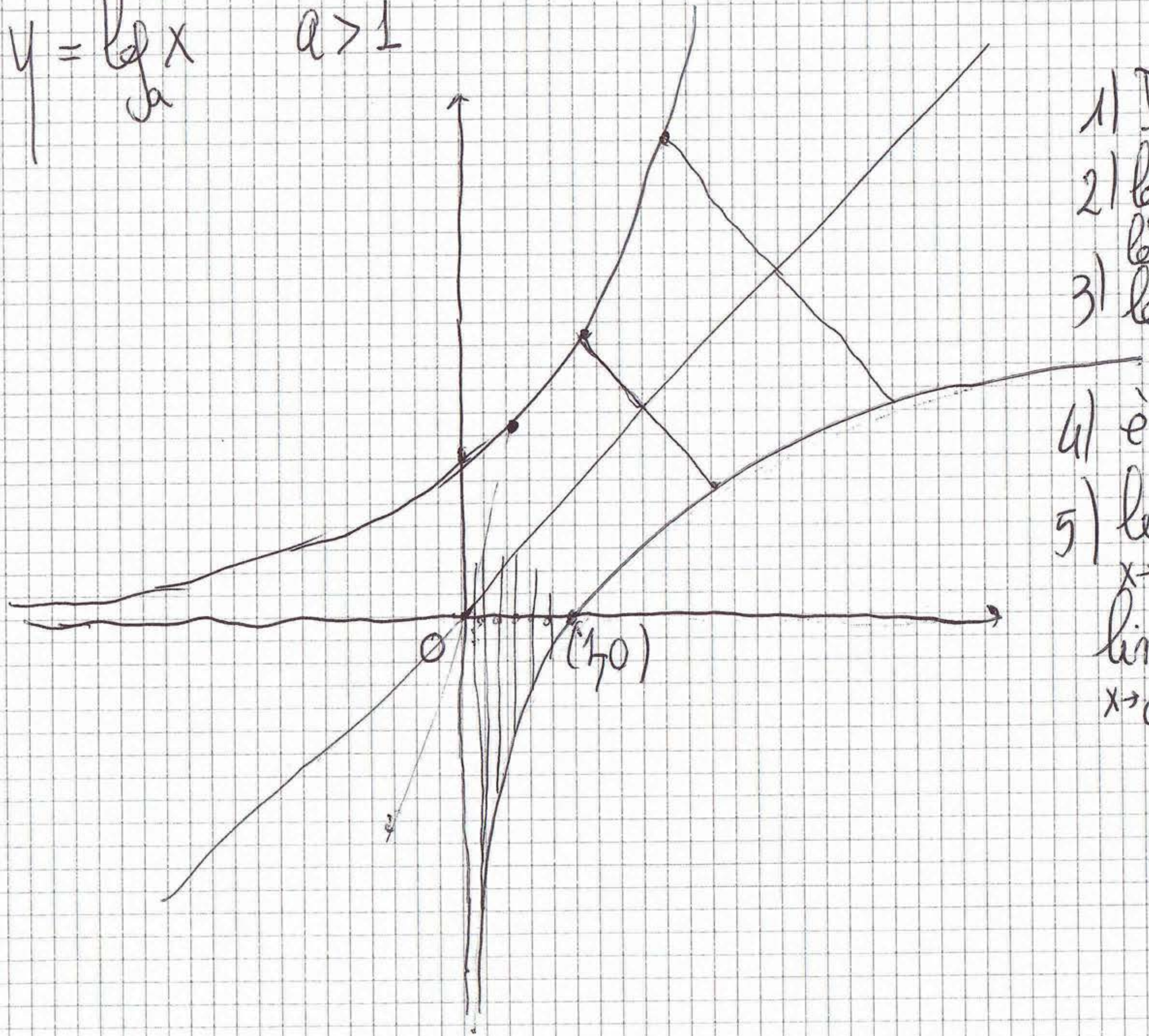
1. $y = \log_a x \quad a > 1$



1) $D = \mathbb{R}^+$

2) $\log_a x > 0$ se $x > 1$
$\log_a x < 0$ se $0 < x < 1$

3) $\log_a 1 = 0$

4) è crescente

5) $\lim\limits_{x \to +\infty} \log_a x = +\infty$
$\lim\limits_{x \to 0^+} \log_a x = -\infty$

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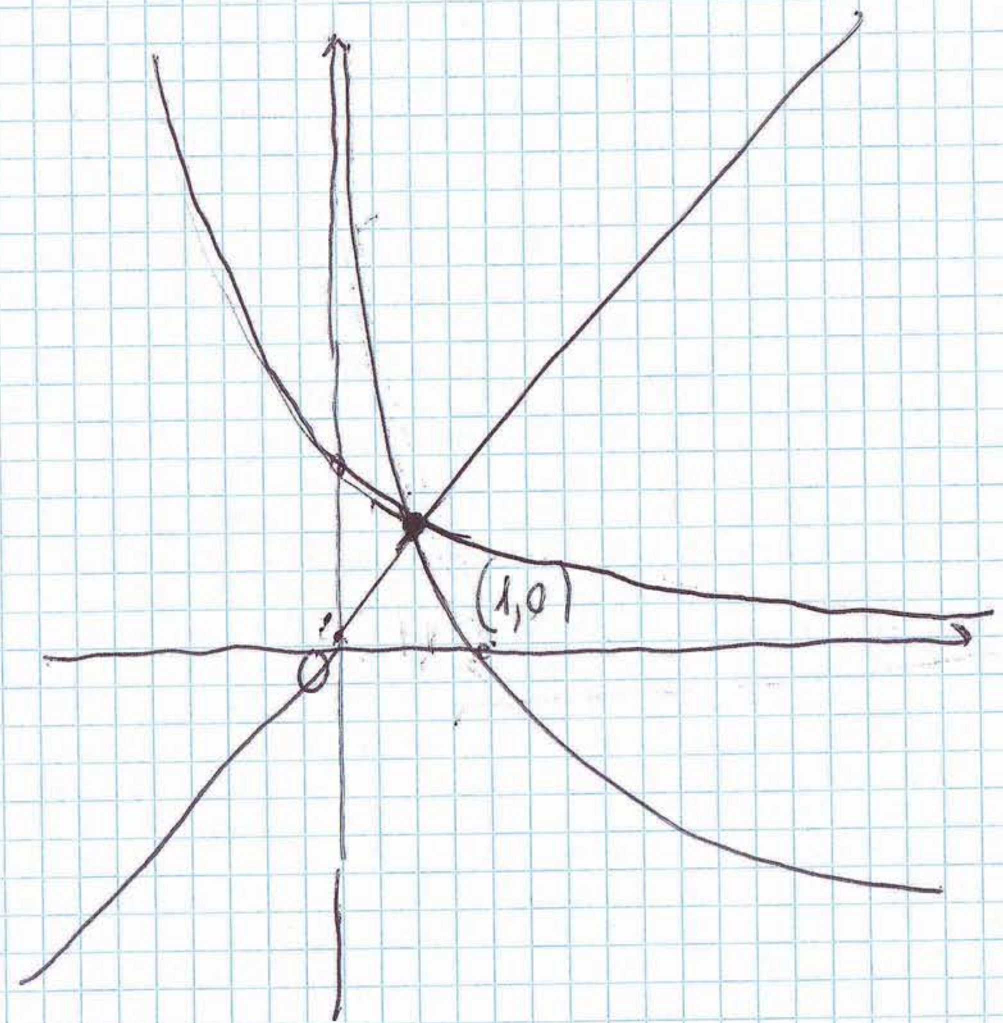
2) $y = \log_a x \qquad 0 < a < 1$



1) $D = \mathbb{R}^+$

2) $\log_a x > 0$ se $0 < x < 1$

3) $\log_a x < 0$ se $x > 1$

$\log_a 1 = 0$

4) è decrescente

5) $\lim_{x \to +\infty} \log_a x = -\infty$

$\lim_{x \to 0^+} \log_a x = +\infty$

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$y = \dfrac{2^x - 2}{2^x + 2}$ $D = \mathbb{R}$ $2^x + 2 \neq 0$

$2^x \neq -2 \quad \forall x \in \mathbb{R}$

$2^x + 2 = 0 \quad \underbrace{2^x}_{+} = \underbrace{-2}_{-} \quad D = \mathbb{R}$

$\begin{cases} x = 0 \\ y = \dfrac{2^0 - 2}{2^0 + 2} = \dfrac{1-2}{1+2} = -\dfrac{1}{3} \end{cases}$

$\overset{P(x)}{a} \gtreqless \overset{Q(x)}{a} \iff P(x) \gtreqless Q(x)$

$\begin{cases} y = 0 \\ \dfrac{2^x - 2}{2^x + 2} = 0 \end{cases}$ $2^x - 2 = 0 \quad 2^x = 2^1 \quad x = 1$

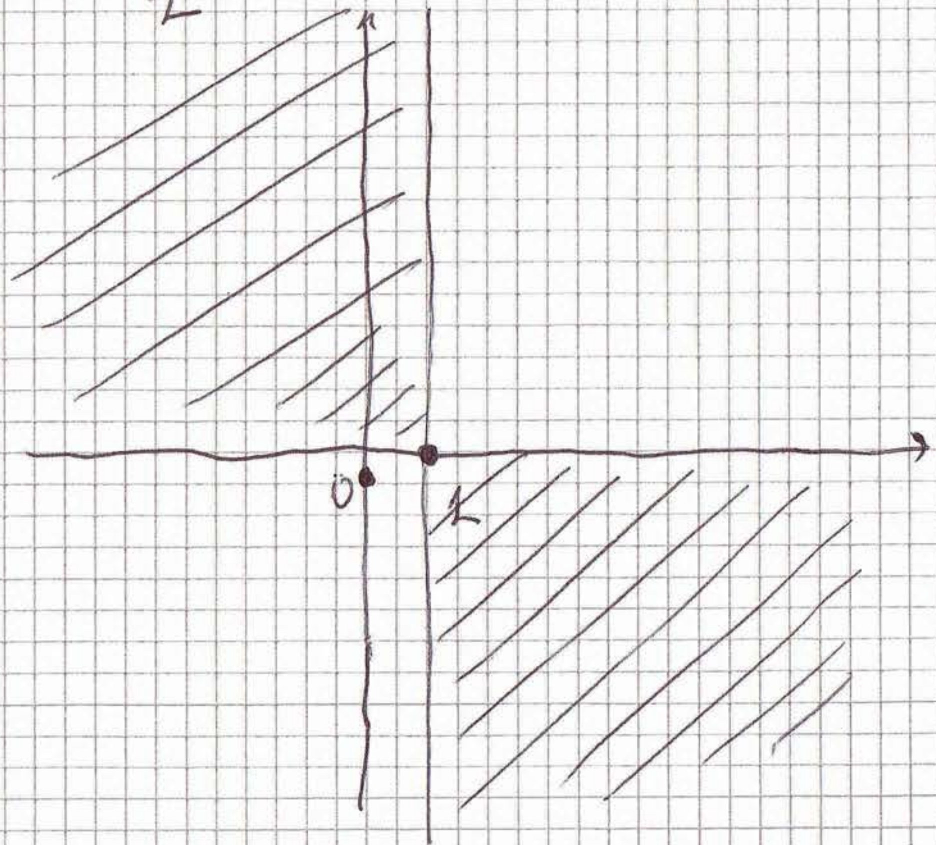
$N > 0 \quad 2^x - 2 > 0 \quad 2^x > 2 \quad x > 1$

<u>Segno</u> $\dfrac{2^x - 2}{2^x + 2} > 0$

$D > 0 \quad 2^x + 2 > 0 \quad 2^x > -2 \quad \forall x$

N	−	+
D	+	+
	−	+

1



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$$f(-x) = \frac{2^{-x}-2}{2^{-x}+2} = \frac{\frac{1}{2^x}-2}{\frac{1}{2^x}+2} = \frac{\frac{1-2\cdot 2^x}{\cancel{2^x}}}{\frac{1+2\cdot 2^x}{\cancel{2^x}}} = \frac{1-2^{x+1}}{1+2^{x+1}}$$

non è né pari né dispari

$$a^{P(x)} \gtreqless a^{Q(x)} \iff P(x) \gtreqless Q(x) \quad \text{se } a>1$$

$$a^{P(x)} \gtreqless a^{Q(x)} \iff P(x) \lesseqgtr Q(x) \quad \text{se } 0<a<1$$

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$y = \log_2 \dfrac{x-1}{x+1}$ D

$\begin{cases} \dfrac{x-1}{x+1} > 0 \\ x+1 \neq 0 \end{cases}$

$N. > 0 \quad x > 1$

$D. > 0 \quad x > -1$

N	−	−	+
D.	−	+	+
	+	−	+
		−1	1

Int. ani

$\cancel{x = 0}$

$\begin{cases} y = 0 \\ \log_2 \dfrac{x-1}{x+1} = 0 = \log_2 1 \end{cases}$

$\dfrac{x-1}{x+1} = 1 \quad \cancel{\dfrac{x-1}{x+1}} = \cancel{\dfrac{x+1}{x+1}}$

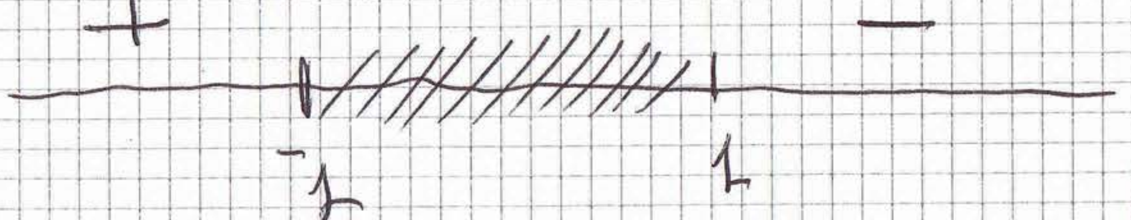
Imp.

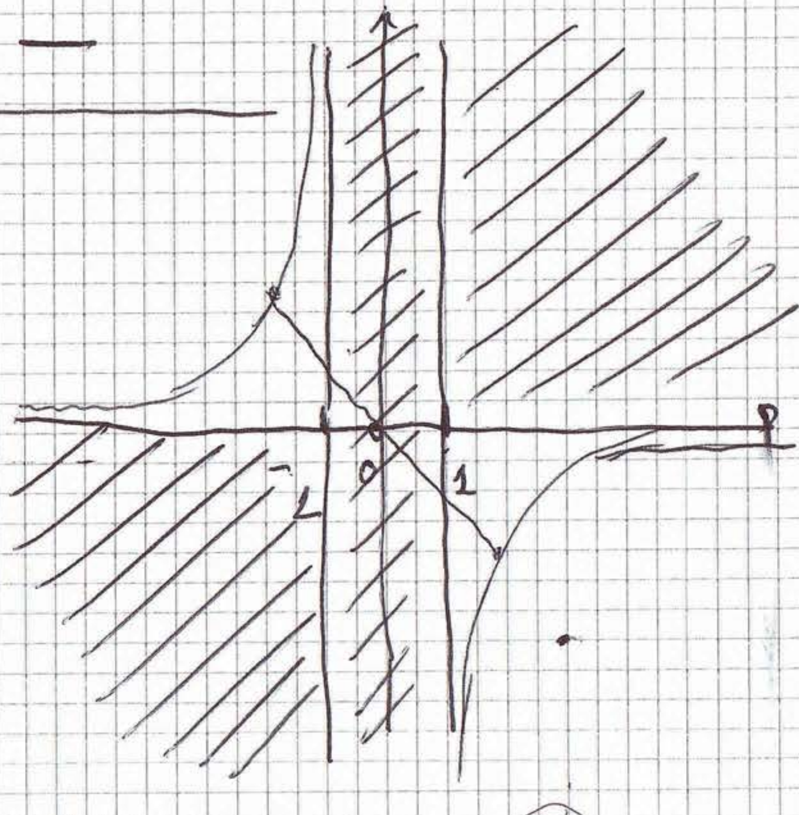
L77

$\lg_2 \frac{x-1}{x+1} > 0 = \lg_2 1 \qquad \frac{x-1}{x+1} > 1$

$\frac{\cancel{x}-1-\cancel{x}-1}{x+1} > 0 \qquad \frac{-2}{x+1} > 0 \qquad x+1 < 0$

$x < -1$





$$f(-x) = \log_2 \frac{-x-1}{-x+1} = \log_2 \frac{x+1}{x-1} = \log_2 \left(\frac{x-1}{x+1}\right)^{-1} = -\log_2 \frac{x-1}{x+1}$$

$$= -f(x)$$

$f(x)$ è dispari

FORMULA DEL CAMBIAMENTO DI BASE

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y \qquad x, y > 0$$

$$\log_a N = \frac{\log_b N}{\log_b a}$$

$$\log_a x^\alpha = \alpha \log_a x$$

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$$

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$$\log_a a = x \longleftrightarrow a^x = a \longleftrightarrow x = 1$$

$$\log_a b = c \leftrightarrow a^c = b$$

$$\log_a \frac{1}{a} = \log_a a^{-1} = -1$$

$$\log_a 1 - \log_a a = 0 - 1 = -1$$

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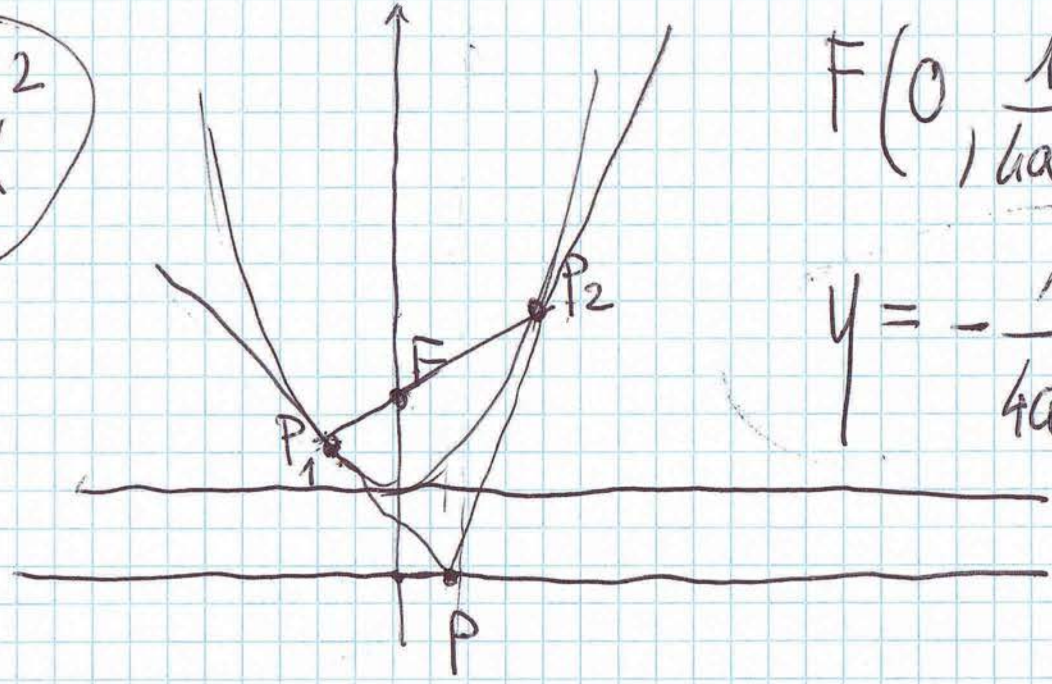
$y = ax^2$



$F\left(0, \frac{1}{4a}\right)$

$y = -\frac{1}{4a}$

$P\left(\alpha, -\frac{1}{4a}\right)$

$$\begin{cases} y + \frac{1}{4a} = m(x-\alpha) \\ y = ax^2 \end{cases}$$

$ax^2 + \frac{1}{4a} = mx - m\alpha$

$ax^2 - mx + \frac{1}{4a} + m\alpha = 0$

$\Delta = 0 \quad m^2 - 4a\left(\frac{1}{4a} + m\alpha\right) = 0 \quad m^2 - 1 - 4am\alpha = 0$

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$P_1(x_1, ax_1^2)$ $\quad P_2(x_2, ax_2^2)$ $\quad F\left(0, \frac{1}{4a}\right)$

$X_1 = \frac{m_1}{2a}$ $\quad X_2 = \frac{m_2}{2a}$

$P_1\left(\frac{m_1}{2a}, a \cdot \frac{m_1^2}{4a^2}\right)$ $\quad P_2\left(\frac{m_2}{2a}, \frac{m_2^2}{4a}\right)$

$P_1\left(\frac{m_1}{2a}, \frac{m_1^2}{4a}\right)$

$$m_{P_1F} = \frac{\frac{m_1^2}{4a} - \frac{1}{4a}}{\frac{m_1}{2a} - 0} = \frac{\frac{m_1^2 - 1}{4a}}{\frac{m_1}{2a}} = \frac{m_1^2 - 1}{2\cancel{4a}} \cdot \frac{\cancel{2a}}{m_1}$$

$$m_{P_1F} = \frac{m_1^2 - 1}{2m_1} = \frac{{}^{2}\cancel{4a}m_1\alpha + \cancel{1} - \cancel{1}}{\cancel{2m_1}} = 2a\alpha$$

$$m_{P_2F} = \frac{m_2^2 - 1}{2m_2} = 2a\alpha$$

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$m^2 - 4am\alpha - 1 = 0$ $m_1 m_2 = \frac{c}{a} = -1$

$m_1^2 - 4am_1\alpha - 1 = 0 \rightarrow m_1^2 = 4am_1\alpha + 1$