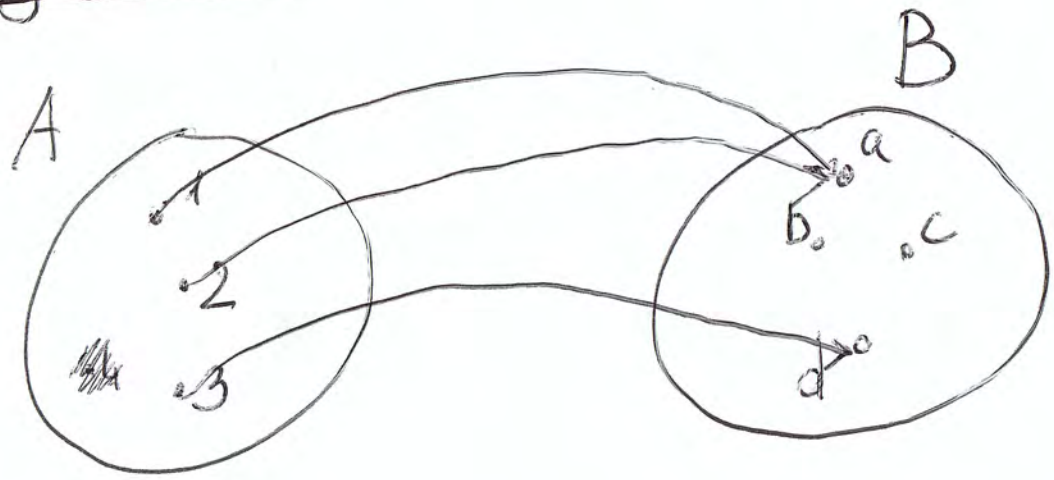
8-10-2018

FUNZIONE

Si ha una funzione dall'insieme A all'insieme B quando è possibile associare ad ogni elemento di A uno e un solo elemento di B.



Si ha una funzione reale di variabile reale quando $A \subseteq \mathbb{R}$ e $B = \mathbb{R}$.

$$f : A \subseteq \mathbb{R} \longrightarrow B = \mathbb{R}$$

$$f(x) = x^2 \qquad f(1) = 1^2 = 1 \qquad f(-3) = (-3)^2 = 9$$

$$f(\sqrt{2}+1) = (\sqrt{2}+1)^2 = 2+1+2\sqrt{2} = 3+2\sqrt{2}$$

Dominio di $f(x)$ è $\mathbb{R}$.

Dominio di $f(x)$ sarà il più grande sottoinsieme di $\mathbb{R}$ nel quale $f(x)$ si può calcolare.

Campo di esistenza | Insieme di definizione

$f(x)$ = polinomio

$$2x^3 - 4x^2 + 5x + \sqrt{2}$$

$D = \mathbb{R}$

$$f(x) = \frac{P(x)}{H(x)}$$

$D = \mathbb{R} \setminus \{\text{zeri di } H\}$

$$f(x) = \frac{2x^2 + 3}{x - 1} \qquad x - 1 \neq 0 \qquad x \neq 1$$

$D = \mathbb{R} \setminus \{1\}$

$$f(x) = \frac{x^2+2x+3}{x^2+x+1}$$

$x^2+x+1=0 \quad \Delta = 1-4 = -3 < 0$

$D = \mathbb{R}$

$$f(x) = \sqrt{x+1} + \sqrt{x-1}$$

$$\begin{cases} x+1 \geq 0 \\ x-1 \geq 0 \end{cases} \quad \begin{cases} x \geq -1 \\ x \geq 1 \end{cases}$$

$[1, +\infty[$

$]1, +\infty[$

$]1,2[$ $\quad [1,2]$

$[1,2[$ $\quad]1,2]$

$$f(x) = \frac{\sqrt{x+1}}{\sqrt{x-1}}$$

$$\begin{cases} x+1 \geq 0 \\ x-1 \geq 0 \\ \sqrt{x-1} \neq 0 \end{cases}$$

$$\begin{cases} x \geq 1 \\ x \neq 1 \end{cases} \quad \boxed{x > 1}$$

$$f(x) = \sqrt{\frac{x+1}{x-1}}$$

$$\begin{cases} \frac{x+1}{x-1} \geq 0 \\ x-1 \neq 0 \quad x \neq 1 \end{cases}$$

$N \geq 0 \quad x \geq -1$

$D > 0 \quad x > 1$

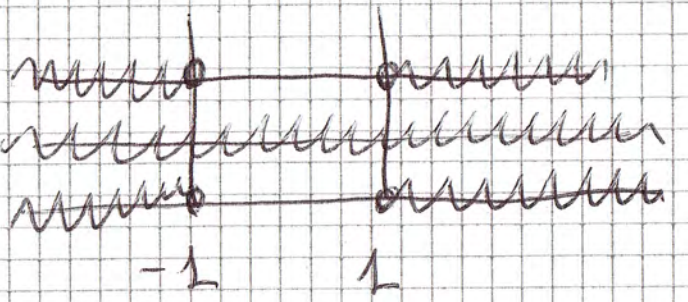
	$-\infty$		-1		1		$+\infty$
N	$-$		$+$		$+$		
D	$-$		$-$		$+$		
	$+$		$-$		$+$		

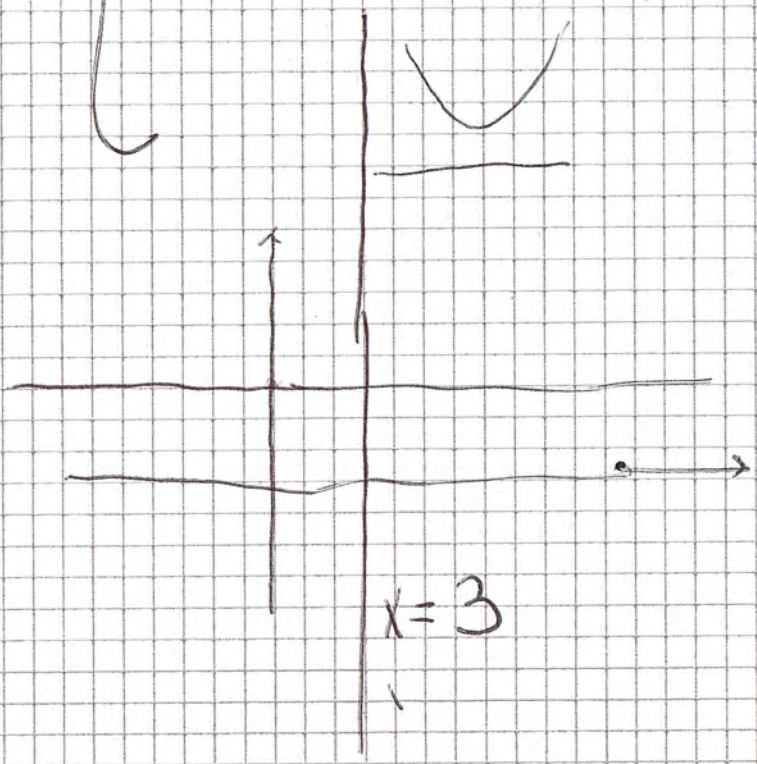
$$]-\infty, -1] \cup]1, +\infty[$$

$$x \leq -1 \vee x > 1$$

$$f(x)=\frac{\sqrt{x^2-1}}{\sqrt{x^2+x+1}}$$

$$\begin{cases} x^2-1\geq 0 & x\leq -1 \vee x\geq 1 \\ x^2+x+1>0 & \Delta<0 \quad \mathbb{R} \end{cases}$$

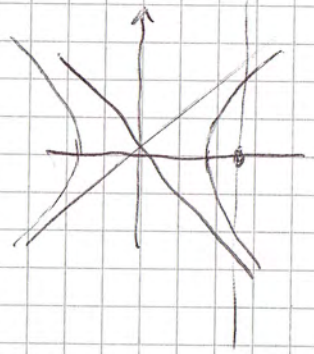


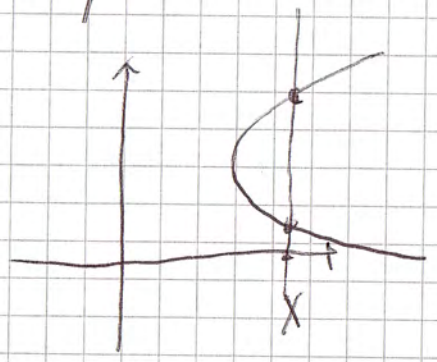


$f(x)=y=2$

$f(x)=y=2x+3$

$y = ax^2 + bx + c$ $\quad$ $x = ay^2 + by + c$





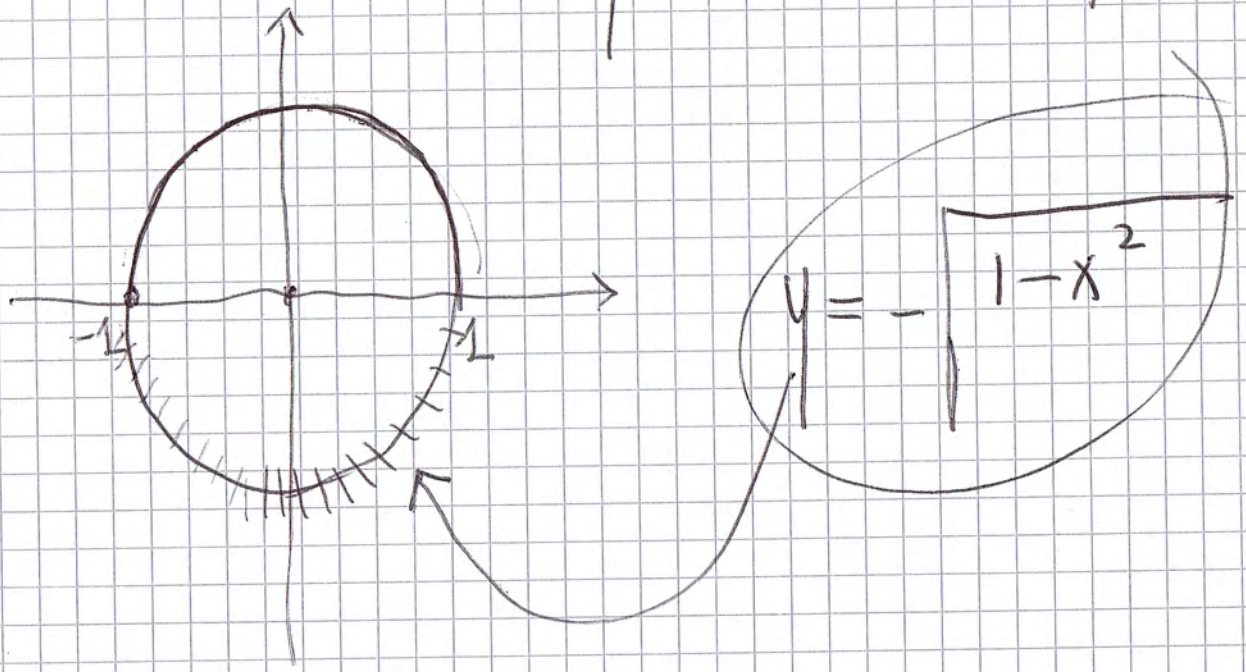
$$y = \frac{ax+b}{cx+d}$$

$$y = \sqrt{1-x^2}$$

$1 - x^2 \geq 0$ $\quad$ $x^2 \leq 1$ $\quad$ $-1 \leq x \leq 1$

$$y = \sqrt{1-x^2}$$

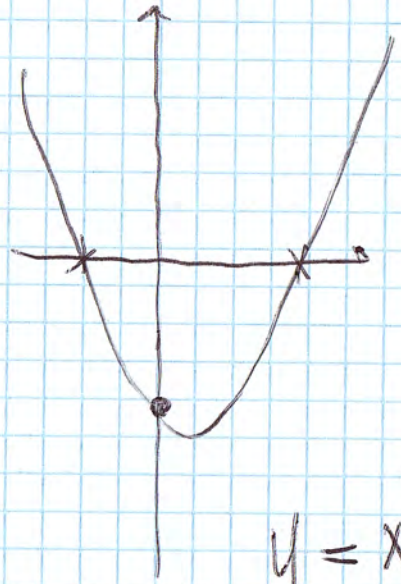
$$\begin{cases} -1 \le x \le 1 \\ y \ge 0 \\ y^2 = 1 - x^2 \end{cases} \longrightarrow x^2 + y^2 = 1$$



Intersezioni con gli assi

$$\begin{cases} y = f(x) \\ x = 0 \end{cases}$$

$$\begin{cases} y = f(x) \\ y = 0 \end{cases} \Leftrightarrow \begin{cases} f(x) = 0 \\ y = 0 \end{cases}$$



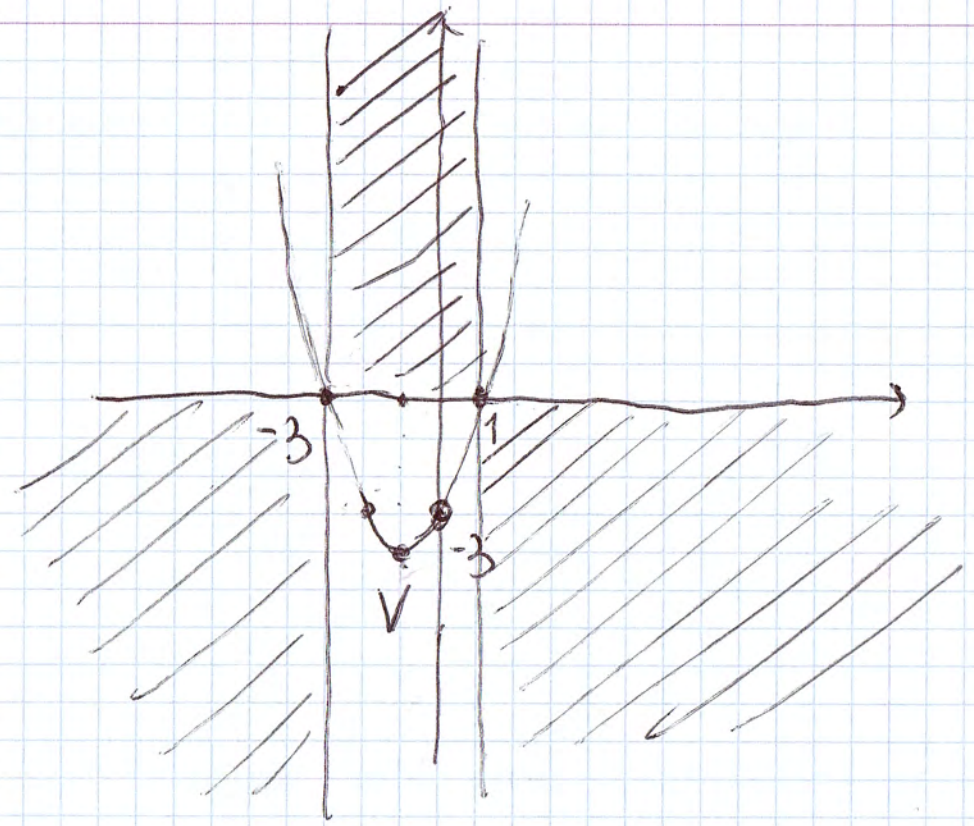
$y = x^2 + 2x - 3$

$D = \mathbb{R}$

$$\begin{cases} x = 0 \\ y = -3 \end{cases} \qquad \begin{cases} y = 0 \\ x^2 + 2x - 3 = 0 \end{cases}$$

$1 \quad -3$

Segno



$y = x^2 + 2x - 3$

$V\left(-\frac{b}{2a}, \frac{-\Delta}{4a}\right)$

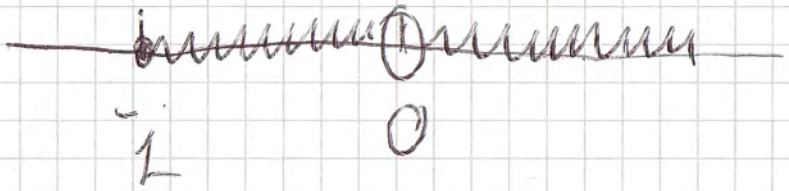
$V\left(\frac{-2}{2} = -1, -4\right)$

$$y = \frac{\sqrt{x+1}}{x}$$

Dominio $\begin{cases} x+1 \geq 0 \\ x \neq 0 \end{cases} \quad \begin{cases} x \geq -1 \\ x \neq 0 \end{cases}$

$\cancel{x=0}$

$\begin{cases} y=0 \\ \frac{\sqrt{x+1}}{x}=0 \end{cases}$

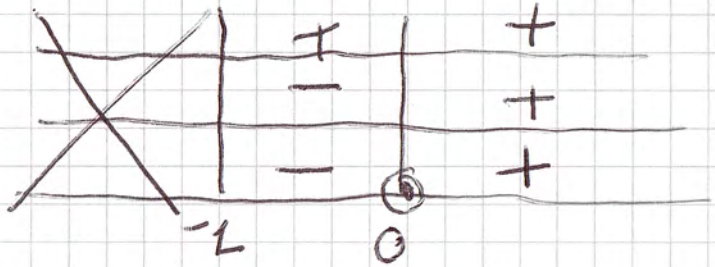


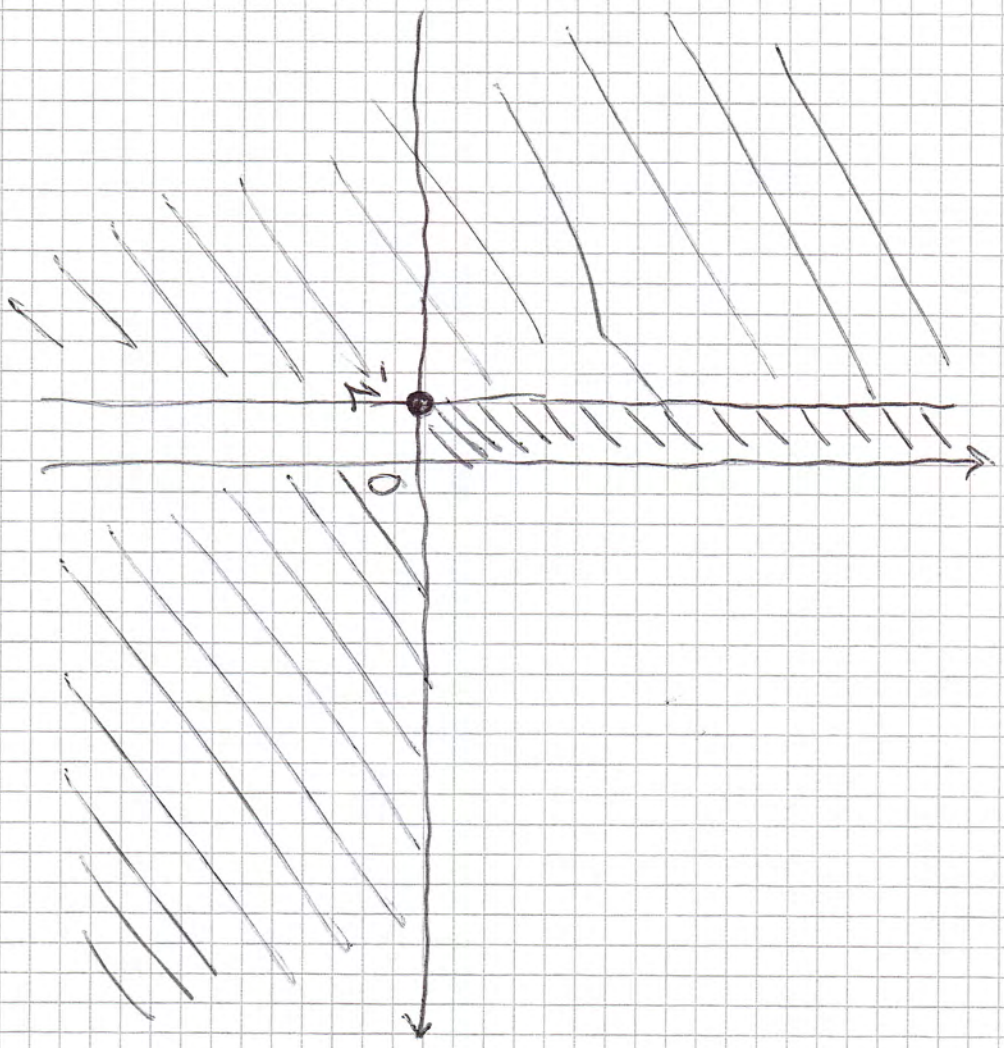
$\sqrt{x+1}=0 \quad x+1=0 \quad x=-1 \quad (-1,0)$

Segno $\frac{\sqrt{x+1}}{x} \geq 0$

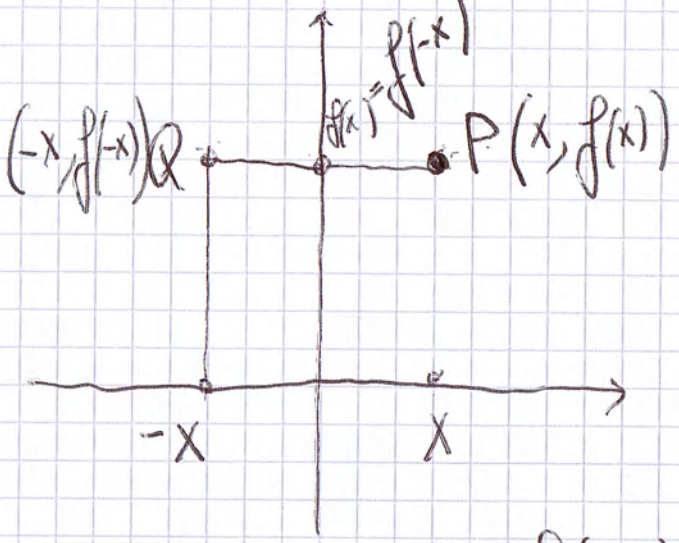
$N. \geq 0 \quad \sqrt{x+1} \geq 0$ sempre nel D.

$x > 0$



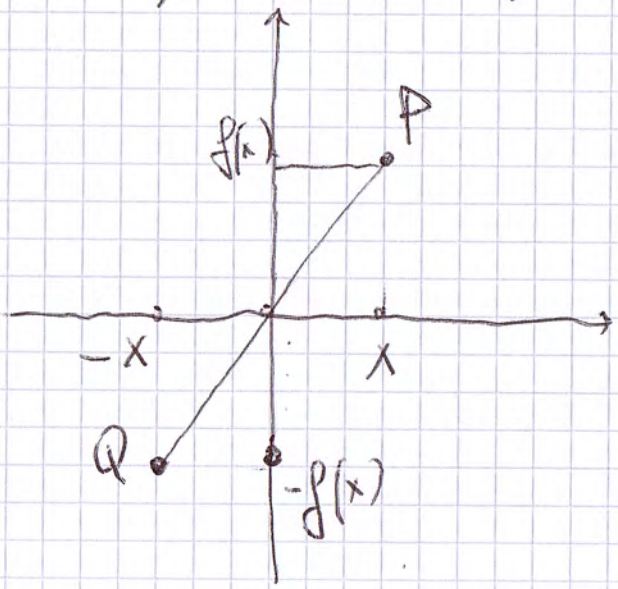

N'
O

① $f(x)$ è pari se $\forall x \in D, -x \in D$ e $f(-x) = f(x)$

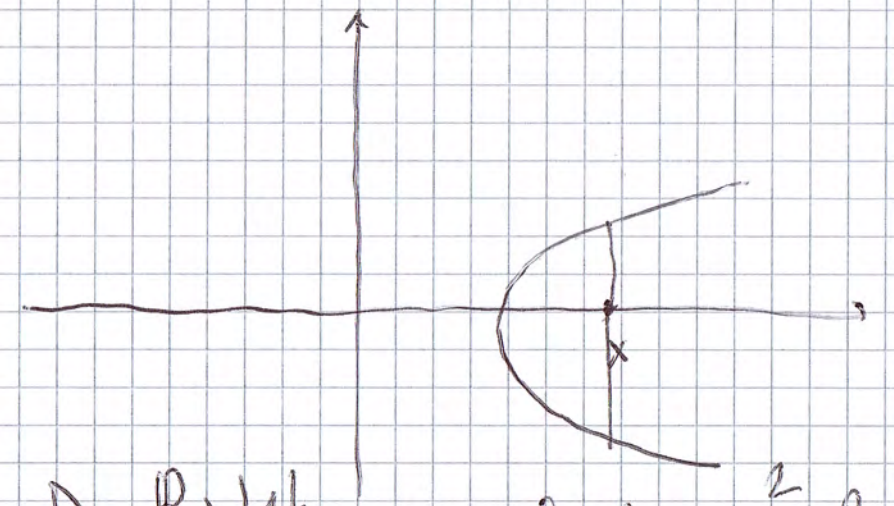


se $f(x)$ è pari
$f(x)$ è simmetrica
rispetto all'asse y

② $f(x)$ è dispari se $\forall x \in D, -x \in D$ e $f(-x) = -f(x)$



se $f(x)$ è dispari
$f(x)$ è simmetrica rispetto
all'origine



$f(x) = \frac{1}{x-1}$ $D = \mathbb{R} \setminus \{1\}$

$f(x) = x^2 + 2x + 1$ $D = \mathbb{R}$

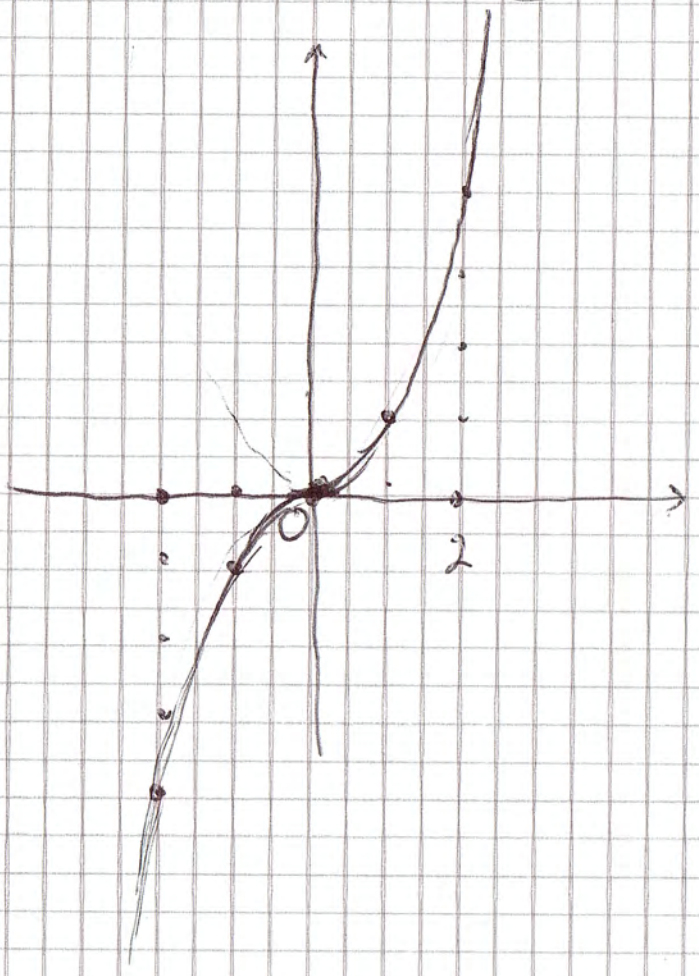
$f(-x) = (-x)^2 + 2(-x) + 1 = x^2 - 2x + 1 \overset{?}{=} x^2 + 2x + 1$ no!

$\overset{?}{=} -x^2 - 2x - 1$

$f(x) = x|x| \qquad D = \mathbb{R}$

$f(-x) = -x|-x| = -(x|x|) = -f(x)$ $f(x)$ è dispari

$f(x) = x^2 \quad x \geq 0$



$f(x)$ è iniettiva se mande elementi distinti in elementi distinti

$\forall x_1, x_2$ se $x_1 \neq x_2 \longrightarrow f(x_1) \neq f(x_2)$

$f(x) = x^2$ $\quad f(1) = f(-1) = 1$

$f(x) = \frac{x}{x+1}$ $\quad D = \mathbb{R} \setminus \{-1\}$ $\quad f(2) = \frac{2}{3}$ $\quad f(1) = \frac{1}{2}$

$$x_1 \neq x_2 \rightarrow f(x_1) \neq f(x_2)$$

se $f(x_1) = f(x_2) \rightarrow x_1 = x_2$

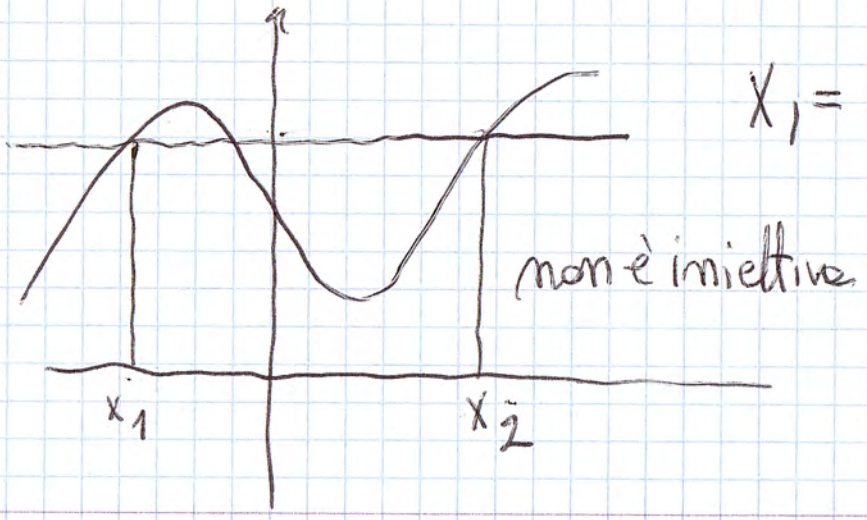
$$f(x) = \frac{x}{x+1} \qquad x \neq -1$$

$$f(x_1) = f(x_2) \qquad \frac{x_1}{x_1+1} = \frac{x_2}{x_2+1}$$

$$\frac{x_1(x_2+1)}{(x_1+1)(x_2+1)} = \frac{x_2(x_1+1)}{(x_2+1)(x_1+1)}$$

$$x_1x_2 + x_1 = x_2x_1 + x_2$$

$$x_1 = x_2$$



non è iniettiva