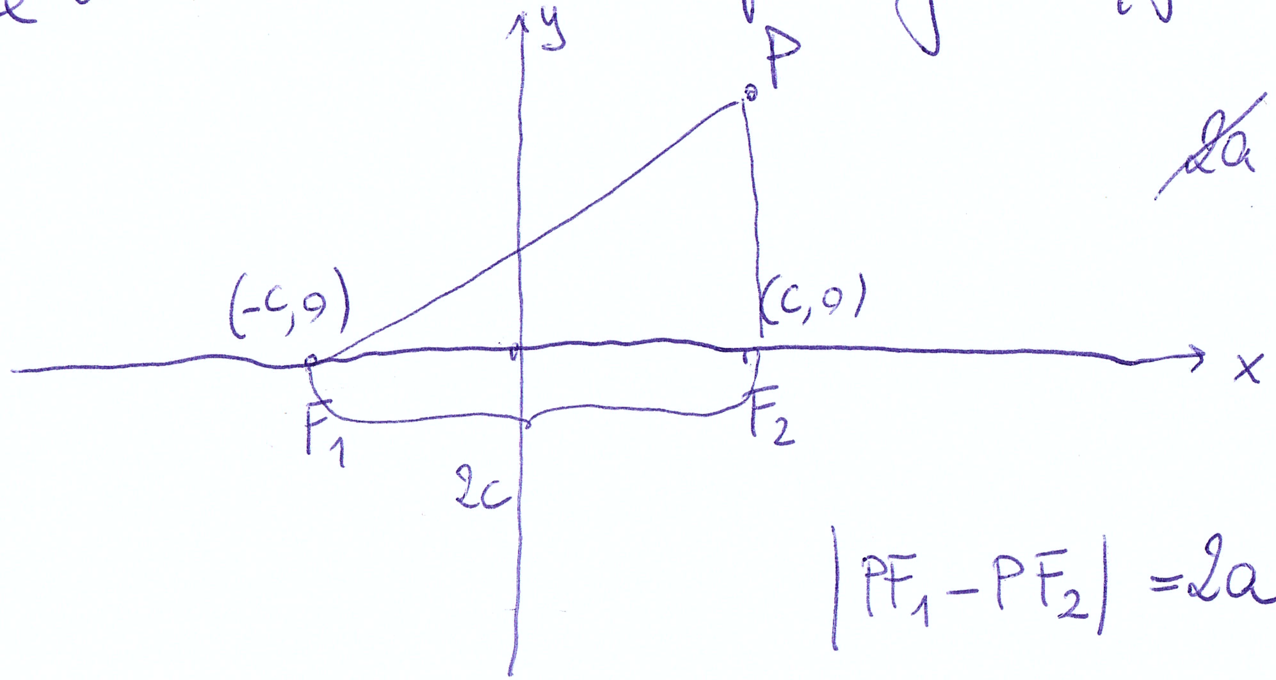
3/10/2018

Iperbole

(36

È il luogo geometrico dei punti del piano per i quali è costante la differenza delle distanze da due punti fissi (fuochi).



$\cancel{2}a < \cancel{2}c$

$|PF_1 - PF_2| = 2a$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

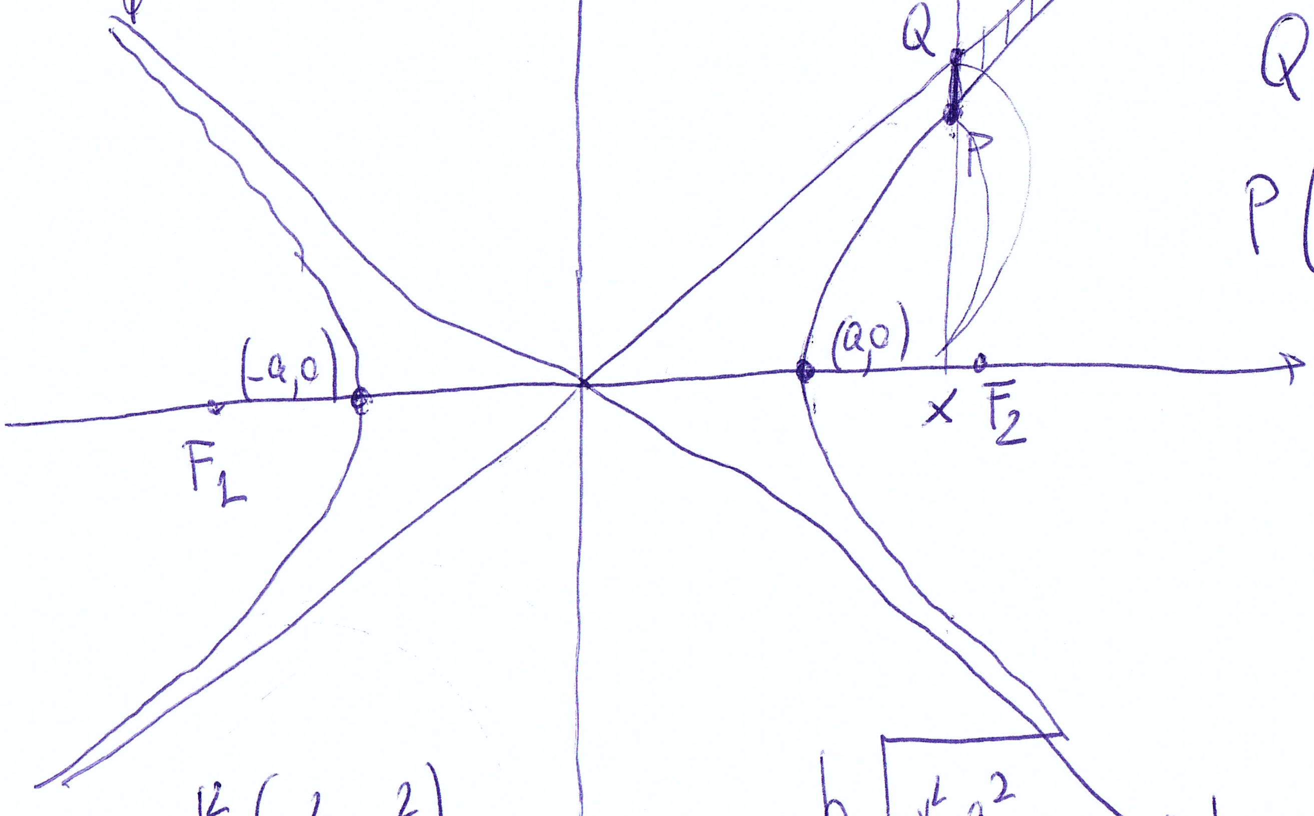
$b^2 = c^2 - a^2$

$a^2 + b^2 = c^2$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad b^2x^2-a^2y^2=a^2b^2$$

$$+a^2y^2=+b^2x^2\mp a^2b^2$$

$$y=\frac{b}{a}x$$



$$Q\left(x, \frac{b}{a}x\right)$$

$$P\left(x, \frac{b}{a}\sqrt{x^2-a^2}\right)$$

$$y^2=\frac{b^2(x^2-a^2)}{a^2} \longrightarrow y=\pm\frac{b}{a}\sqrt{x^2-a^2} \qquad y=-\frac{b}{a}x$$

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$$\overline{PQ} = \frac{b}{a}x - \frac{b}{a}\sqrt{x^2-a^2} = \frac{b}{a}\left(x - \sqrt{x^2-a^2}\right)$$

$$\frac{b}{a}\left(x - \sqrt{x^2-a^2}\right) \xrightarrow{x \to +\infty} \frac{b}{a} \cdot \frac{\left(x - \sqrt{x^2-a^2}\right)\left(x + \sqrt{x^2-a^2}\right)}{x + \sqrt{x^2-a^2}} =$$

$$= \frac{b}{a} \cdot \frac{x^2 - (x^2-a^2)}{x + \sqrt{x^2-a^2}} = \frac{b}{\cancel{a}} \cdot \frac{\cancel{x^2} - \cancel{x^2} + \cancel{a^2}}{x + \sqrt{x^2-a^2}} = \frac{ab}{x + \sqrt{x^2-a^2}}$$

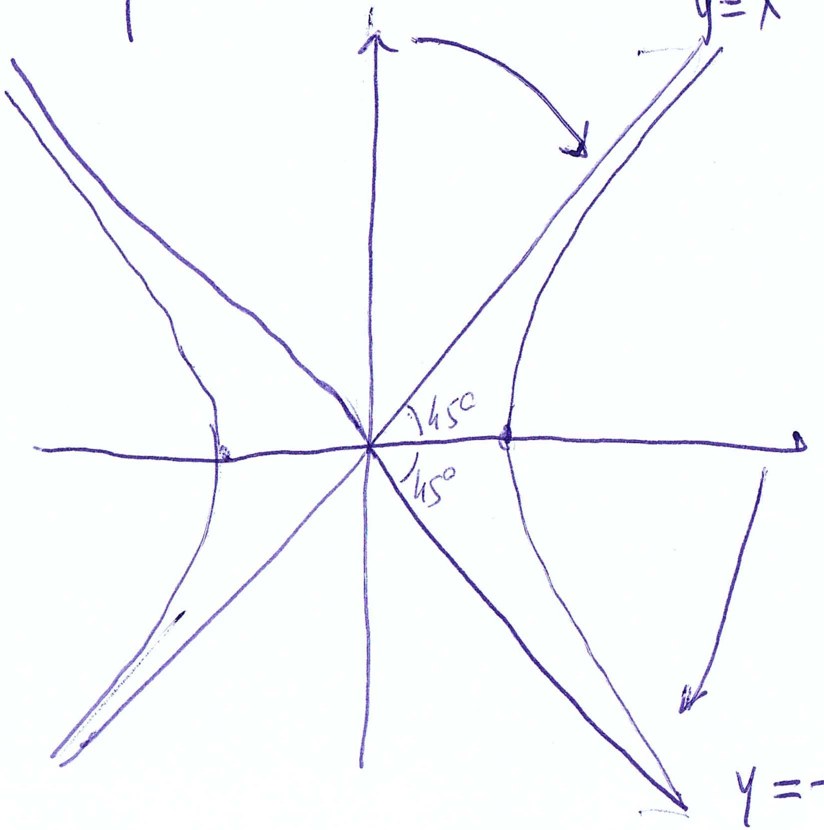
$$\downarrow$$

$$0$$

Se $a=b$ l'iperbole si dice equilatera.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad x^2-y^2=a^2$$

$$y=\pm\frac{b}{a}x \qquad y=\pm x$$



$y=x$ $m=1$

$y=-x$ $m=-1$

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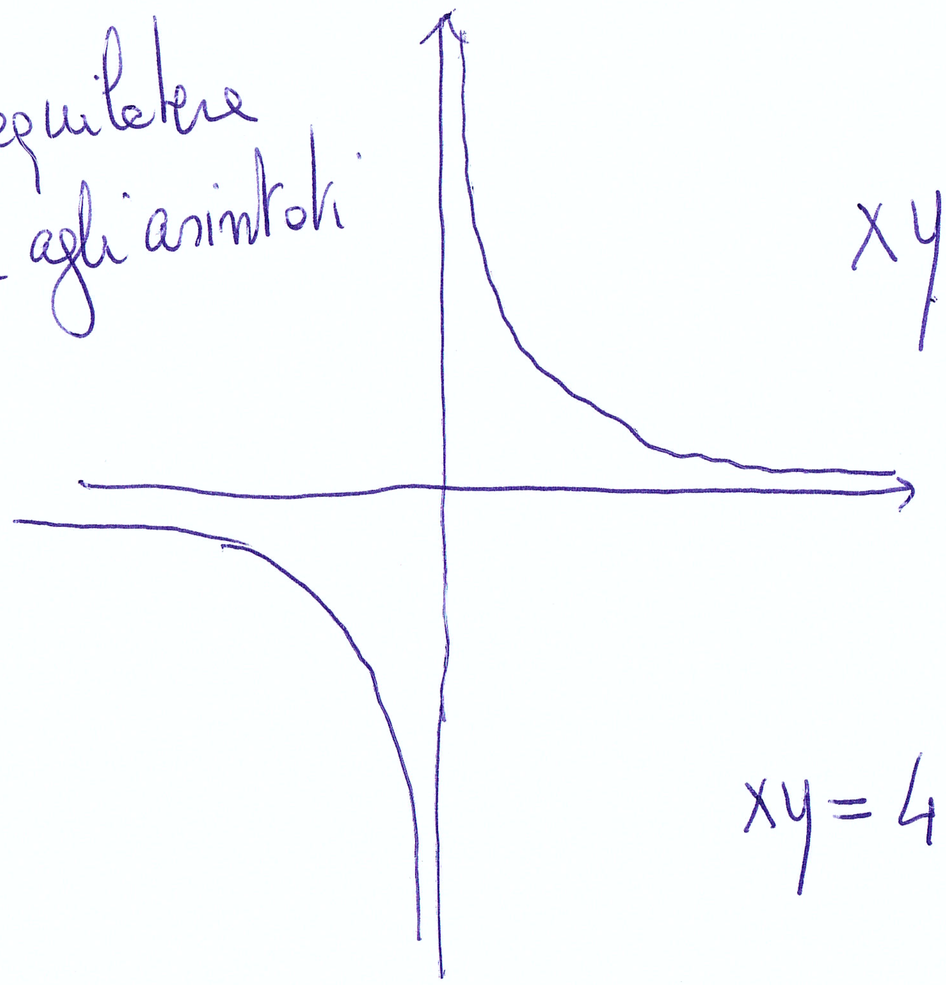
iperbole equilatera riferita agli asintoti



$$xy = \frac{a^2}{2}$$

$$xy = 4$$

x	y
1	4
2	2
4	1
8	$\frac{1}{2}$

$$y = 2x$$

x	y
1	2
2	4
3	6

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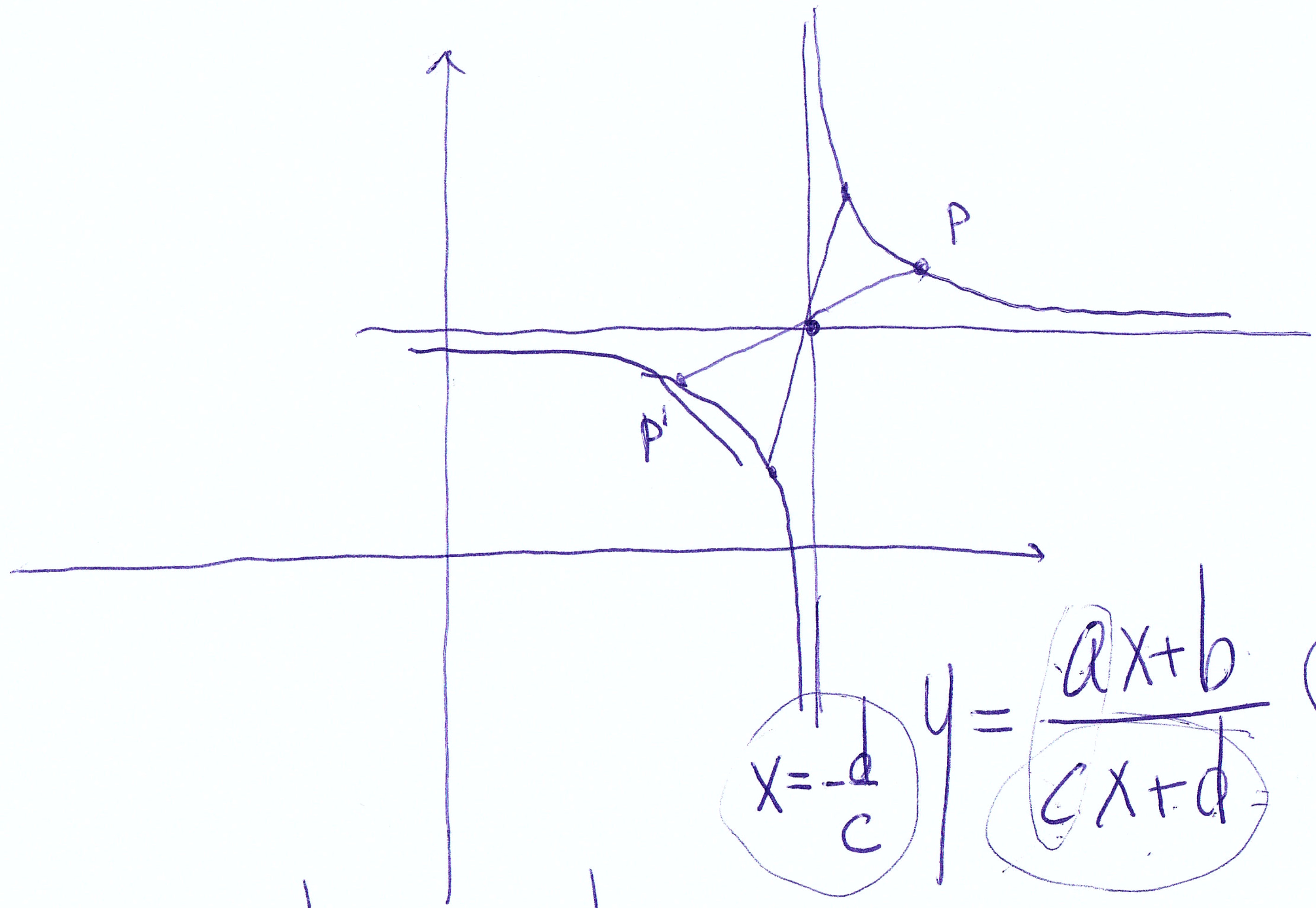


$c \neq 0$

$ad \neq bc$

$$y=\frac{ax+b}{cx+d}$$

$$y=\frac{ax+b}{d}=\frac{a}{d}x+\frac{b}{d}$$

$$y=\frac{3x+2}{6x+4}=\frac{\cancel{3x+2}}{2\cancel{(3x+2)}}=\frac{1}{2}$$

$x \neq -\frac{2}{3}$

$\lfloor 4^2$

$$y = \frac{2x-4}{x+1}$$

$c=1 \quad a=2 \quad b=-4$

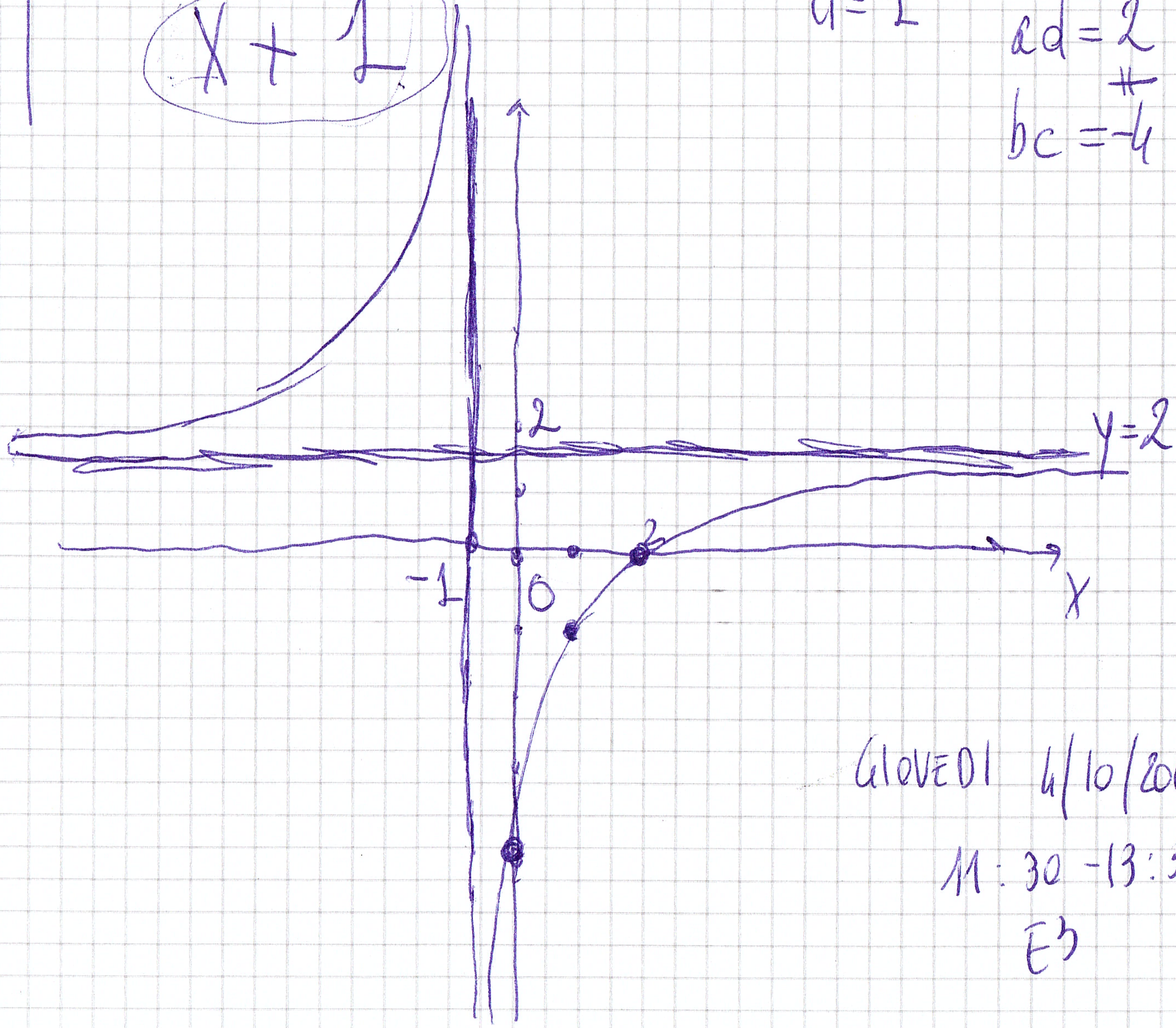
$d=1$

$ad = 2$
$\neq$
$bc = -4$

$x = -1$

$y = 2$

x	y
1	-1
2	0
0	-4



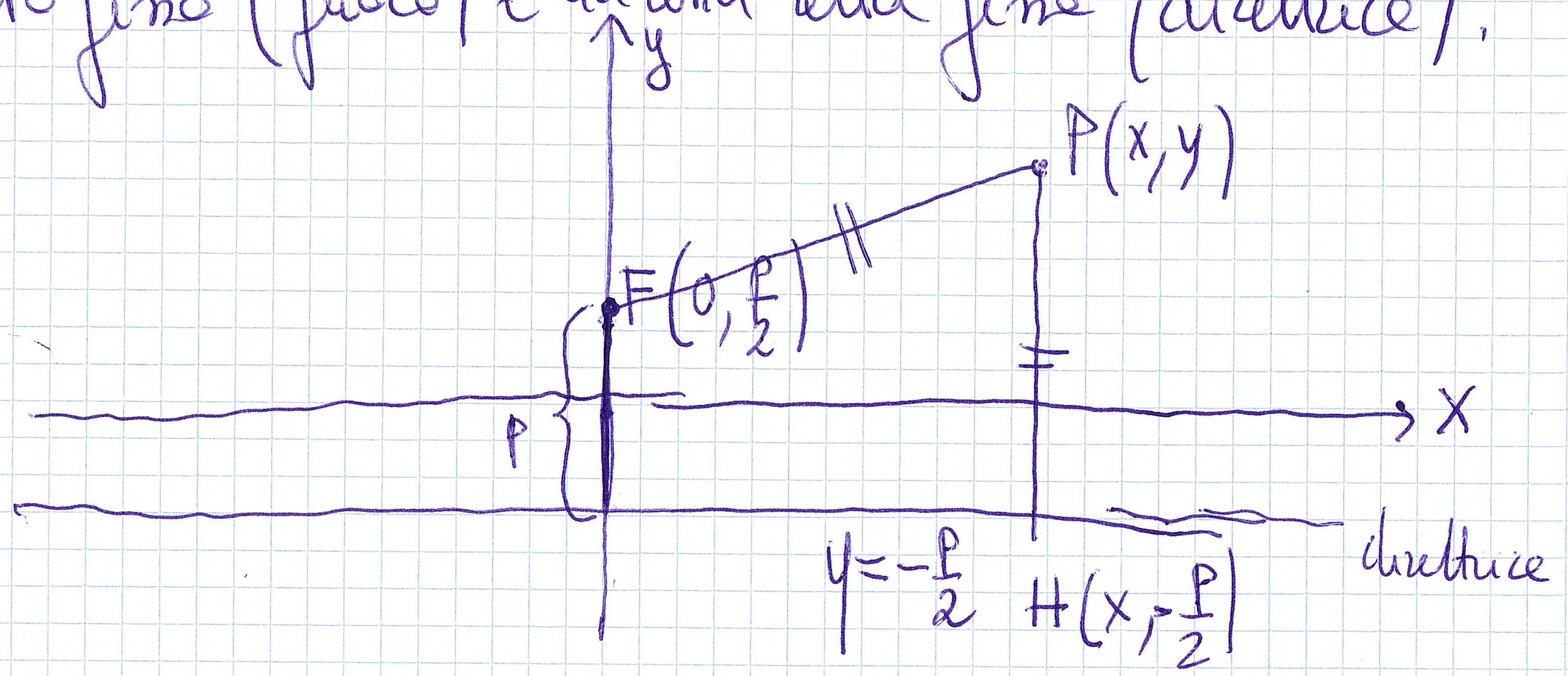
GIOVEDI 4/10/2018

11:30 - 13:30

E3

PARABOLA

È il luogo geometrico dei punti del piano equidistanti da un punto fisso (fuoco) e da una retta fissa (direttrice).



$$\overline{PF} = \overline{PH}$$

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$P(x, y) \quad F\left(0, \frac{p}{2}\right) \quad H\left(x, -\frac{p}{2}\right) \quad \overline{PF}^2 = \overline{PH}^2$

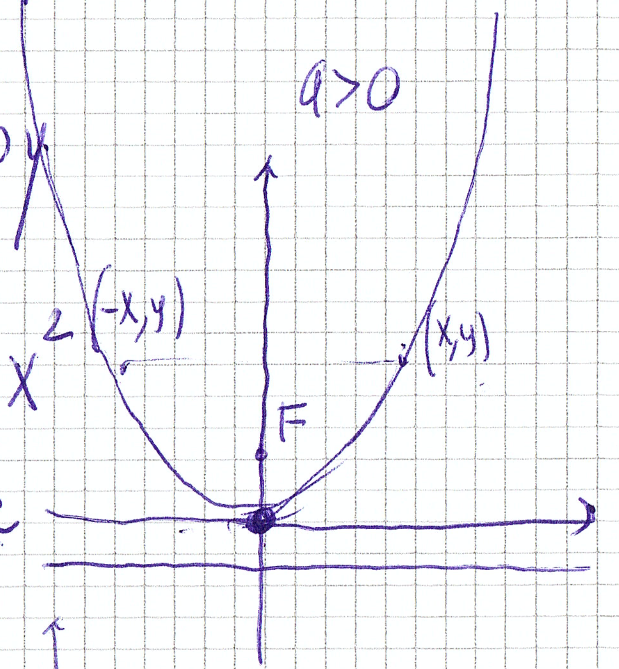
$$(x-0)^2 + \left(y - \frac{p}{2}\right)^2 = \cancel{(x-x)^2} + \left(y + \frac{p}{2}\right)^2$$

$$x^2 + \cancel{y^2} + \cancel{\frac{p^2}{4}} - py = \cancel{y^2} + \cancel{\frac{p^2}{4}} + py$$

$$x^2 = 2py \qquad y = \frac{1}{2p} x^2$$

a

$$y = a x^2$$



$y = -2x^2$

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$y = ax^2 + bx + c \qquad a \neq 0$

$\Delta = b^2 - 4ac$

$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$ vertice

$F\left(-\frac{b}{2a}, \frac{1-\Delta}{4a}\right)$ fuoco

$x = -\frac{b}{2a}$ asse

$y = \frac{-1-\Delta}{4a}$ direttrice

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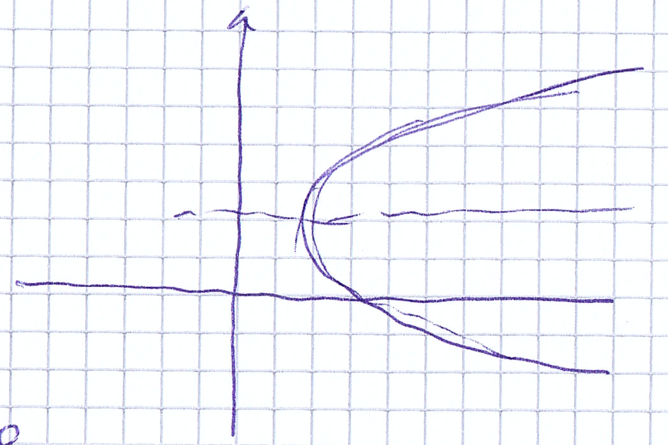
$$x = ay^2 + by + c$$

$V\left(\frac{-\Delta}{4a}, -\frac{b}{2a}\right)$ vertice

$F\left(\frac{1-\Delta}{4a}, -\frac{b}{2a}\right)$ fuoco

$y = -\frac{b}{2a}$ asse

$x = \frac{-1-\Delta}{4a}$ direttrice



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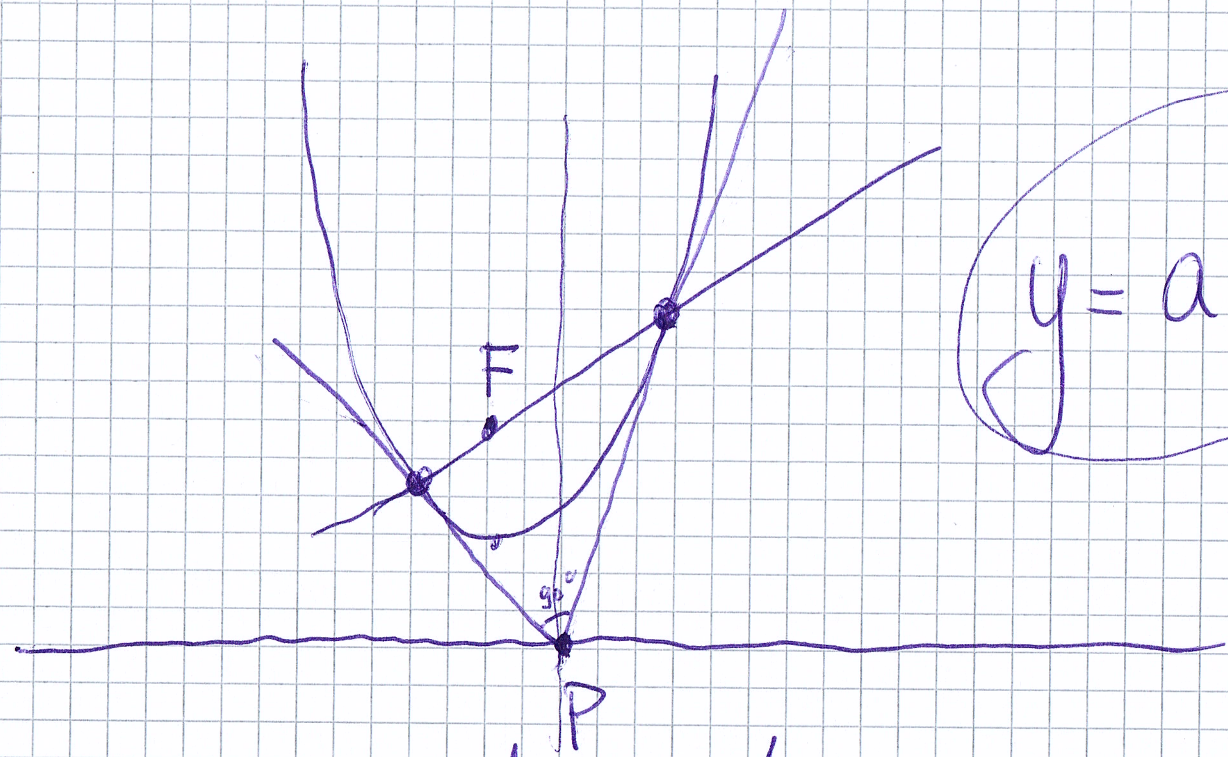

$$y = ax^2$$

$$y = \frac{-1-\Delta}{4a} = -\frac{1}{4a} \qquad P\left(\alpha, -\frac{1}{4a}\right)$$

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$$\begin{cases} y = ax^2 \\ y + \frac{1}{4a} = m(x-\alpha) \end{cases} \qquad \begin{cases} y = ax^2 \\ ax^2 + \frac{1}{4a} = mx - m\alpha \end{cases}$$

$$ax^2 - mx + \frac{1}{4a} + m\alpha = 0 \qquad \Delta = 0$$

$$\Delta = m^2 - 4a\left(\frac{1}{4a} + m\alpha\right) = 0$$

$$m^2 - 1 - 4am\alpha = 0 \qquad m^2 - 4a\alpha m - 1 = 0$$

$$\Delta = 16a^2\alpha^2 + 4 > 0$$

$$m_1 = -\frac{1}{m_2} \qquad m_1 m_2 = -1 \qquad m_1 m_2 = -1$$

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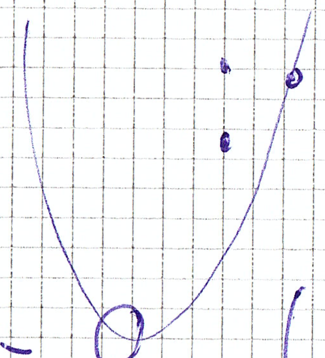
$A(1,0)$

$B(2,4)$

$C(3,8)$

$y = ax^2 + bx + c$



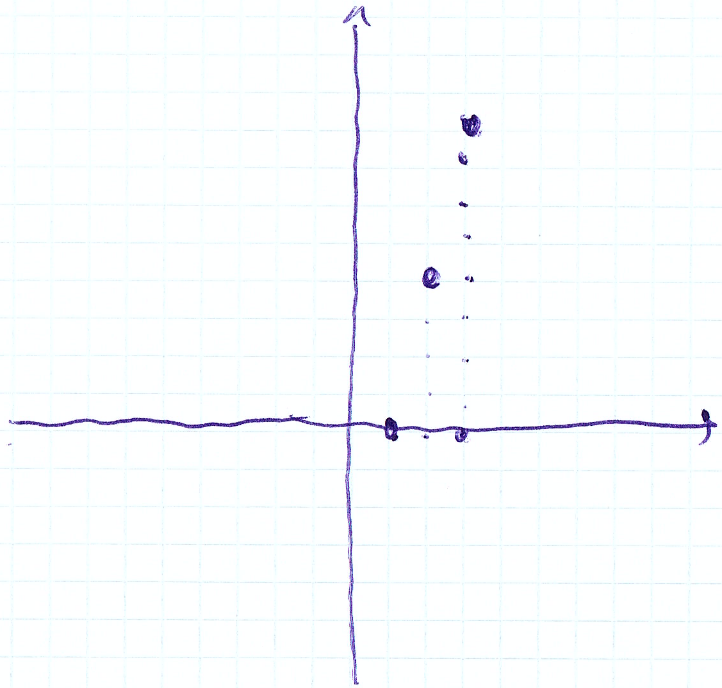
$$\begin{cases} \cancel{0} = a + b + c = 0 \\ \cancel{4} = 4a + 2b + c = 4 \\ \cancel{8} = 9a + 3b + c = 8 \end{cases}$$

$$\begin{cases} c = -a - b \\ 4a + 2b - a - b = 4 \\ 9a + 3b - a - b = 8 \end{cases}$$

$$\begin{cases} 3a + b = 4 \\ 8a + 2b = 8 \end{cases}$$

$$\begin{cases} b = 4 - 3a \\ 8a + \cancel{8} - 6a = \cancel{8} \end{cases} \quad 2a = 0 \quad a = 0$$

(50



$(1,0)$ $(2,4)$

$$\frac{x-1}{2-1} = \frac{y-0}{4-0}$$

$(3,8)$

$$\frac{x-1}{1} = \frac{y}{4}$$

$4x-4=y$ $y=4x-4$