STATISTICA

Y	y_1	y_2	....	y_m
Freq. ASS.	f_1	f_2	....	f_m
freq. REL	p_1	p_2		p_m

Y voto	6	7	8	9	10
Freq. ASS					

$$N = f_1 + f_2 + \dots + f_m$$

$$\text{media} = E(Y) = \frac{f_1 \cdot y_1 + f_2 \cdot y_2 + \dots + f_m \cdot y_m}{\underbrace{f_1 + f_2 + \dots + f_m}_{N}} = p_1 \cdot y_1 + p_2 \cdot y_2 + \dots \; p_m y_m$$

MEDIANA

$$\underbrace{y_1 \dots y_1}_{f_1 \text{ volte}} \quad \underbrace{y_2 \dots y_2}_{f_2 \text{ volte}} \quad \dots \quad \underbrace{y_m}_{f_m \text{ volte}}$$

Se N è dispari, ossia $N = 2k+1$

$$\underbrace{\cdot \; \cdot \; \cdot \; \cdot \; \cdot}_{k} \; \cdot \; \underbrace{\cdot \; \cdot \; \cdot \; \cdot \; \cdot}_{k}$$

MEDIANA

Se N è pari ossia $N = 2k$

FACCIO LA MEDIA

$$\underbrace{\cdot \; \cdot \; \cdot \; \cdot}_{k} \quad \underbrace{\cdot \; \cdot \; \cdot \; \cdot}_{k}$$

in mezzo non resta nulla

dal + piccolo 10 11 al + grande

$$\underbrace{6\,6}_{2}\;\underbrace{7\,7\,7\,7\,7}_{5}\;\underbrace{8\,8\,8\,8\,8\,8}_{6}\;\underbrace{9\,9\,9\,9\,9}_{5}\;\underbrace{10\,10}_{2}$$

20

$$\frac{8+8}{2} = 8$$

Quindi

Voto	6	7	8	9	10
Freq. Ass	2	5	6	5	2

$$N = 2+5+6+5+2 = 20$$

$$\mu = \text{media} = \frac{2 \cdot 6 + 5 \cdot 7 + 6 \cdot 8 + 5 \cdot 9 + 2 \cdot 10}{20} = \frac{160}{20} = 8 \longrightarrow \text{2punti!}$$

MEDIANA

$N = 20 \longrightarrow$ PARI $\longrightarrow$ la metà è 10

Devo fare la media tra ~~10 e 11~~ valore $\boxed{\frac{8+8}{2}}$

ENTRAMBI SONO $= 8$

VARIANZA

$$\sigma^2(Y) = \frac{\sum_{i=1}^{m} f_i \cdot (y_i - \mu)^2}{N} =$$

$$= \frac{2 \cdot (6-8)^2 + 5 \cdot (7-8)^2 + 6 (8-8)^2 + \dots}{20}$$

$$= \frac{26}{20} = 1{,}3$$

$$\underbrace{\cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot}_{10} \; \underbrace{\cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot}_{10}$$

$N = 5$ $\quad \cdot \; \cdot \; \cancel{\cdot} \; \cdot \; \cdot$

$N = 6$ $\quad \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot$

Se N è pari, prendete

$$\frac{X_{\frac{N}{2}} + X_{\frac{N}{2}+1}}{2}$$

FUNZIONE

$$f(x) = \frac{1+3e^{2x}}{1-e^{2x}}$$

Se N è dispari $x^{\frac{N+1}{2}}$ (se $N=5 \rightarrow \frac{N+1}{2} = \frac{5+1}{2} = 3$)

DOMINIO

$1-e^{2x} \neq 0$ $\quad e^{2x} \neq \underbrace{1}_{=e^0} \iff e^{2x} \neq e^0 \iff 2x \neq 0$

Quindi $1-e^{2x} \neq 0 \iff$

$\iff x \neq 0$

Quindi il dominio è $\mathbb{R}-\{0\}$ (oppure $(-\infty,0)\cup(0,+\infty)$)

Intersezioni

- con l'asse y (che ha eq. $x=0$) non si possono trovare intersezioni perché $x=0$ è escluso dal dominio
- con l'asse x (che ha eq. $y=0$) trovo

$$\begin{cases} y=0 \\ y=\frac{1+3e^{2x}}{1-e^{2x}} \end{cases} \Rightarrow 0=\frac{1+3e^{2x}}{1-e^{2x}} \iff 1+3e^{2x}=0$$

$\iff \boxed{e^{2x}=-\frac{1}{3}}$

MAI perché $e^{2x} \geq 0 \ \forall x \in \mathbb{R}$

$e \ -\frac{1}{3} < 0$

SEGNO

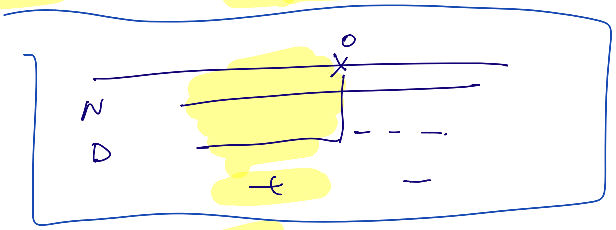
$$\frac{1+3e^{2x}}{1-e^{2x}} \geq 0$$

N: $1+3e^{2x} \geq 0 \iff 3e^{2x} \geq -1 \iff \underbrace{e^{2x}}_{\geq 0} \geq \underbrace{-\frac{1}{3}}_{<0}$ $\quad \forall x \in \mathbb{R}$ (perché $e^x \geq 0 \ \forall x \in \mathbb{R}$)

D: $1-e^{2x} > 0 \iff e^{2x} < 1 \iff e^{2x} < e^0 \iff 2x < 0 \iff x < 0$

$-e^{2x} > -1$

$e^{2x} < 1$



LIMITI

$$\lim_{x\to+\infty} \frac{1+3\overset{\to+\infty}{e^{2x}}}{1-\underset{\downarrow +\infty}{e^{2x}}} = \frac{+\infty}{-\infty}$$

$\boxed{e^{2x} \xrightarrow[x\to+\infty]{} +\infty}$

$\Rightarrow$ F. I

$$\lim_{x\to+\infty} \frac{3\cancel{e^{2x}}}{-\cancel{e^{2x}}} = -3 \Longrightarrow y=-3 \quad \text{AS. OR. } (a +\infty)$$

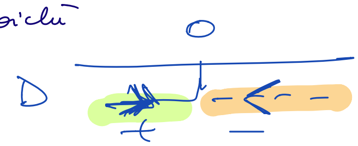
$$\lim_{x\to-\infty} \frac{1+3\overset{\to 0}{e^{2x}}}{1-\underset{\downarrow 0}{e^{2x}}} = 1$$

$e^{2x} \xrightarrow[x\to-\infty]{} e^{-\infty} = 0$

$\Rightarrow y=1$ AS. OR. $(a -\infty)$

$$\lim_{x\to 0^+} \frac{1+3e^{2x}}{\underbrace{1-e^{2x}}_{D}} = \frac{1+3}{0^-} = -\infty \quad \text{perché}$$

$1-e^0=0$



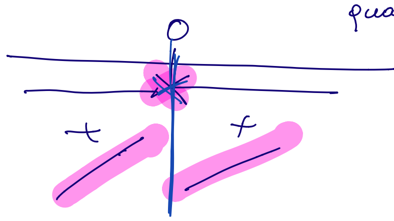
$$\lim_{x\to 0^-} \frac{1+3e^{2x}}{1-e^{2x}} = \frac{1+3}{0^+} = +\infty$$

$$\frac{1+3e^{2x}}{1-e^{2x}}$$

$$f'(x) = \frac{3\cdot 2e^{2x}(1-e^{2x}) - (1+3e^{2x})(-2e^{2x})}{(1-e^{2x})^2} =$$

$$= \frac{6e^{2x} - \cancel{6e^{4x}} + 2e^{2x} + \cancel{6e^{4x}}}{(1-e^{2x})^2} = \frac{8e^{2x}}{(1-e^{2x})^2} \geq 0$$

- $8e^{2x} > 0$ $\forall x \in \mathbb{R}$ perché l'esp. è sempre positivo
- $\underbrace{(1-e^{2x})^2}_{\text{quadrato}} > 0$ $\forall x \in \mathbb{R}$ tale che $1-e^{2x} \neq 0$ ovvero $\forall x$ t.c. $e^{2x} \neq 1 = e^0$ $\Rightarrow$ $\forall x \neq 0$ ($\forall x \in$ dominio di f)



$\Rightarrow$ $\nexists$ MAX e MIN (relativi) perché 0 è escluso!

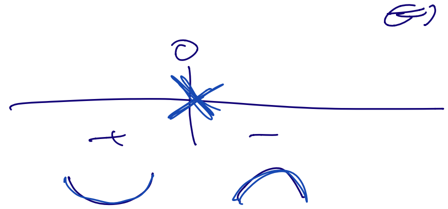
$$f''(x) = \frac{8\cdot 2e^{2x}(1-e^{2x})^2 - 8e^{2x}\cdot 2(1-e^{2x})(-2e^{2x})}{(1-e^{2x})^4}$$

$$= \frac{16e^{2x}\cancel{(1-e^{2x})}\left[1-e^{2x}+2e^{2x}\right]}{(1-e^{2x})^{\cancel{4}\,3}} = \frac{16e^{2x}(1+e^{2x})}{(1-e^{2x})^3} \geq 0$$

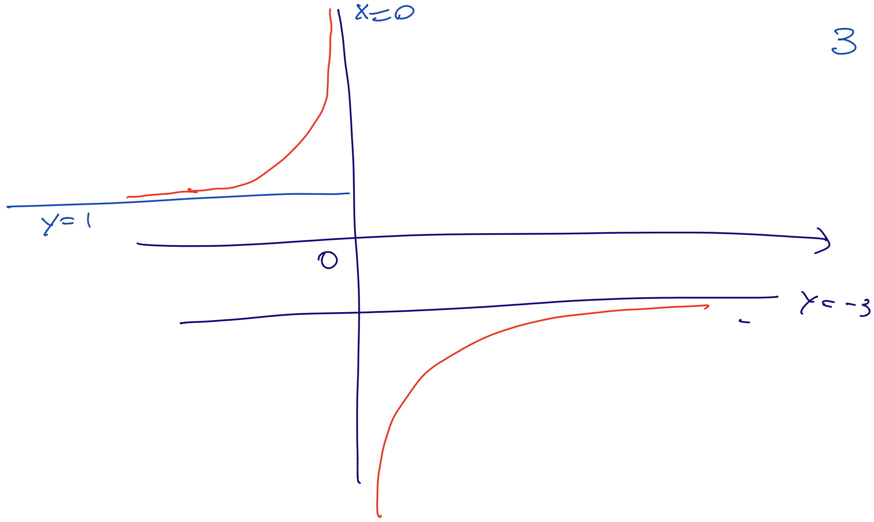
$\Leftrightarrow$ $(1-e^{2x})^3 > 0$

$\Leftrightarrow$ $1-e^{2x} > 0 \Leftrightarrow e^{2x} < 1$

$\Leftrightarrow$ $\underline{x < 0}$



$\nexists$ punti di flesso



3

Integrali

$\frac{N}{D} = Q + \frac{R}{D}$

$\int \frac{4x^2+4x-3}{4x^2-4x+1}\,dx$ ha grado 2 / ha grado 2

I POSSIBILITÀ: DIVIDO NUMERAT. per DENOM.

$$\begin{array}{l|l} N:\ 4x^2+4x-3 & 4x^2-4x+1 = D \\ -4x^2+4x-1 & 1 = Q \\ \hline //\ //\ 8x-4 = R & \end{array}$$

$$= \int \underset{Q}{1} + \frac{\overset{R}{8x-4}}{\underset{D}{4x^2-4x+1}}\,dx = \underbrace{\int 1 + \frac{4\cancel{(2x-1)}}{(2x-1)^{\cancel{2}}}\,dx}_{} = \int dx + \int \frac{4}{2x-1}\,dx =$$

$2\int \frac{2}{2x-1}$

$$= \underbrace{x + 2\log|2x-1|}_{} + C$$

$\int \frac{f'}{f} = \log|f(x)|$

oppure $\frac{8x-4}{\underbrace{4x^2-4x+1}} = \frac{f'(x)}{f(x)} \Rightarrow \int \frac{8x-4}{4x^2-4x+1}\,dx = \log|4x^2-4x+1|$

$f' = 8x-4$

$$= \log|(2x-1)|^2 =$$
$$= 2\log|2x-1| + C$$

$-4x+1\ +4x-1$

oppure $\frac{4x^2+4x-3}{4x^2-4x+1} = \frac{4x^2-4x+1 + 4x-1+4x-3}{4x^2-4x+1}$

$$= \frac{\cancel{4x^2-4x+1}\ 1}{\cancel{4x^2-4x+1}} + \frac{8x-4}{4x^2-4x+1} = 1 + \frac{8x-4}{4x^2-4x+1}$$

$$\Rightarrow \int_0^{1/4} \frac{4x^2+4x-3}{4x^2-4x+1}\,dx = \Big[\underbrace{x + 2\log|2x-1|}\Big]_0^{1/4} = [\quad]_a^b$$

$= f(b) - f(a)$ $\quad = \frac{1}{4} + 2\log\left|\frac{1}{\cancel{4}_2}\cdot\cancel{8}^1 - 1\right| - 0 - \cancel{2\log|-1|}$

$$= \frac{1}{4} + 2\log\left|\frac{1}{2}\right| = \frac{1}{4} - 2\log 2$$