INTEGRALI

$$\int_{-1}^{0} \frac{x}{x^2+x-2}\, dx$$

$$x^2+x-2=0 \qquad x_1=1 \quad x_2=-2$$

$$x^2+x-2=(x-1)(x+2)$$

$$\frac{1\cdot x+0}{x^2+x-2}=\frac{A}{x-1}+\frac{B}{x+2}=$$ Vogliamo trovare A e B che soddisfano l'uguaglianza

$$=\frac{A(x+2)+B(x-1)}{(x-1)(x+2)}=\frac{(A+B)x+(2A-B)}{(x-1)(x+2)}$$

$$\begin{cases} A+B=1 \\ 2A-B=0 \end{cases}$$
$$3A=1 \Rightarrow A=\frac{1}{3}$$

$$B=2A=\frac{2}{3}$$

$$\frac{x}{x^2+x-2}=\frac{\frac{1}{3}}{x-1}+\frac{\frac{2}{3}}{x+2}$$

$$\int_{-1}^{0}\frac{x}{x^2+x-2}\,dx=\int_{-1}^{0}\frac{\frac{1}{3}}{x-1}\,dx+\int_{-1}^{0}\frac{\frac{2}{3}}{x+2}\,dx=\frac{1}{3}\Big[\log|x-1|\Big]_{-1}^{0}+\frac{2}{3}\Big[\log|x+2|\Big]_{-1}^{0}=$$

$$=\frac{1}{3}\big(\log|-1|-\log|-2|\big)+\frac{2}{3}\big(\log|2|-\log|1|\big)=$$

$$=\frac{1}{3}\left(\underbrace{\log(1)}_{=0}-\log(2)\right)+\frac{2}{3}\left(\log(2)-\underbrace{\log(1)}_{=0}\right)=$$

$$=-\frac{1}{3}\log(2)+\frac{2}{3}\log(2)=\frac{1}{3}\log(2)=\log\left(2^{\frac{1}{3}}\right)=\log\left(\sqrt[3]{2}\right)$$

DISCRIMINANTE $\Delta<0$

$$\left(\arctan\left(\frac{x}{2}\right)\right)'=\frac{1}{\left(\frac{x}{2}\right)^2+1}\cdot\frac{1}{2}$$

$$\int\frac{1}{1+x^2}\,dx=\arctan(x)+C$$

$$\int\frac{1}{x^2+4}\,dx=\int\frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx=\frac{1}{4}\int\frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx=\frac{1}{2}\int\frac{\frac{1}{2}}{\left(\frac{x}{2}\right)^2+1}\,dx=$$

Per sostituzione $\frac{x}{2}=t$

$x=2t$

$dx=2dt$

$$=\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$$

$$\int\frac{1}{x^2+x+1}\,dx=$$

$$x^2+x+1=\left(x^2+x+\frac{1}{4}\right)+\frac{3}{4}=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$=\int\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}\,dx=\int\frac{1}{\frac{3}{4}\left(\frac{4}{3}\left(x+\frac{1}{2}\right)^2+1\right)}\,dx=\frac{4}{3}\int\frac{1}{\left(\frac{2x+1}{\sqrt{3}}\right)^2+1}\,dx=$$

SOSTITUZIONE

$$= \frac{4}{3} \int \frac{1}{t^2+1} \frac{\sqrt{3}}{2} dt =$$

$$\frac{2x+1}{\sqrt{3}} = t \qquad 2x+1 = \sqrt{3}t$$

$$2x = \sqrt{3}t - 1$$

$$x = \frac{\sqrt{3}}{2}t - \frac{1}{2}$$

$$dx = \frac{\sqrt{3}}{2} dt$$

$$= \frac{\cancel{4}^{2}}{\sqrt{3}\,\cancel{3}} \cdot \frac{\cancel{\sqrt{3}}}{\cancel{2}} \int \frac{1}{t^2+1} dt = \frac{2}{\sqrt{3}} \arctan(t) + c =$$

$$= \frac{2}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + c$$

$$\Delta = b^2 - 4ac < 0$$

$$a=1 \quad b=1 \quad c=1$$

$$\Delta = -3$$

$$\int \frac{1}{ax^2+bx+c} dx =$$

$$= \frac{2}{\sqrt{-\Delta}} \arctan\left(\frac{2ax+b}{\sqrt{-\Delta}}\right) + \text{costante}$$

$$\int \frac{1}{x^2+x+1} dx = \frac{2}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + \text{costante}$$

ESEMPIO

$$\int \frac{1}{2x^2+5x+9} dx = \frac{2}{\sqrt{47}} \arctan\left(\frac{4x+5}{\sqrt{47}}\right) + \text{costante}$$

$$a=2 \quad b=5 \quad c=9$$

$$\Delta = 25 - 72 = -47$$

$$\int \frac{3x+5}{x^2+x+1}\,dx = \int \frac{3x}{x^2+x+1}\,dx + \int \frac{5}{x^2+x+1}\,dx =$$

$$= \underbrace{\frac{3}{2}\int \frac{2x}{x^2+x+1}\,dx}_{=} + \underbrace{5\int \frac{1}{x^2+x+1}\,dx}$$

lo conosciamo già

$$\frac{3}{2}\int \frac{2x+1-1}{x^2+x+1}\,dx = \underbrace{\frac{3}{2}\int \frac{2x+1}{x^2+x+1}\,dx}_{=} - \overbrace{\frac{3}{2}\int \frac{1}{x^2+x+1}\,dx}$$

$$\frac{3}{2}\log|x^2+x+1| + c$$

$$=$$

$$\frac{3}{2}\log(x^2+x+1) + c$$

PER PARTI

$$\int f'(x)\,g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

$$\int \underbrace{x^2}_{=g(x)} \underbrace{\cos(x)}_{=f'(x)}\,dx =$$

$f'(x) = \cos(x) \Rightarrow f(x) = \operatorname{sen}(x)$

$g(x) = x^2 \Rightarrow g'(x) = 2x$

$$= x^2 \operatorname{sen}(x) - \int 2x \operatorname{sen}(x)\,dx =$$

$$= x^2 \operatorname{sen}(x) - 2\int \underbrace{x}_{g(x)} \underbrace{\operatorname{sen}(x)}_{f'(x)}\,dx =$$

$f'(x) = \operatorname{sen}(x) \Rightarrow f(x) = -\cos(x)$

$g(x) = x \Rightarrow g'(x) = 1$

$$= x^2 \operatorname{sen}(x) - 2\left(-x\cos(x) + \int \cos(x)\,dx\right) =$$

$$= x^2 \operatorname{sen}(x) + 2x\cos(x) - 2\int \cos(x)\,dx = x^2 \operatorname{sen}(x) + 2x\cos(x) - 2\operatorname{sen}(x) + C$$

FATE LA VERIFICA!

$$\int \underbrace{x^2}_{=g(x)} \underbrace{\operatorname{sen}(3x)}_{=f'(x)}\,dx =$$

$f'(x) = \operatorname{sen}(3x) \Rightarrow f(x) = -\dfrac{\cos(3x)}{3}$

$g(x) = x^2 \Rightarrow g'(x) = 2x$

$$= -\frac{x^2}{3}\cos(3x) + \int \frac{2x}{3}\cos(3x)\,dx =$$

$$f'(x) = \cos(3x) \Rightarrow f(x) = \frac{\operatorname{sen}(3x)}{3}$$
$$g(x) = x \Rightarrow g'(x) = 1$$

$$= -\frac{x^2}{3}\cos(3x) + \frac{2}{3}\int x\cos(3x)\,dx =$$

$$= -\frac{x^2}{3}\cos(3x) + \frac{2}{3}\left(x\frac{\operatorname{sen}(3x)}{3} - \int \frac{\operatorname{sen}(3x)}{3}\,dx\right) =$$

$$= -\frac{x^2}{3}\cos(3x) + \frac{2}{9}x\operatorname{sen}(3x) - \frac{2}{9}\int \operatorname{sen}(3x)\,dx =$$

$$= -\frac{x^2}{3}\cos(3x) + \frac{2}{9}x\operatorname{sen}(3x) + \frac{2}{27}\cos(3x) + C$$

$$\int 1\cdot\arctan(x)\,dx = \quad \text{PER PARTI}$$

$$f'(x) = 1 \Rightarrow f(x) = x$$
$$g(x) = \arctan(x) \Rightarrow g'(x) = \frac{1}{x^2+1}$$

$$= x\arctan(x) - \int \frac{x}{x^2+1}\,dx =$$

$$= x\arctan(x) - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = x\arctan(x) - \frac{1}{2}\log|x^2+1| + C =$$

$$= x\arctan(x) - \frac{1}{2}\log(x^2+1) + C$$

$$\int \tan(x)\,dx = \int \frac{\operatorname{sen}(x)}{\cos(x)}\,dx = -\int \frac{-\operatorname{sen}(x)}{\cos(x)}\,dx = -\log|\cos(x)| + c$$

ESERCIZIO calcola l'area della regione di piano, finita, delimitata dalla curva

$$f(x) = \frac{1-e^x}{e^x+2}$$

dell'asse x e della retta $x=1$.

Dominio $= \mathbb{R}$ Né pari né dispari $\cap$ assi: $(0,0)$

Segno: $f(x)>0$ per $x<0$ e $f(x)<0$ per $x>0$

Asintoto orizzontale destro (per $x \to +\infty$) $y=-1$

" " sinistro (per $x \to -\infty$) $y=\frac{1}{2}$

$$f'(x) = -\frac{3e^x}{(e^x+2)^2}$$

$f'(x)>0$ mai

$f(x)$ monotona decrescente

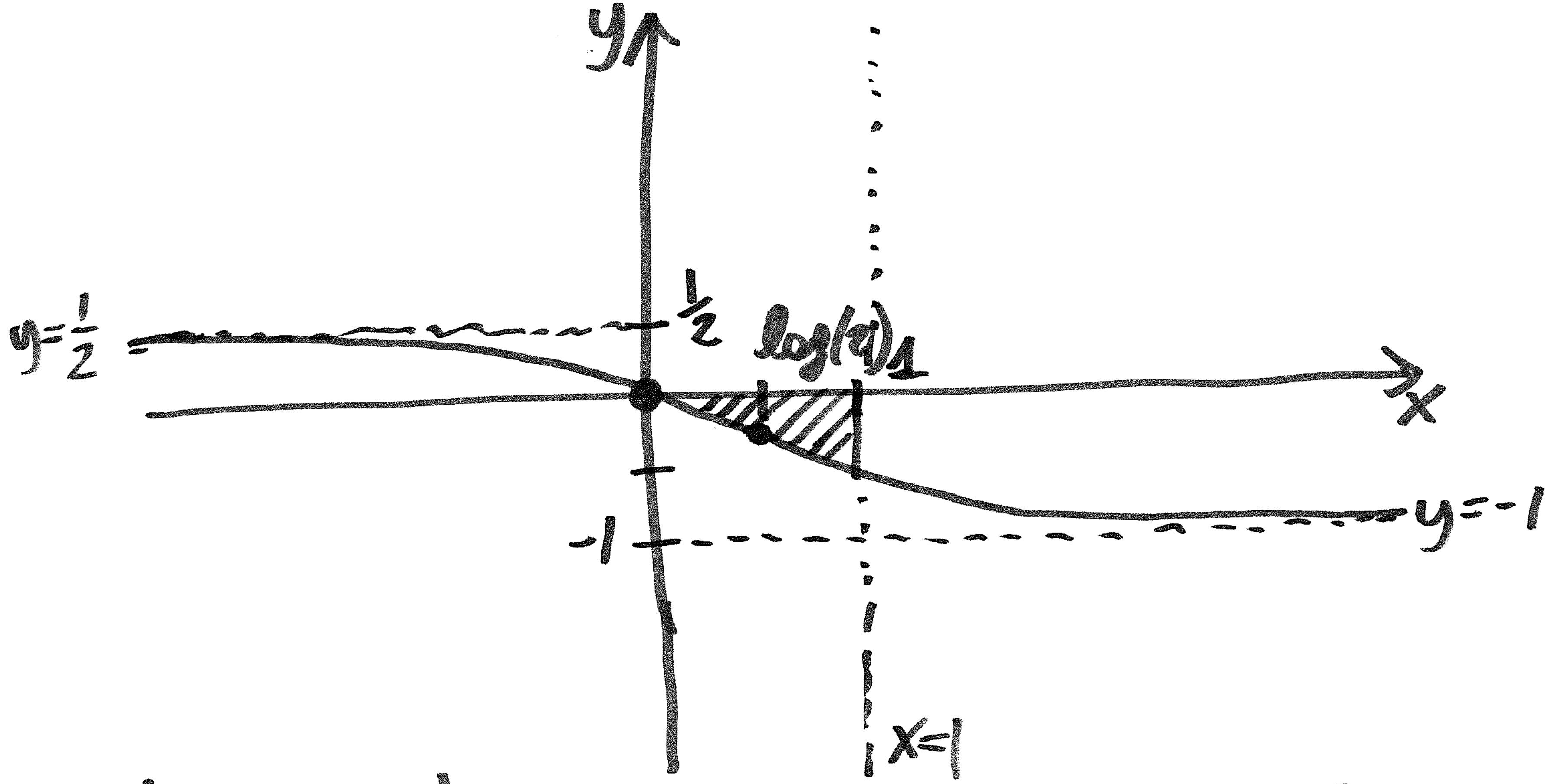
Non ci sono massimi o minimi

$$f''(x) = \frac{3e^x(e^x-2)}{(e^x+2)^3}$$

$f(x)$ concavità verso il basso per $x < \log(2)$

" " " l'alto per $x > \log(2)$

in $x_1 = \log(2)$ c'è un flesso dove $f(\log(2)) = -\frac{1}{4}$



$$-\int_0^1 \frac{1-e^x}{e^x+2}\,dx = \int_0^1 \frac{e^x-1}{e^x+2}\,dx$$

SOSTITUZIONE

$e^x = t$ $\quad x = \log(t)$

$dx = \frac{1}{t}\,dt$

$$\int_0^1 \frac{e^x-1}{e^x+2}\,dx = \int_1^e \frac{t-1}{t+2}\cdot\frac{1}{t}\,dt =$$

$x=0 \Rightarrow t=e^0=1$

$x=1 \Rightarrow t=e^1=e$

$$=\int_1^e \frac{t-1}{t(t+2)}\,dt =$$

$$=\frac{t-1}{t(t+2)} = \frac{A}{t}+\frac{B}{t+2} = \frac{At+2A+Bt}{t(t+2)} =$$

$$=\frac{(A+B)t+2A}{t(t+2)}$$

$$=\int_1^e \frac{-\frac{1}{2}}{t}\,dt + \int_1^e \frac{\frac{3}{2}}{t+2}\,dt =$$

$$\begin{cases} A+B=1 \\ 2A=-1 \end{cases} \Rightarrow A=-\frac{1}{2} \qquad B=1-A=1+\frac{1}{2}=\frac{3}{2}$$

$$=-\frac{1}{2}\Big[\log|t|\Big]_1^e + \frac{3}{2}\Big[\log|t+2|\Big]_1^e =$$

$$=-\frac{1}{2}\Big(\underbrace{\log(e)}_{=1} - \cancel{\log(1)}\Big) + \frac{3}{2}\log(e+2) - \frac{3}{2}\log(3) =$$

$$=-\frac{1}{2}+\frac{3}{2}\log(e+2)-\frac{3}{2}\log(3)$$