INTEGRALI INDEFINITI

$$\int f'(x)\,dx = f(x) + c$$

$$\int x^{\alpha}\,dx = \frac{x^{\alpha+1}}{\alpha+1} + c \quad \text{per } \alpha \neq -1$$

$$\int 3x^5\,dx = 3\int x^5\,dx = \cancel{3}\,\frac{x^6}{\cancel{6}\,2} + c = \frac{x^6}{2} + c$$

$$\int \frac{1}{x}\,dx = \log|x| + c$$

$$\int e^x\,dx = e^x + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$$

$$\int f'(x)\, e^{f(x)}\,dx = e^{f(x)} + c$$

per $\alpha \neq -1$

$$\int f'(x)\left(f(x)\right)^{\alpha} dx = \frac{\left(f(x)\right)^{\alpha+1}}{\alpha+1} + c$$

$$\int \frac{1}{5x-4}\,dx = \frac{1}{5}\int \frac{5}{5x-4}\,dx = \frac{1}{5}\log|5x-4| + c$$

$$\int x e^{-x^2}\,dx = -\frac{1}{2}\int -2x e^{-x^2}\,dx = -\frac{1}{2}e^{-x^2} + c$$

$$\int (3x-1)(3x^2-2x+5)^3\,dx =$$

$$(3x^2-2x+5)' = 6x-2 = 2(3x-1)$$

$$= \frac{1}{2}\int 2(3x-1)(3x^2-2x+5)^3\,dx =$$

$$= \frac{1}{2}\,\frac{(3x^2-2x+5)^4}{4} + c =$$

$$= \frac{1}{8}(3x^2-2x+5)^4 + c$$

INTEGRAZIONE PER PARTI

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

TRUCCHETTO: MOLTIPLICARE PER 1

$$\int \log(x)\,dx = \int \underbrace{1}_{f'(x)} \cdot \underbrace{\log(x)}_{g(x)}\,dx = x\log(x) - x + c$$

FUNZIONI TRIGONOMETRICHE

$$\int \cos(x)\,dx = \operatorname{sen}(x) + c$$

$$\int \operatorname{sen}(x)\,dx = -\cos(x) + c$$

$$\int \frac{1}{\cos^2(x)}\,dx = \tan(x) + c$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{arcsen}(x) + c$$

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + c$$

$$\int x \,\mathrm{sen}(x)\, dx = \qquad f'(x) = \mathrm{sen}(x) \Rightarrow f(x) = -\cos(x)$$

$$g(x) = x \qquad \Rightarrow g'(x) = 1$$

$$= -x\cos(x) + \int \cos(x)\, dx =$$

$$= -x\cos(x) + \mathrm{sen}(x) + C$$

VERIFICA PER ESERCIZIO!

$$\int \mathrm{sen}^2(x)\, dx = \int \mathrm{sen}(x)\,\mathrm{sen}(x)\, dx =$$

$$f'(x) = \mathrm{sen}(x) \Rightarrow f(x) = -\cos(x)$$

$$g(x) = \mathrm{sen}(x) \Rightarrow g'(x) = \cos(x)$$

$$= -\cos(x)\,\mathrm{sen}(x) + \int \cos^2(x)\, dx =$$

$$\boxed{\mathrm{sen}^2(x) + \cos^2(x) = 1}$$

$$= -\cos(x)\,\mathrm{sen}(x) + \int 1\, dx - \int \mathrm{sen}^2(x)\, dx =$$

$$\cos^2(x) = 1 - \mathrm{sen}^2(x)$$

$$= -\cos(x)\,\mathrm{sen}(x) + x \underbrace{- \int \mathrm{sen}^2(x)\, dx}$$

portiamo questo termine dall'altro membro

$$\cancel{\int \mathrm{sen}^2(x)\, dx} \overset{2}{+} \int \mathrm{sen}^2(x)\, dx = -\cos(x)\,\mathrm{sen}(x) + x + C$$

$$\int \mathrm{sen}^2(x)\, dx = \frac{1}{2}\left(x - \cos(x)\,\mathrm{sen}(x)\right) + C$$

ESERCIZI VERIFICATE CHE

$$\int \cos^2(x)\,dx = \frac{1}{2}\left(x + \sin(x)\cos(x)\right) + C$$

INTEGRALI DI FUNZIONI RAZIONALI

$$\int \frac{5x^2+3x-2}{x+2}\,dx =$$

$$\begin{array}{rr|l} 5x^2+3x-2 & & x+2 \\ \hline -5x^2-10x & & 5x-7 = Q(x) \\ \hline // \quad -7x-2 & & \\ +7x+14 & & \\ \hline 12 = R(x) & & \end{array}$$

$$= \int (5x-7)\,dx + \int \frac{12}{x+2}\,dx =$$

$$= \frac{5}{2}x^2 - 7x + 12\log|x+2| + C$$

$$\int \frac{3x+2}{x^2+2x-3}\,dx$$

$$\frac{3x+2}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} =$$

$x_1 = 1$

$x_2 = -3$

$$x^2+2x-3 = (x-1)(x+3)$$

$$= \frac{A(x+3)+B(x-1)}{(x-1)(x+3)} =$$

$$= \frac{(A+B)x + (3A-B)}{(x-1)(x+3)}$$

$$\begin{cases} A+B=3 \\ 3A-B=2 \end{cases}$$

$$4A=5 \Rightarrow A=\frac{5}{4}$$

$$B=3-\frac{5}{4}=\frac{7}{4}$$

$$\int \frac{3x+2}{x^2+2x-3}\,dx = \frac{5}{4}\int \frac{1}{x-1}\,dx + \frac{7}{4}\int \frac{1}{x+3}\,dx = \frac{5}{4}\log|x-1| + \frac{7}{4}\log|x+3| + C$$

$$\int \frac{x^2-5}{3x-2}\,dx =$$

$$\begin{array}{l|l} x^2 \qquad -5 & 3x-2 \\ \hline -x^2+\frac{2}{3}x & \frac{1}{3}x+\frac{2}{9} \\ \hline \qquad \frac{2}{3}x-5 & \\ \quad -\frac{2}{3}x+\frac{4}{9} & \\ \hline \quad // \quad -\frac{41}{9} & \end{array}$$

$$\frac{x^{\cancel{2}}}{3\cancel{x}} = \frac{1}{3}x \qquad \frac{\frac{2}{3}\cancel{x}}{3\cancel{x}} = \frac{2}{9}$$

$$= \int\left(\frac{x}{3}+\frac{2}{9}\right)dx - \frac{41}{3\cdot 9}\int \frac{1\cdot 3}{3x-2}\,dx =$$

$$= \frac{1}{6}x^2 + \frac{2}{9}x - \frac{41}{27}\log|3x-2| + c$$

$$\int \frac{3x-1}{4x^2-4x+1}\,dx$$

$$4x^2-4x+1=0 \qquad x_1=x_2=\frac{1}{2}$$

$$\parallel$$

$$4\left(x-\frac{1}{2}\right)^2 = (2x-1)^2$$

$$\frac{3x-1}{4x^2-4x+1} = \frac{A}{2x-1} + \frac{B}{(2x-1)^2} = \frac{A(2x-1)+B}{(2x-1)^2} = \frac{2Ax-A+B}{(2x-1)^2} \Rightarrow \begin{cases} 2A=3 \\ -A+B=-1 \end{cases}$$

$$A=\frac{3}{2} \qquad B=-1+A=-1+\frac{3}{2}=\frac{1}{2} \qquad \frac{3x-1}{4x^2-4x+1} = \frac{3}{2}\cdot\frac{1}{2x-1} + \frac{1}{2}\,\frac{1}{(2x-1)^2}$$

$$\int \frac{3x-1}{4x^2-4x+1}\,dx = \int \frac{3}{2}\cdot\frac{1}{2x-1}\,dx + \int \frac{1}{2}\,\frac{1}{(2x-1)^2}\,dx =$$

$$= \frac{3}{2\cdot 2}\int \frac{1\cdot 2}{2x-1}\,dx + \frac{1}{2\cdot 2}\int 2(2x-1)^{-2}\,dx =$$

$$= \frac{3}{4}\log|2x-1| + \frac{1}{4}\,\frac{(2x-1)^{-1}}{-1} + c =$$

$$= \frac{3}{4}\log|2x-1| - \frac{1}{4}\cdot\frac{1}{2x-1} + c$$

INTEGRAZIONE PER SOSTITUZIONE

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx =$$

Poniamo $\sqrt{x} = t \Rightarrow x = t^2$

$dx = 1\cdot dx = 2t\,dt$

$$= \int \frac{t}{1+t}\cdot 2t\,dt = \int \frac{2t^2}{1+t}\,dt = 2\int \frac{t^2}{1+t}\,dt =$$

DIVISIONE

$t^2 = (t-1)(t+1)+1$

7^2

$$= 2\left(\int (t-1)\,dt + \int \frac{1}{t+1}\,dt\right) = 2\left(\frac{t^2}{2} - t + \log|t+1|\right) + C =$$

$$= t^2 - 2t + 2\log|t+1| + C = \quad \text{SOSTITUENDO } t = \sqrt{x}$$

$$= x - 2\sqrt{x} + 2\log|\sqrt{x}+1| + C = x - 2\sqrt{x} + 2\log(\sqrt{x}+1) + C$$

$$\int \frac{e^x+1}{e^x-1}\,dx = \quad \text{Poniamo } \boxed{e^x = t} \quad x = \log(t) \quad dx = \frac{1}{t}\,dt$$

$$= \int \frac{t+1}{t-1}\cdot\frac{1}{t}\,dt = \int \frac{t+1}{t(t-1)}\,dt =$$

$$\frac{t+1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} = \frac{A(t-1)+Bt}{t(t-1)} = \frac{(A+B)t - A}{t(t-1)}$$

$$\begin{cases} A+B=1 \\ -A=1 \end{cases}$$

$$\Rightarrow A=-1,\ B=2$$

$$= \int \frac{-1}{t}\,dt + \int \frac{2}{t-1}\,dt =$$

$$= -\log|t| + 2\log|t-1| + C =$$

$$= -\log(e^x) + 2\log|e^x-1| + C =$$

$$= -x + 2\log|e^x-1| + C$$

$$\int f(x)\,dx \qquad x = g(t) \Rightarrow dx = g'(t)\,dt$$

$$\left(f(g(t))\right)' = f'(g(t))\,g'(t)$$

$$\int f(x)\,dx = \int f(g(t))\,g'(t)\,dt$$

$$\int \log(3x-5)\,dx =$$

Poniamo $3x-5=t \Leftrightarrow 3x=t+5$

$\Leftrightarrow x=\frac{t}{3}+\frac{5}{3}$

$\Rightarrow dx=\frac{1}{3}dt$

$$=\int \log(t)\cdot\frac{1}{3}dt = \frac{1}{3}\int \log(t)\,dt =$$

$$=\frac{1}{3}\left(t\log(t)-t\right)+c = \frac{1}{3}\left((3x-5)\log(3x-5)-(3x-5)\right)+c$$