MASSIMI E MINIMI

Segno di f' $\rightarrow$ max. e min. di f

- sia f derivabile in x_0

$$\boxed{f'(x_0) > 0}$$

Poiché $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$

$$\frac{f(x_0+h) - f(x_0)}{h} > 0$$

per $|h|$ abbastanza piccolo

In base al segno di h:

$$\boxed{f(x_0-h) < f(x_0) < f(x_0+h)}$$

se $h > 0$ e h vicino a 0

• se $\boxed{f'(x_0) < 0}$

$$\frac{f(x_0+h) - f(x_0)}{h} < 0$$

per $|h|$ piccolo.

• In base al segno di h:

$$\boxed{f(x_0-h) > f(x_0) > f(x_0+h)}$$

Def. f è <u>crescente</u> in x_0

se $x_1 < x_0 < x_2 \Rightarrow$

$$f(x_1) < f(x_0) < f(x_2)$$

Def. f è <u>decrescente</u> in x_0

se $x_1 < x_0 < x_2 \Rightarrow$

$$f(x_1) > f(x_0) > f(x_2)$$

Prop.

$f'(x_0) > 0 \Rightarrow f$ crescente in x_0.

$f'(x_0) < 0 \Rightarrow f$ decrescente in x_0.

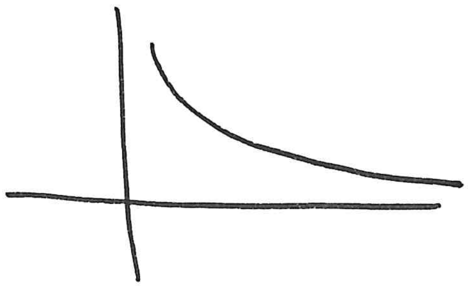
Viceversa:

- Supponiamo di sapere f è crescente in x_0 $\Rightarrow f(x_0 - h) < f(x_0) < f(x_0 + h)$ con $h > 0$ e piccolo

$$\Rightarrow \boxed{\frac{f(x_0+h) - f(x_0)}{h} > 0}$$

$$\Rightarrow \boxed{f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} \geq 0}$$

es.:



Analogamente:

f decrescente in x_0 $\Rightarrow$ $\boxed{f'(x_0) \leq 0}$

Riassumendo:

- $\boxed{f'(x_0) > 0} \Rightarrow f$ crescente in x_0 $\Rightarrow$ $\boxed{f'(x_0) \geq 0}$;
- $\boxed{f'(x_0) < 0} \Rightarrow f$ decrescente in x_0 $\Rightarrow$ $\boxed{f'(x_0) \leq 0}$.

Def.

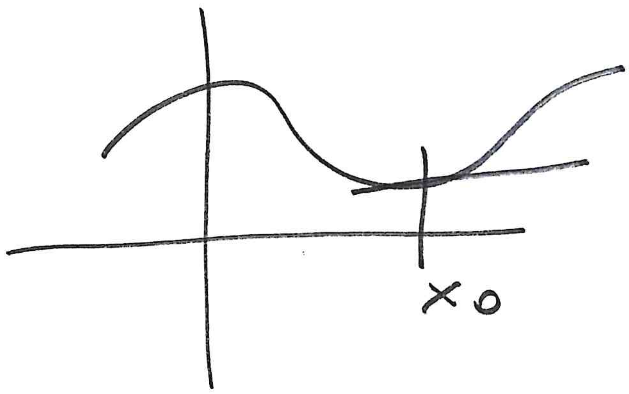
se f è crescente (decrescente) su tutto (a, b) $\Rightarrow$ $f'(x) \geq 0$ (risp. $f'(x) \leq 0$) $\forall x \in (a, b)$.

Def.

Se x_0 t.c. $f'(x_0) = 0$ $\Rightarrow$ (x_0) è p.to critico, stazionario, o estremo

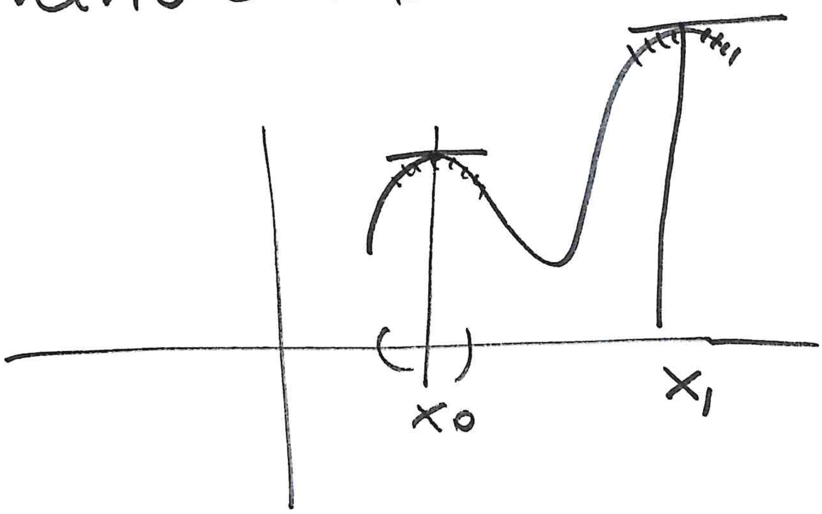
Def.

Se x_0 è p.to critico $\Rightarrow$
$f(x_0)$ = <u>valore critico</u>

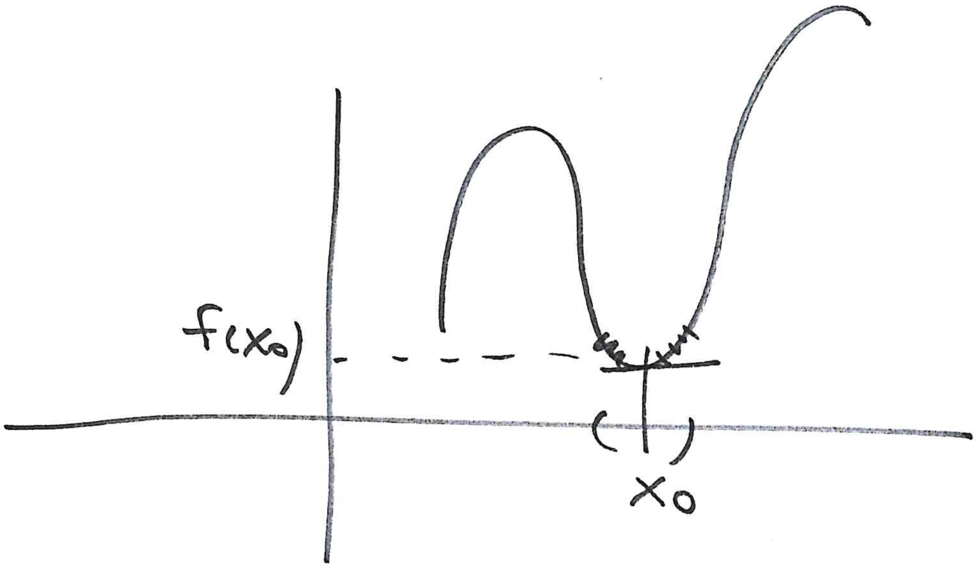


Def.

x_0 è <u>p.to</u> di <u>massimo</u> <u>locale</u> (o relativo) se $f(x) \leq f(x_0)$
$\forall x$ vicino a x_0

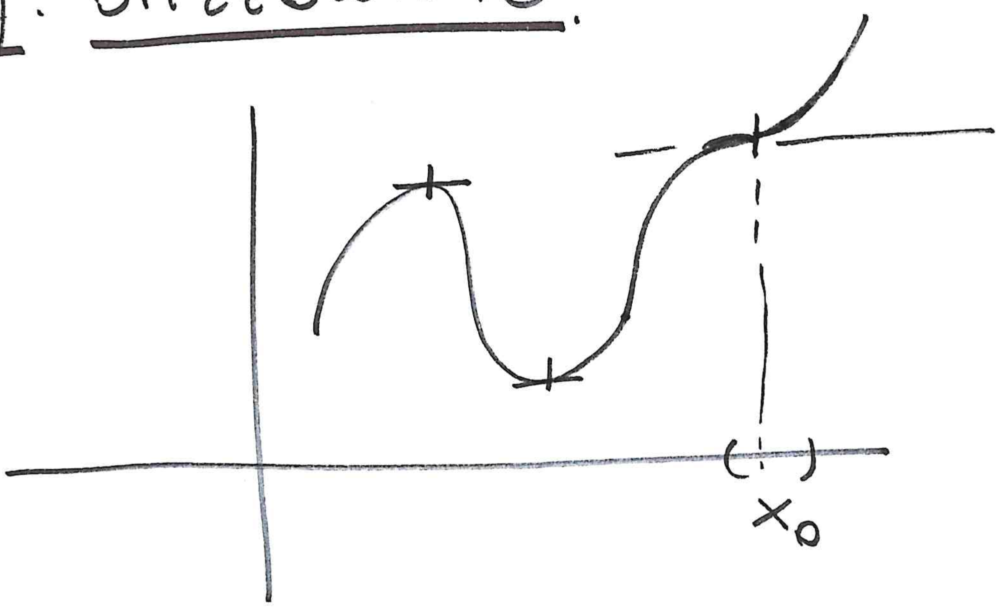


Def. x_0 = minimo locale (relativo)
se $f(x) \geq f(x_0)$



⚠ Non tutti i pti critici sono min o max. relativi.

Es.
Se $f'(x_0) = 0 \Rightarrow x_0$ sia FLESSO a tg. orizzontale.



Def.

x_0 = p.to <u>massimo</u> <u>globale</u> (assoluto)

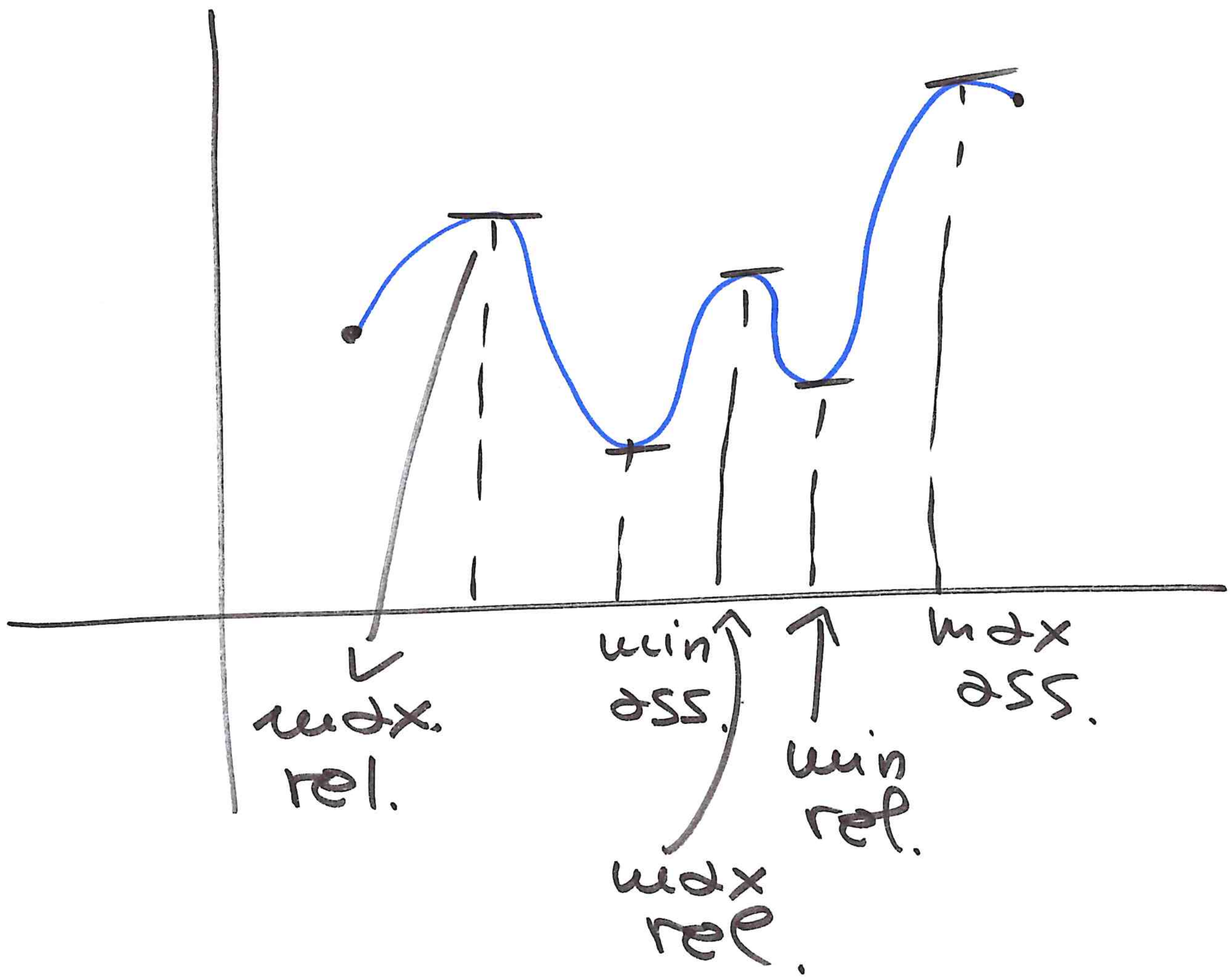
se $f : I \to \mathbb{R}$

$$f(x_0) \geq f(x), \ \forall x \in I$$

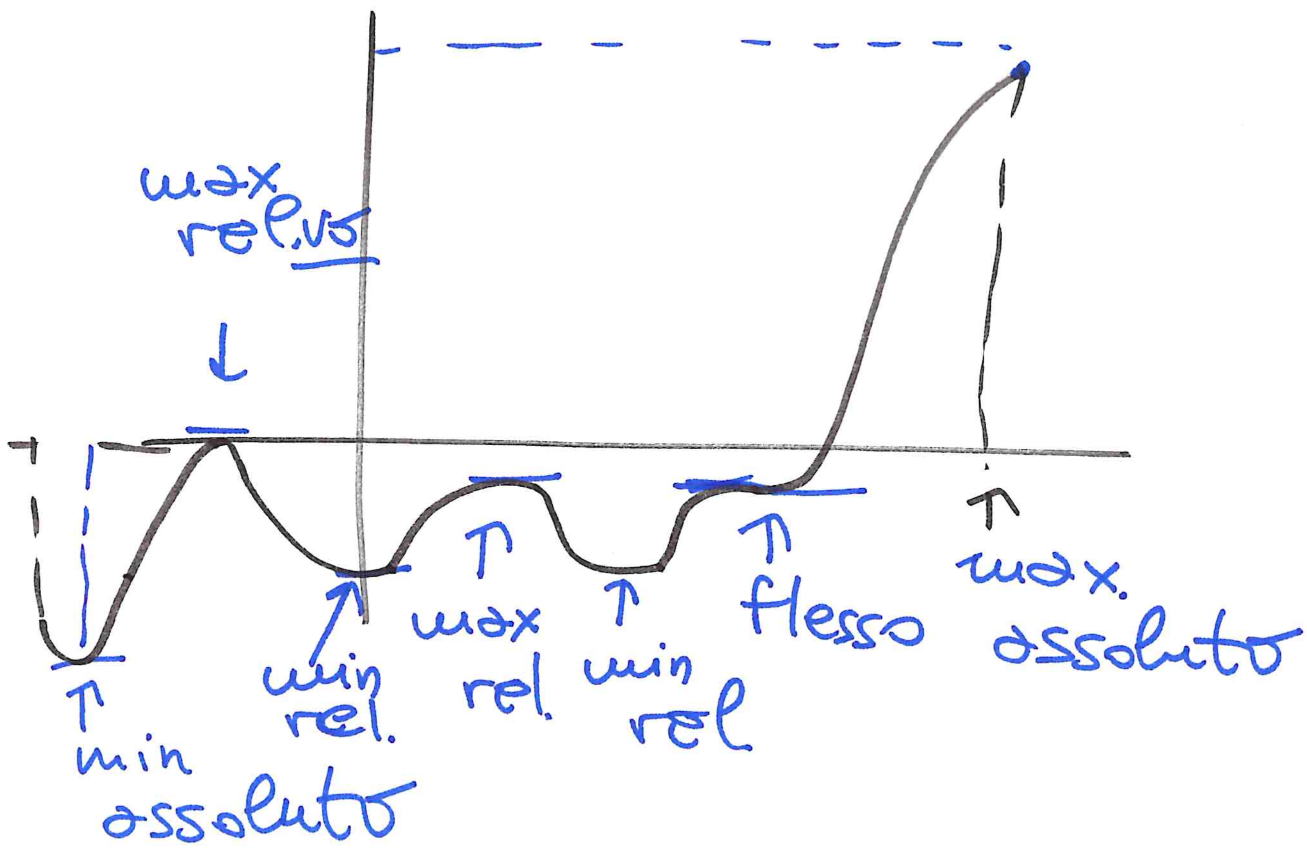
x_0 = p.to <u>minimo</u> <u>globale</u> (assoluto)

se $f : I \to \mathbb{R}$

$$f(x_0) \leq f(x), \ \forall x \in I.$$



Oss.



si può usare la <u>derivata</u> per distinguere max, min e flessi:

(1) sia $\boxed{f'(x_0) = 0}$

$f'(x) > 0$ per $x > x_0 \Rightarrow f$ cresc.

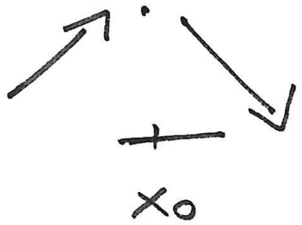
$f'(x) < 0$ per $x < x_0 \Rightarrow f$ decresc.

$\Rightarrow \searrow . \nearrow \Rightarrow x_0$ <u>min. relativo</u>

2) $f'(x_0)=0$

$f'(x)<0$ per $x>x_0 \Rightarrow f$ decr.

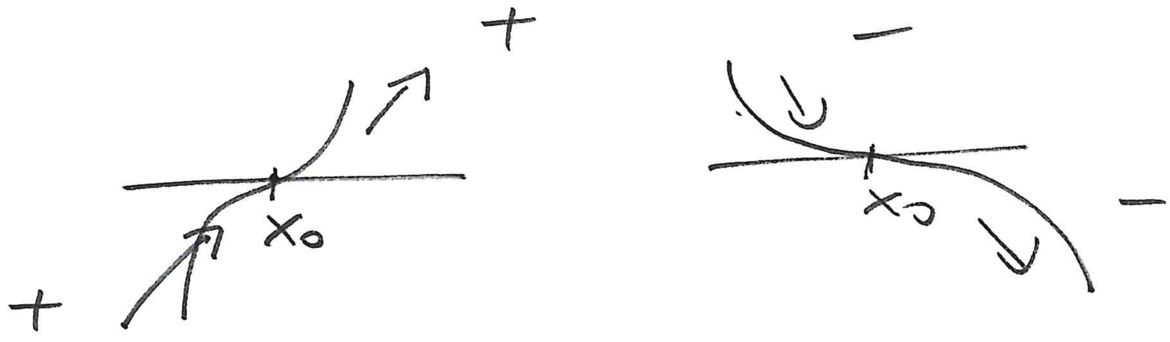
$f'(x)>0$ per $x<x_0 \Rightarrow f$ cresc.



$\Rightarrow x_0$ <u>max. rel.vo</u>

3) $f'(x_0)=0$

sign $f'(x)$ è lo stesso prima e dopo $x_0 \Rightarrow x_0$ <u>flesso a tg. orizz.le</u>



studio $f''(x_0)$ aiuta a capire meglio.

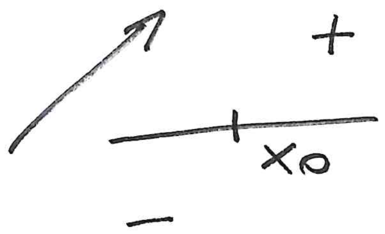
f'' = è la derivata prima di f'

• $f'(x_0) = 0$, $\boxed{f''(x_0) > 0}$

$\Downarrow$

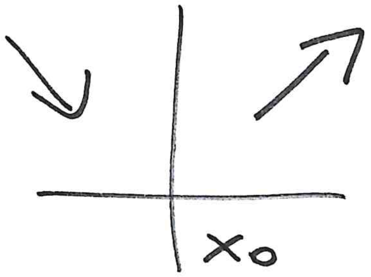
f' è <u>crescente</u>

f'



$\Rightarrow x_0 = \boxed{\text{min. relat.}}$

f

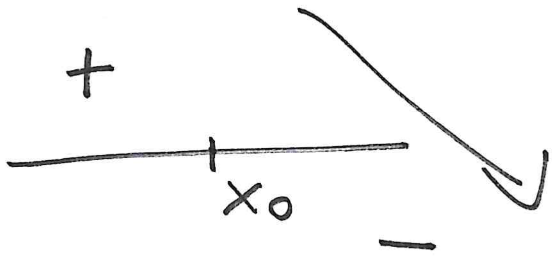


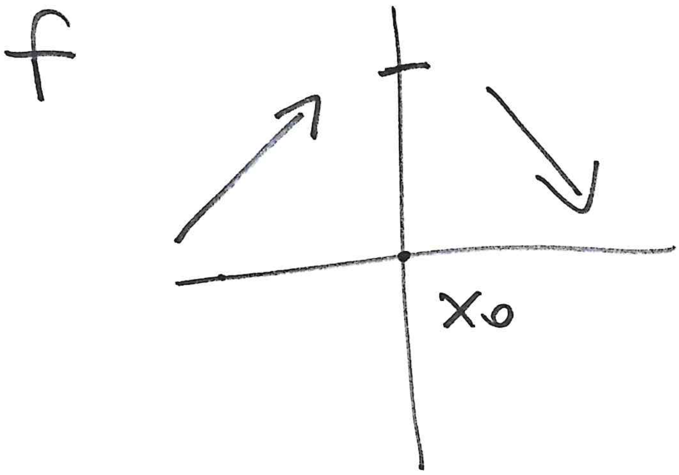
• $f'(x_0) = 0$, $\boxed{f''(x_0) < 0}$

$\Downarrow$

f' è <u>decresc.</u>

f'

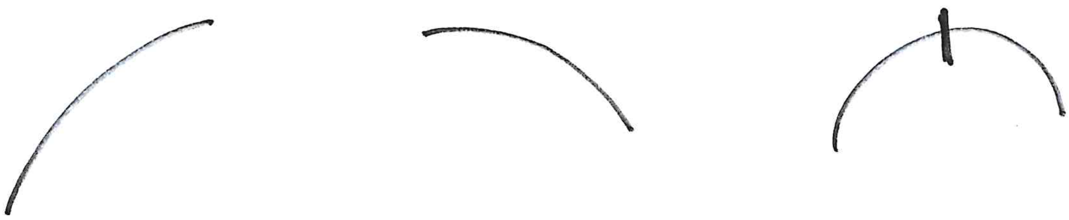




x_0 = max. rel.vo

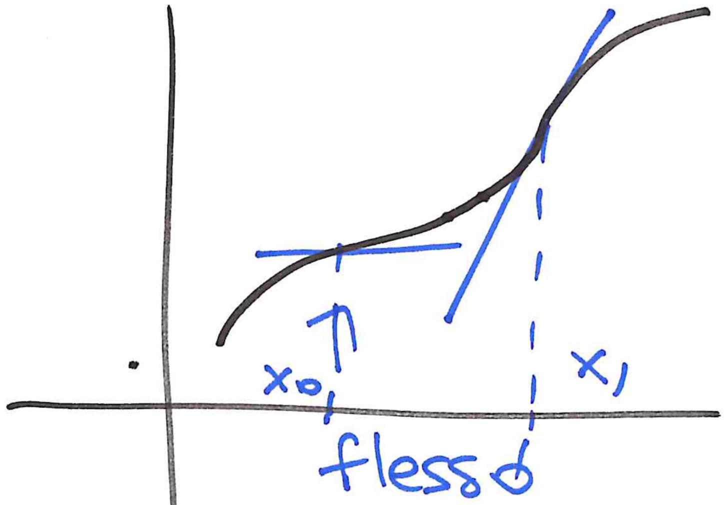
Def. $f''(x_0) > 0 \Rightarrow$ f.ne convessa in x_0.

Def. $f''(x_0) < 0 \Rightarrow$ f.ne concava in x_0



Def. x_0 t.c. $f''(x_0) = 0$

$\Rightarrow x_0$ FLESSO (· se $f'(x_0) \neq 0$ è obliquo, · se $f'(x_0) = 0$ è a tg. orizz.)



$f''(x_1)=0$

$f'(x_1)>0$

tg. orizz.

$f'(x_0)=f''(x_0)=0$

Ex.

1) $f(x)=x^3$

$g(x)=x^4$

$h(x)=-x^4$

o che pto è per le 3 f.ni ?

$h=-g$

$f'(x)=3x^2\big|_0=0$

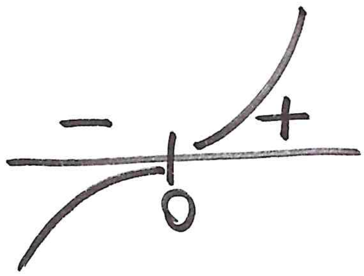
f': $+$ | $+$ (0)

$g'(x)=4x^3\big|_0=0$

$h'(x)=-4x^3\big|_0=0$

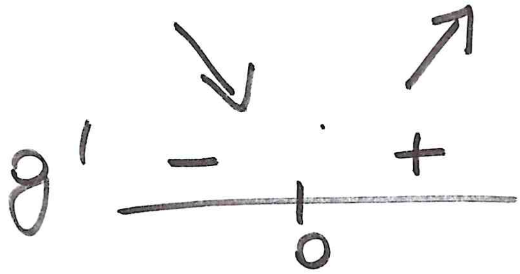
$f''(x)=6x\big|_0=0$

f''



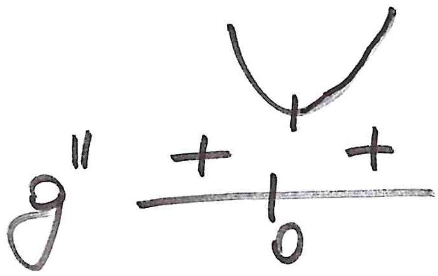
1) 0 è flesso a tg. orizz. per f.

2) $g'(x) = 4x^3$



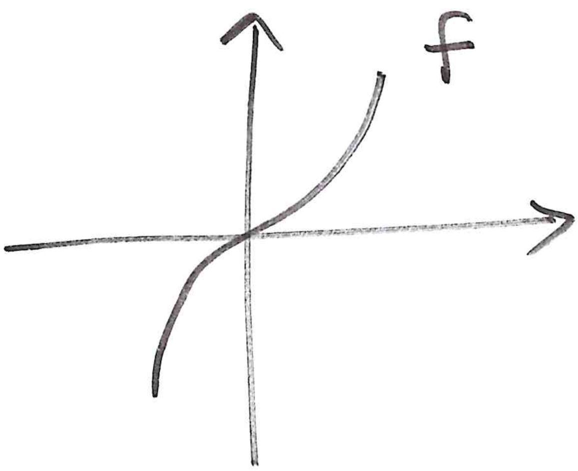
0 min. rel.vd

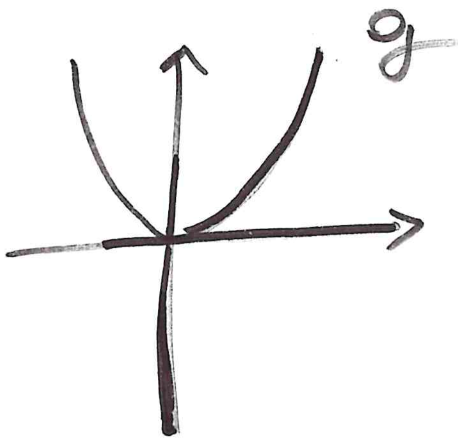
$g''(x) = 12 \cdot x^2$

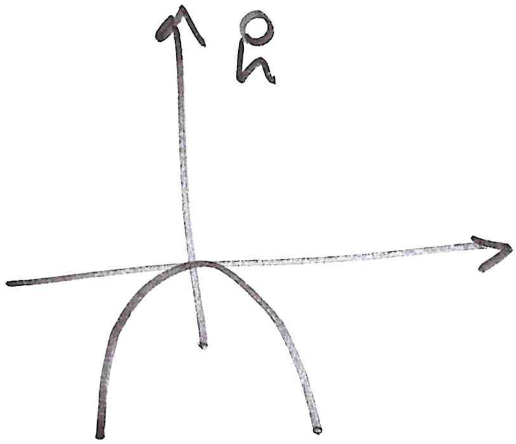


3) $h = -g$

0 max. relvd







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• studio qualitativo di una f.ne

(1) capire chi è il dominio : levare i pti singolari, se ci sono.

(2) f è pari? è dispari? nessuna delle 2?

(3) Se $0 \in Dom(f) \Rightarrow f(0)$ così so dove grafico di $f \cap$ asse delle ordinate.

$x / f(x) = 0 \Rightarrow$ i pti in cui grafico di $f \cap$ asse delle ascisse.

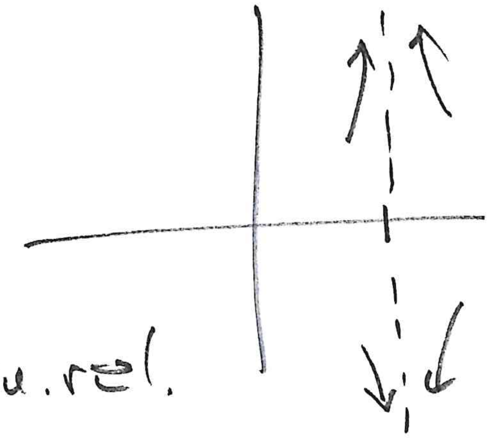
(4) sign(f) $\Rightarrow$ grafico di f sta in semi-piano sup. o semi-piano inferiore.

(5) se Dom(f) illimitato:

$\lim_{x \to \pm\infty} f(x)$ così conosco grafico di f se $x >>$, $x <<$

(6) se x_0 pto sing.

$$\lim_{x \to x_0^{\pm}} f(x)$$



(7) sign f' → min. rel.

→ max. rel.

→ flessi a tg. orizz.

(8) sign f'' → $\cup$

→ $\cap$

flessi a tg. obliqua.

Ex.

$$f(x) = \frac{2x^2-1}{x^2+1}$$

Studio qualitativo f.ne.

(1) Dom f $= \mathbb{R}$

$x^2+1 \neq 0$ su $\mathbb{R}$

$x^2+1 \geq 1$

(2) $f(-x) = \dfrac{2(-x)^2-1}{(-x)^2+1} = f(x)$

f è pari

Il grafico è simmetrico risp. asse ~~x~~ y.

(4) $0 \in \mathrm{Dom}(f)$

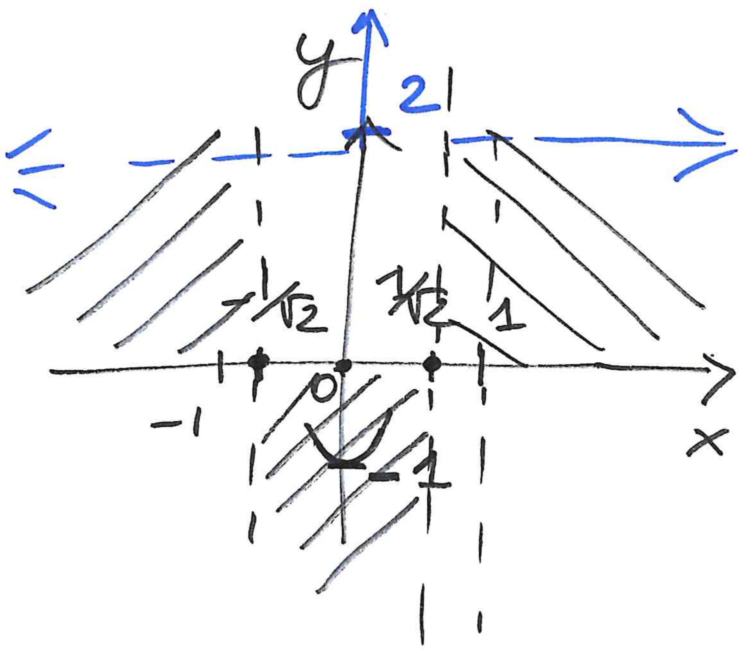
$f(0) = -1$

$f(x) = 0$

$2x^2-1 = 0 \iff$

$x = \pm \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} < 1$



$2x^2-1>0$ se $x>1/\sqrt{2}$

$x<-1/\sqrt{2}$

$2x^2-1<0$ se $-\frac{1}{\sqrt{2}}<x<1/\sqrt{2}$

(5) $\lim\limits_{x \to \pm\infty} \frac{2x^2-1}{x^2+1} = +2$

(6) $f'(x) = \frac{4x(x^2+1)-2x(2x^2-1)}{(x^2+1)^2} = \frac{6 \cdot x}{(x^2+1)^2}$

$f'(0)=0$

$f'(x)>0$ per $x>0$

$f'(x)<0$ per $x<0$

f': $-$ $\searrow$ | $\nearrow$ $+$

0 pto min. relativo

(7) $f''(x) = \frac{6(x^2+1)^2-2(x^2+1)(2x)(6x)}{(x^2+1)^4} = \frac{-6(3x^4+2x^2-1)}{(x^2+1)^4}$

Pongo $\boxed{x^2 = y}$

$$N_{f''} = -\cancel{6}(3y^2 + 2y - 1) = 0$$

$$y = -1, \quad y = 1/3$$

"x^2" non è accettabile

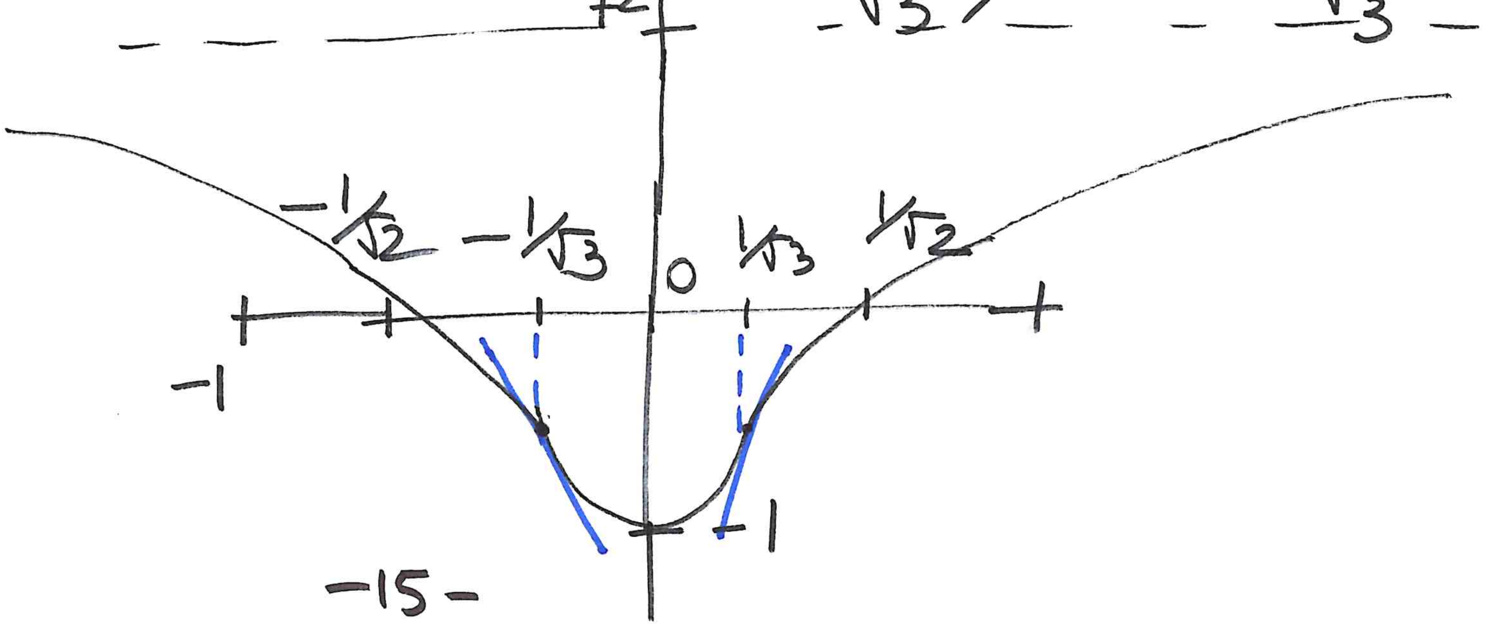
$$x^2 = 1/3 \Rightarrow x = \pm 1/\sqrt{3}$$

$$1/\sqrt{3} < 1/\sqrt{2}$$

$x = \pm 1/\sqrt{3}$ 2 flessi a tg. obliqua.

$f'' > 0$ $\quad x \in \left(-\frac{1}{\sqrt{3}}, 1/\sqrt{3}\right)$

$f'' < 0$ $\quad x > \frac{1}{\sqrt{3}}, \quad x < -\frac{1}{\sqrt{3}}$



Ex. studio qualitativo di:

1) $f(x) = \dfrac{x^2+1}{x^2-1}$

2) $g(x) = \dfrac{x^4-x^2+1}{x^3-x}$

Ex.
studiare cresc., decresc. ed eventuali max e min rel di:

1) $f(x) = 3x^2 - 6x$

2) $f(x) = 6 \sin x + 6 \cos x$.