Chapter
6

LIGHT TRAPPING

6.1 INTRODUCTION

To maximize the open-circuit voltage of a solar cell, recombination must be minimized throughout the cell, including recombination in the bulk of the cell. An effective way of reducing bulk recombination is to reduce the volume of the cell. It follows that, to obtain the best possible photovoltaic performance from silicon, every possible advantage must be taken of the light absorbing capabilities of a given cell volume. This means taking advantage of schemes which trap the light into the cell so that, if not absorbed during its first passage through the cell volume, light will have additional opportunities for absorption.

Such pathlength enhancement in the bulk regions of the cell is referred to as "light trapping". The same term is sometimes used to refer to the multiple reflections of externally incident light from the surface of a cell with surface texture, which reduce overall reflection loss. Reflection control in both unencapsulated and encapsulated cells is discussed in Appendix F. In the present chapter, such surface reflection will be assumed negligible and "light trapping" refers solely to bulk processes.

Figures 6.1(a) and (b) show two different types of light trapping schemes [6.1]. The first is based upon the randomization of the direction of light once it has been coupled into the semiconductor.

Si

~17°

To be coupled out of the silicon, a light ray has to strike the surface of silicon within an angle to the normal equal to *arcsin(1/n)* where *n* is the refractive index of silicon. This angle is only about 16 - 17° for silicon. The rest of the light is reflected by total internal reflection. This means that only a fraction of the light will escape from the silicon on each passage across it. In the second scheme, a regular geometrical structure is built into the bulk region to steer the light away from the normal to the front cell surface. This chapter describes the light-trapping properties of both the ***randomizing*** and ***geometrical*** schemes of Fig. 6.1.

6.2 RANDOMIZING SCHEMES

The light trapping properties of the randomizing or ***Lambertian*** scheme of Fig. 6.1(a) can be relatively simply analyzed [6.2]. The randomizing or Lambertian rear reflector is defined as reflecting light with uniform brightness (radiance) in all directions, regardless of the original angle of incidence. (Uniform "brightness" corresponds to a radiant intensity which varies as $\cos\theta$ where θ is the angle between the direction of observation and the normal to the surface reflecting the light [6.2]).

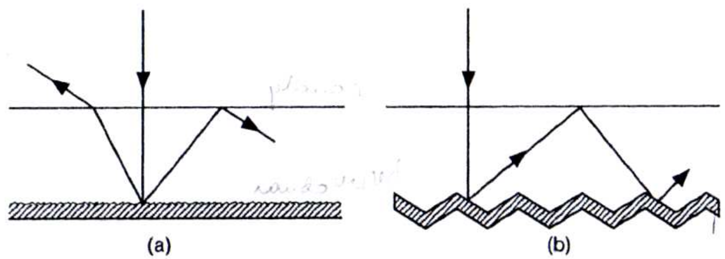


Figure 6.1: Two schemes for trapping light into a solar cell;(a) scheme based upon internal randomization of the light direction; (b) geometrically based scheme.

The key result of this analysis is that, in the case of weak absorption, a fraction of the light, equal to $1/n^2$, will be coupled out of the top surface after each reflection from the rear surface, assuming that the top surface has unity transmittivity for the light involved. The light coupled out will be that travelling in the cell in a direction close to the normal to the top surface. Its pathlength across the cell to this surface will approximately equal the cell thickness. The light which is not coupled out will be reflected to the rear of the cell where its direction will be re-randomized. On each passage across the cell, this light travels an average distance which is equal to twice the cell thickness, due to its being oblique to the normal (a total of four cell thicknesses between rear surface reflections).

One possible figure of merit for a light trapping scheme is the pathlength enhancement which is possible in the limit of weakly absorbed light. This is the ratio of the average pathlength of such a light beam to the minimum possible pathlength, which equals the average cell thickness, W. By calculating the pathlength of the light which is coupled out after each successive rear reflection, the following power series can be constructed for the average pathlength of the scheme of Fig. 6.1(a).

$$\bar{P} = 2W/n^2 + 6W(n^2-1)/n^4 + 10W(n^2-1)^2/n^6 + \ldots \qquad (6.1)$$

The power series expansion:

$$1/(1-x) = 1 + x + x^2 + x^3 + x^4 + \ldots \qquad (6.2)$$

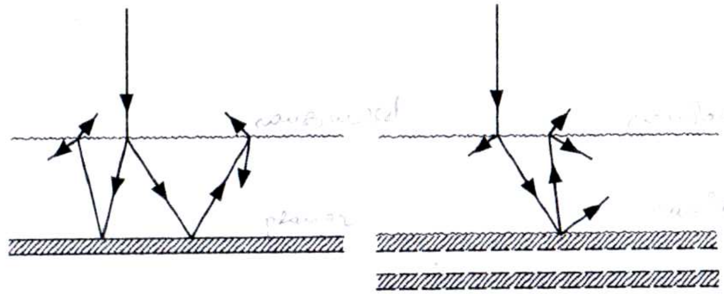
can be used to simplify Eq. (6.1) to give the following expression for the pathlength enhancement factor B:

$$B = 4n^2 - 2 \qquad (6.3)$$

This approximately equals the value of $4n^2$ which is also the approximate result of a more comprehensive analysis of this structure by Sheng [6.3], and that of a more general analysis by Yablonovitch and Cody [6.4] and Yablonovitch [6.5]. The value of the pathlength enhancement factor is close to 50 for silicon, indicating the substantial benefits which light trapping can provide.

The more general structures of Figs. 6.2(a) and (b) can be analyzed using a similar approach. The work of Yablonovitch and Cody [6.4] indicates that a fraction, $1/n^2$, of the randomized light striking the top surface will be coupled out even when it is randomly textured as in Fig. 6.2(a). This allows the same type of analysis as that above to be applied to this structure.

Such an analysis gives a pathlength enhancement equal to the previously mentioned value of $4n^2$. The structure of Fig. 6.2(b) also has the same value when an ideal reflector is used at the rear surface, either in intimate contact or detached, as shown. If no rear reflector is used, the pathlength enhancement factor has half of the previous value (assuming unity transmittivity for rays capable of escaping from the rear surface). This reduction by two would be expected from symmetry [6.4].



(a) (b)

Figure 6.2: Other randomizing light trapping schemes; (a) planar rear surface; (b) both surfaces randomizing. The broken lines at the rear surface indicate two different locations of the rear reflector

The previous expressions assume perfect rear reflectors. With a variable rear surface reflectance, R, the expression for pathlength enhancement for the schemes of Figs. 6.2(a) and (b) becomes:

$$B = 2(1+R)/[1 - R(1 - 1/n^2)] \qquad (6.4)$$

From this, the value of reflectance required to maintain more than a fraction, f, of the light trapping ability of the idealized case is found to be:

$$R > (2fn^2 - 1)/[2f(n^2-1)+1] \qquad (6.5)$$

For silicon, the reflectance must be greater than 91% to do at least half as well as the idealized case. If a metal reflectance better than this cannot be obtained, it would be more effective to rely upon total internal reflection from the rear surface. Although previously stated that this approach would give performance limited to half that of the idealized reflector case, the approach could do better than this in practice by designing for a lower than unity transmittivity of rays within the escape cone at the rear, or by using a simple detached rear reflector (Fig. 6.2(b)). The white polymer layer normally used at the rear of present solar cell modules could be used as such a detached reflector.

Experimentally, it is possible to obtain rear reflectance higher than the figure of 91% previously mentioned by interposing a dielectric layer between the metal and the silicon. Figure 6.3 shows the calculated reflectance, at a wavelength of 1.2 μm, from a rear Al layer separated by a variable thickness of SiO_2 from a polished silicon substrate, for various internal angles of incidence, θ, upon this combination.

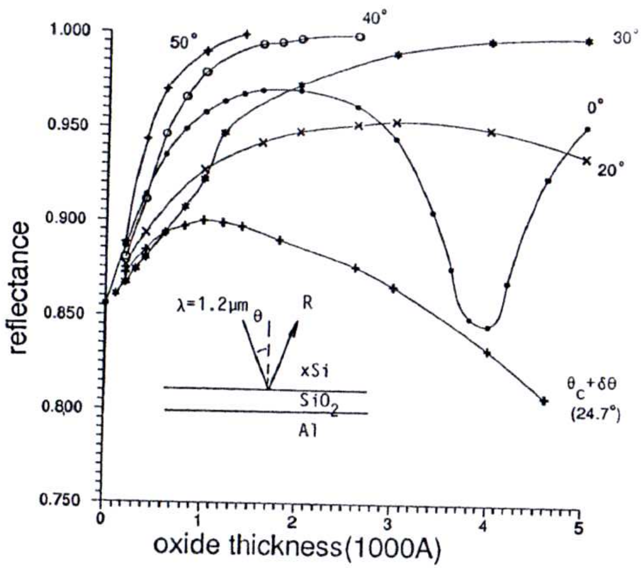


Figure 6.3: Calculated reflectance from a rear Al reflector at the rear of a silicon wafer as a function of the thickness for an intervening layer of SiO_2, for various internal angles of incidence. The reflectance for the $\theta_c + \delta\theta$ case reaches a minimum for about 1 mm oxide thickness (not shown), then increases towards unity (after Campbell [6.6]).

The combination of Fig. 6.2 is of interest for the high efficiency laboratory cells of Chapter 10. For angles of incidence below the critical angle for total internal reflection from the Si/SiO_2 interface (24.7°), cyclic interference effects predominate as the thickness of the oxide layer increases. Above the critical angle, reflectance approaches 100% as the oxide thickness increases and the detached reflector situation, described above, is approached. Reflectance is lowest near the critical angle. Just below the critical angle, the ray refracted at the Si/SiO_2 interface travels almost parallel to this interface, providing plenty of opportunity for energy transfer to the rear metal. More surprisingly, reflection is also low for angles of incidence just above the critical angle.

6.3 GEOMETRICAL LIGHT TRAPPING

6.3.1 Introduction

The levels of light trapping provided by the previous randomizing (Lambertian) schemes have sometimes been considered as limits on obtainable performance [6.7]. It appears that there may well be limits for cells with an ***isotropic*** response. (That is, for a cell which gives the same response, regardless of the angle of incidence of the light). However, the direction of most incident sunlight is predictable and, consequently, an isotropic response is not likely to be optimal. A cell design would be superior where the response at the less important oblique angles of incidence is traded off against the response for light incident at directions closer to the perpendicular.

Some ways of achieving such a trade-off for randomizing schemes are outlined in Section 6.6. Another possibility is that a geometrical light trapping scheme such as in Figure 6.1(b), designed for perpendicularly incident light, would also exhibit a superior performance to that of randomizing schemes for a range of angles of incidence close to the perpendicular.

The theory of geometrical light trapping is more difficult than that for randomizing approaches. However, such schemes can be analyzed on an individual basis by ray tracing.

LIGHT TRAPPING

53

Un modo per migliorare l'intrappolamento della luce in una cella solare è quello di mettere un diffusore lambertiano nella parte posteriore della cella. La superficie d'ingresso della luce è immaginata liscia.

Fig. 1.

Lo schema di Fig. 1 (Green, "Silicon Solar Cells") è detto: "Randomizing scheme" o "Lambertian scheme".

Randomizing or lambertian rear reflector.

Ipotizziamo un buon assorbimento.
Una porzione pari a $1/n^2$ della luce riflessa dal riflettore sarà disaccoppiata dal semiconduttore (vedi raggio 1), nell'ipotesi che la trasmittanza della superficie sia unitaria. La luce che esce dal semiconduttore è ~ normale alla superficie e quindi il suo cammino ottico sarà pari allo spessore della cella, W. La luce non disaccoppiata verrà riflessa indietro (TIR) e sarà di nuovo randomizzata. Ad ogni passaggio attraverso la cella questa luce percorrerà una distanza media pari a ~ 2W. (ovvero quella che viene disaccoppiata)
Dimostriamo per prima cosa che la porzione di luce che viene disaccoppiata è pari a $1/n^2$.

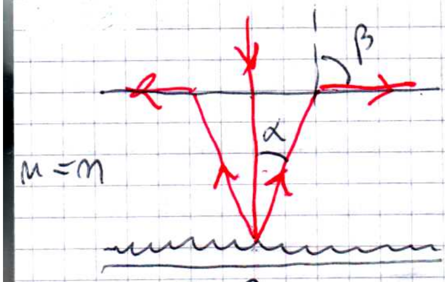


Fig. 2

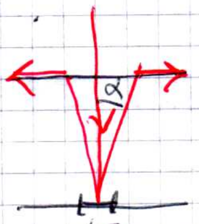
Il disaccoppiamento della luce si ha per un valore di $\alpha = \sin^{-1}(1/n)$. Abbiamo infatti che, ci corrisponde della riflessione totale:

$$\frac{\sin\alpha}{\sin\beta} = \frac{1}{n} \xrightarrow{TIR} \beta = \frac{\pi}{2} \rightarrow \boxed{\sin\alpha = \frac{1}{n}} \quad (1)$$

Ad esempio, nel caso di una superficie di Silicio in aria avremo:

$$\sin\alpha \simeq \frac{1}{3,4} \rightarrow \alpha \simeq 17°.$$

Fig. 3.



Consideriamo ora una superficie dS che riflette il flusso $d\phi$. Se il diffusore è lambertiano avremo:

$$\boxed{d\phi_{tot} = M\cdot dS = \pi\cdot L\cdot dS} \quad (2)$$

Il flusso riflesso entro l'angolo α sarà:

$$d\phi_\alpha = 2\pi\cdot\int_0^\alpha d\alpha'\cdot\sin\alpha'\cdot L\cdot dS\cdot\cos\alpha' = 2\pi\cdot L\cdot dS\int_0^\alpha d\alpha'\sin\alpha'\cos\alpha'$$

$$d\phi_\alpha = 2\pi L\cdot dS\cdot\frac{1}{2}\cdot\sin^2\alpha = \pi L\cdot dS\cdot\sin^2\alpha$$

$$d\phi_\alpha = \pi\cdot L\cdot dS\cdot 1/n^2 \Rightarrow \boxed{\frac{d\phi_\alpha}{d\phi_{tot}} = \frac{1}{n^2}} \quad (3)$$

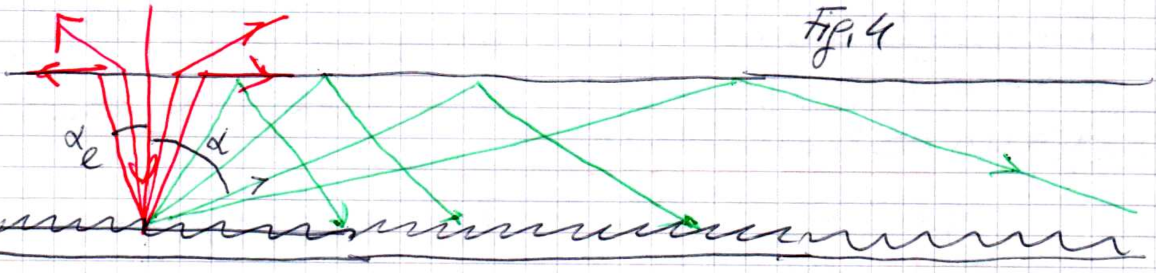
Il flusso non disaccoppiato dal semiconduttore sarà una frazione

$$\boxed{1-\frac{1}{n^2} = \frac{(n^2-1)}{n^2}} \quad (4)$$

← frazione di flusso non disaccoppiato dopo una riflessione sul back.

Di questo flusso non disaccoppiato, dobbiamo calcolare la distanza media percorsa prima di una nuova riflessione nel back:

Fig. 4



ho chiamato α_e il valore di α al di sotto del quale si ha disaccoppiamento. I raggi rossi ($\alpha < \alpha_e$) sono disaccoppiati dal silicio e sono una frazione $\frac{1}{n^2}$ di tutti quelli che sono riflessi dal back. Questi raggi avranno percorso

il semiconduttore in un tratto medio di $\sim 2W$ (andata e ritorno quasi perpendicolare attraverso lo strato). Quindi, il percorso $2W$ sarà pesato con $1/n^2$. Consideriamo ora i raggi (verdi) non disaccoppiati alla prima riflessione sul back della cella e supponiamo che essi siano disaccoppiati alla 2ª riflessione nel back. Un raggio generico di questi farà allora un percorso come rappresentato in fig. 5.

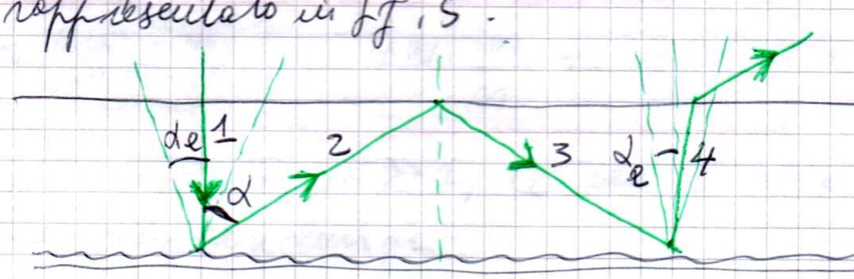


Fig. 5

Considerando che α_e è molto piccolo (α_e per Silicio $\simeq 17^o$)
il percorso del raggio verde sarà:

$$\begin{cases} \text{Tratto } 1: \sim W \\ \text{tratto } 2: \; ? \\ \text{Tratto } 3 = \text{Tratto } 2: \; ? \\ \text{Tratto } 4: \sim W \end{cases}$$

I tratti 1 e 4 sono pari a $\sim W$, mentre i tratti 2 e 3, uguali fra loro, dipendono dall'angolo α

e sono uguali a: $\boxed{\text{Tratto } 2(3) = \dfrac{W}{\cos\alpha}}$ (5)

Ora, i raggi non disaccoppiati alla 1ª riflessione avranno una pendenza compresa fra α e $\pi/2$ -
Il percorso 2+3 dovrà essere calcolato pesandolo in funzione dell'intensità radiante $I(\alpha)$ in direzione α: $I(\alpha) = I_0 \cdot \cos\alpha$ - Avremo allora:
Percorso medio 2+3 $= P_{2+3}$:

$$P_{2+3} = \frac{\cancel{2\pi} \cdot \int_\alpha^{\pi/2} d\alpha' \cdot \sin\alpha' \cdot \cos\alpha' \cdot \left(\frac{2W}{\cos\alpha'}\right) \cdot \cancel{I_0}}{\cancel{2\pi} \int_\alpha^{\pi/2} d\alpha' \cdot \sin\alpha' \cdot \cos\alpha' \cdot \cancel{I_0}} =$$

$$P_{2+3} = \frac{2W \cdot \left[-\cos\alpha'\right]_\alpha^{\pi/2}}{\left[\frac{1}{2}\sin^2\alpha'\right]_\alpha^{\pi/2}} = \frac{4 \cdot W \cdot \cos\alpha}{1-\sin^2\alpha} = \frac{4W}{\sqrt{1-\sin^2\alpha}}$$

$$\boxed{P_{2+3} = \frac{4W \cdot n}{\sqrt{n^2-1}} \simeq 4W \cdot \frac{n}{n} = 4W} \quad (6)$$

Abbiamo supposto $n^2 \gg 1$, il che è vero per i comuni semiconduttori -

57

Quindi, il percorso medio di un raggio (verde) disaccoppiato dopo 2 riflessioni nel back è pari a: Tratto 1 + (tratto 2 + tratto 3) + tratto 4 5 ...

... $\simeq W + 4W + W \simeq 6W$. La porzione di raggi che è disaccoppiata alla 2^a riflessione sarà pari a:

$$\boxed{\frac{1}{n^2}\cdot\left[1-\frac{1}{n^2}\right] = \frac{1}{n^2}\left[\frac{n^2-1}{n^2}\right] = \frac{(n^2-1)}{n^4}} \quad (7)$$

Quindi avremo che il percorso medio dei raggi disaccoppiati alla 1^a e alla 2^a riflessione sarà:

$$\boxed{\bar{P}_{1,2} = \frac{2W}{n^2} + 6W\cdot\frac{(n^2-1)}{n^4}} \quad (8)$$

Se iteriamo il discorso fatto finora a tutti i raggi, è facile verificare che il percorso medio $\bar{P}$ sarà uguale a:

$$(9) \quad \boxed{\bar{P} = \frac{2W}{n^2} + 6W\cdot\frac{(n^2-1)}{n^4} + 10W\cdot\frac{(n^2-1)^2}{n^6} + 14W\cdot\frac{(n^2-1)^3}{n^8} + \dots}$$

La (9) è una serie di potenze del tipo:

$$\bar{P} = \frac{2W}{n^2}\left\{1 + 3\frac{(n^2-1)}{n^2} + 5\cdot\frac{(n^2-1)^2}{n^4} + 7\frac{(n^2-1)^3}{n^6} + \dots\right\} =$$

$$= \frac{2W}{n^2} + \left[2W\cdot\frac{(n^2-1)}{n^4} + 4W\cdot\frac{(n^2-1)}{n^4}\right] + \left[2W\,\frac{(n^2-1)^2}{n^6} + 8W\cdot\frac{(n^2-1)^2}{n^6}\right] + \dots$$

$$\dots + \left[2W\cdot\frac{(n^2-1)^3}{n^8} + 12W\cdot\frac{(n^2-1)^3}{n^8}\right] + \dots$$

(58)

$$\bar{p} = \left\{ \frac{2w}{m^2} + 2w\cdot\frac{(m^2-1)}{m^4} + 2w\frac{(m^2-1)^2}{m^6} + 2w\frac{(m^2-1)^3}{m^8} + \cdots \right\} + \cdots$$

$$\cdots + \left\{ 4w\cdot\frac{(m^2-1)}{m^4} + 8w\cdot\frac{(m^2-1)^2}{m^6} + 12w\cdot\frac{(m^2-1)^3}{m^8} + \cdots \right\}$$

$$\boxed{\bar{p} = \Sigma_1 + \Sigma_2} \quad (10)$$

$$\Sigma_1 = 2w\left[\frac{1}{m^2} + \frac{(m^2-1)}{m^4} + \frac{(m^2-1)^2}{m^6} + \frac{(m^2-1)^3}{m^8} + \cdots \right] =$$

$$= \frac{2w}{m^2}\left[1 + \frac{(m^2-1)}{m^2} + \frac{(m^2-1)^2}{m^4} + \frac{(m^2-1)^3}{m^6} + \cdots \right] = \cdots$$

$$\cdots = \frac{2w}{m^2}\left[1 + \left(\frac{m^2-1}{m^2}\right) + \left(\frac{m^2-1}{m^2}\right)^2 + \left(\frac{m^2-1}{m^2}\right)^3 + \cdots \right] = \cdots$$

$$\boxed{\Sigma_1 = \frac{2w}{m^2}\cdot\frac{1}{1-\frac{(m^2-1)}{m^2}} = \frac{2w}{m^2}\cdot m^2 = 2w} \quad (11)$$

$$\Sigma_2 = 4w\left\{ \frac{m^2-1}{m^4} + 2\frac{(m^2-1)^2}{m^6} + 3\frac{(m^2-1)^3}{m^8} + \cdots \right\} = \cdots$$

$$\cdots = 4w\cdot\frac{m^2-1}{m^4}\cdot\left\{ 1 + 2\frac{m^2-1}{m^2} + 3\cdot\frac{(m^2-1)^2}{m^4} + 4\cdot\frac{(m^2-1)^3}{m^6} + \cdots \right\}$$

$$\boxed{\Sigma_2 = 4w\cdot\frac{(m^2-1)}{m^4}\cdot\Sigma_3} \quad (12)$$

$$\Sigma_3 = \sum_{i=0}^{\infty} (i+1)\cdot\frac{(m^2-1)^i}{m^{2i}} = \sum_{i=0}^{\infty}\frac{(m^2-1)^i}{m^{2i}} + \sum_{i=0}^{\infty} i\cdot\frac{(m^2-1)^i}{m^{2i}}$$

$$\boxed{\Sigma_3 = m^2 + \Sigma_4} \quad (13)$$

(59)

$$\Sigma'_4 = \frac{(n^2-1)}{n^2} + \frac{2(n^2-1)^2}{n^4} + \frac{3(n^2-1)^3}{n^6} + \frac{4\cdot(n^2-1)^4}{n^8} + \cdots$$

$$\Sigma'_u = \frac{(n^2-1)}{n^2} + \frac{(n^2-1)^2}{n^4} + \frac{(n^2-1)^3}{n^6} + \frac{(n^2-1)^4}{n^8} + \cdots \infty \quad \cdots +$$

$$\cdots + \frac{(n^2-1)^2}{n^4} + \frac{(n^2-1)^3}{n^6} + \frac{(n^2-1)^4}{n^8} + \frac{(n^2-1)^5}{n^{10}} + \cdots \infty \quad +$$

$$\cdots + \frac{(n^2-1)^3}{n^6} + \frac{(n^2-1)^4}{n^8} + \frac{(n^2-1)^5}{n^{10}} + \frac{(n^2-1)^6}{n^{12}} + \cdots \infty \quad +$$

$$\cdots$$

$$\Sigma'_u = \frac{n^2-1}{n^2}\left[1 + \frac{n^2-1}{n^2} + \frac{(n^2-1)^2}{n^4} + \frac{(n^2-1)^3}{n^6} + \cdots\right] + \cdots$$

$$\cdots + \frac{(n^2-1)^2}{n^4}\left[1 + \frac{n^2-1}{n^2} + \frac{(n^2-1)^2}{n^4} + \frac{(n^2-1)^3}{n^6} + \cdots\right] + \cdots$$

$$\cdots + \frac{(n^2-1)^3}{n^6}\left[1 + \frac{n^2-1}{n^2} + \frac{(n^2-1)^2}{n^4} + \frac{(n^2-1)^3}{n^6} + \cdots\right] + \cdots$$

$\cdots + \cdots \infty$ Poniamo: $x = (n^2-1)/n^2$

$$\Sigma'_u = x\left[1 + x + x^2 + x^3 + x^4 + \cdots\right] + \cdots$$

$$\cdots + x^2\left[1 + x + x^2 + x^3 + x^4 + \cdots\right] + \cdots$$

$$\cdots + x^3\left[1 + x + x^2 + x^3 + x^4 + \cdots\right] + \cdots$$

$$\Sigma'_u = \left[1 + x + x^2 + x^3 + \cdots\right]\cdot\left[x + x^2 + x^3 + \cdots\right] =$$

$$= \left[1 + x + x^2 + x^3 + \cdots\right]\cdot x\left[1 + x + x^2 + \cdots\right] = \cdots$$

$$\boxed{\Sigma'_4 = x\cdot\frac{1}{(1-x)^2} = \frac{n^2-1}{n^2}\cdot(n^2)^2 = (n^2-1)n^2} \quad (14)$$

$$\Sigma'_3 = n^2 + \Sigma'_4 = n^2 + n^2(n^2-1) = n^2(1+n^2-1) = n^4 \quad (15)$$

$$\Sigma'_2 = 4W \cdot \frac{(n^2-1)}{n^4} \cdot \Sigma'_3 = 4W \cdot (n^2-1) \quad (16)$$

$$\Sigma'_1 = 2W \quad (17)$$

$$\bar{P} = \Sigma'_1 + \Sigma'_2 = 2W + 4W(n^2-1) = W\,[2+4(n^2-1)]$$

$$\bar{P} = W \cdot [4n^2-2] \quad (18)$$

$$\bar{P} = B \cdot W \; ; \quad \boxed{B = 4n^2-2}$$

Il parametro B è detto "Pathlength enhancement factor" e rappresenta il fattore con il quale il cammino ottico W è innalzato per effetto della presenza nel back della cella di un diffusore lambertiano.