Theory of the "Inverse Method" for Characterization of Solar Concentrators

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Abstract: The theory of "inverse method" applied to the optical characterization of solar concentrators is revisited. New optical quantities are introduced and the experimental procedure for measuring the on-axis absolute "direct" transmission efficiency is reported.

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1. Introduction

The "inverse method" (IM) of characterization of solar concentrators (SC) is used to obtain the angle-resolved transmission efficiency [1,2]. It is an advantageous alternative to the so called "direct method" (DM), which requires long measurements and an expensive apparatus. The IM consists in irradiating the concentrator in inverse way, that is from the output aperture, therefore reversing the light path occurring in common operation. The IM has been already presented and the method to derive the "relative" angle-resolved transmission efficiency has been discussed [1,2]. In this paper we revisit the theory introducing new quantities and presenting the method for measuring the on-axis "direct" transmission efficiency, necessary to calculate the "absolute" angle-resolved transmission efficiency, previously achievable only by the "direct" method.

2. Theory of "inverse method"

The "inverse method" is applied by producing a lambertian irradiation at the output aperture of the SC [1,2]. Light rays therefore will follow paths which are inverted respect to the those occurring during normal operation. The light projected from the SC is intercepted by a planar screen at adequate distance and the image there produced is recorded by a CCD and analyzed at a computer. From the intensity distribution of the image the irradiance distribution on the screen, $E(\theta, \varphi)$, is derived as function of emission direction [3]. The irradiance, corrected in any point by the factor $\cos^{-4}(\theta)$, gives the angular distribution of radiance $L_C^{inv}(\theta,\varphi)$ of input aperture of the concentrator, which is proportional to the angular distribution of the "direct" transmission efficiency $\eta_{dir}(\theta, \varphi)$. This affirmation will be carefully discussed in this paper. Let us go to an *ab initio* investigation of "direct" and "inverse" methods by considering the variety of optical paths that light can follow inside a concentrator from (ia) to (oa) and vice versa. We distinguish between "coupling" and "uncoupling" paths depending if the paths connect or not the two apertures (see Fig. 1). The reversibility principle in optics establishes that the attenuation undergone by an unpolarized beam on the same path at opposite directions is the same. Eq.s (1) show the attenuation for single reflection *R*, and refraction, *T*.

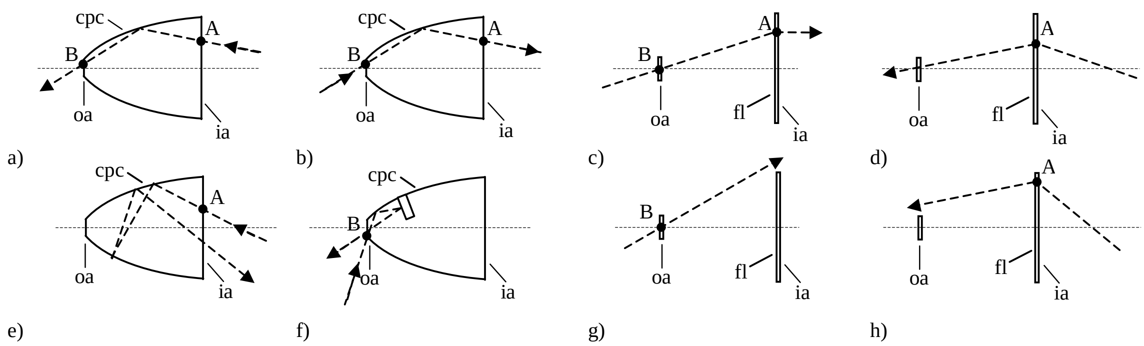

Figure 1. Examples of light paths connecting (a-d) and non connecting (e-h) input (ia) and output (oa) apertures in nonimaging (cpc) and imaging (fresnel lens, fl) concentrators. Beams: (a, d) direct transmitted; (b, c) inverse transmitted; (e) direct reflected; (f) inverse reflected; (g) inverse absorbed (h) direct absorbed.

$$R = \frac{1}{2}\cdot\sin^2(\varphi-\varphi')\cdot\left[\frac{\cos^2(\varphi+\varphi')+\cos^2(\varphi-\varphi')}{\sin^2(\varphi+\varphi')\cdot\cos^2(\varphi-\varphi')}\right] \quad T = 2\cdot\sin\varphi\cdot\sin\varphi'\cdot\cos\varphi\cdot\cos\varphi'\cdot\left[\frac{1+\cos^2(\varphi-\varphi')}{\sin^2(\varphi+\varphi')\cdot\cos^2(\varphi-\varphi')}\right] \quad (1)$$

It's easy to verify that factors R and T remain unchanged at exchanging incidence and transmitted angles, φ, φ'. Eq.s (1) can be extended to any multi step path inside the concentrator. The transmission of flux from input to output apertures and vice versa takes place only through the coupling paths and is attenuated as dictated by Eq.s (1). The non coupling paths, both direct and inverse, determine the loss of flux for reflection and full absorption. In the coupling, overlapped paths (a) and (b) of Fig. 1, for example, the attenuation factor of A→B path is equal to the attenuation factor of B→A path: $T_{AB} = T_{BA}$, then the relative loss of radiance of "direct" beam in (a) is the same experienced by the "inverse" beam in (b), if the starting "direct" and "inverse" beams are unpolarized. The attenuation factor for the radiance of "direct" beam incident on A at (θ, φ) direction is the "local direct" transmission efficiency $\eta_{dir}(A, \theta, \varphi)$, whereas the attenuation factor for the radiance of the "inverse" beam emitted by A at (θ, φ) direction is the "local inverse" transmission (emission) efficiency $\eta_{inv}(A, \theta, \varphi)$. We extend now these concepts to all the points of (ia) "directly" irradiated at (θ, φ) direction and to the same points of (ia) "inversely" emitting at (θ, φ) direction. If the input "inverse" radiance L_{inv} is kept constant for any direction, that means if we put a lambertian, source at output aperture, then the output "inverse" radiance $L_C^{inv}(\theta, \varphi)$, averaged for all the points of (ia), must have the same angular distribution of the "inverse" transmission efficiency $\eta_{inv}(\theta, \varphi)$, averaged for all the points of (ia). We can affirm therefore that: ″the output "inverse" radiance $L_C^{inv}(\theta, \varphi)$ of a concentrator irradiated on the backside by a lambertian, unpolarized source, is proportional to the "direct" transmission efficiency $\eta_{dir}(\theta, \varphi)$ of a collimated, unpolarized beam″. Equivalently, being $L_C^{inv}(\theta, \varphi) = L_C^{inv}(0) \cdot L_C^{inv,rel}(\theta, \varphi)$ and $\eta_{dir}(\theta, \varphi) = \eta_{dir}(0) \cdot \eta_{dir}^{rel}(\theta, \varphi)$, we can write: $L_C^{inv,rel}(\theta, \varphi) = \eta_{dir}^{rel}(\theta, \varphi)$, as affirmed at the beginning of this paragraph. The theory so far exposed establishes the validity of "inverse" method to give the full information about the relative transmission efficiency of concentrator in "direct" irradiation $\eta_{dir}^{rel}(\theta, \varphi)$ (see Fig. 2a).

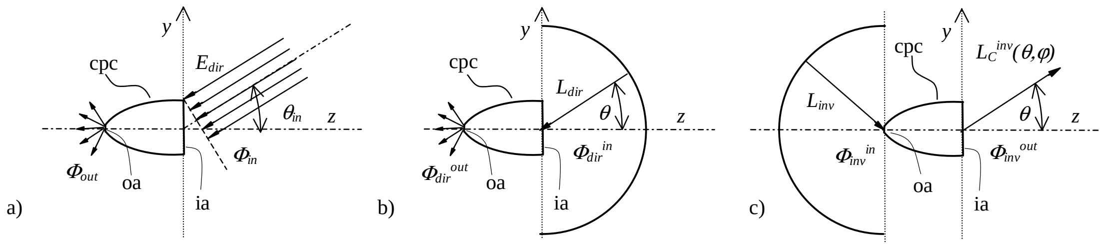

Figure 2. a) Scheme of "direct method" (DM). b) Scheme of "integral direct method" (IDM). c) Scheme of "inverse method" (IM).

Let's now consider the "isotropic direct" or "integral direct" irradiation of SC with constant radiance L_{dir} as illustrated in Fig. 2b (integral direct method or IDM). The total input flux is:

$$\Phi_{dir}^{in} = L_{dir}\cdot A_{in}\cdot\int_0^{2\pi} d\phi\cdot\int_0^{\pi/2} d\theta\cdot\sin\theta\cdot\cos\theta = \pi\cdot L_{dir}\cdot A_{in} \quad (2)$$

where we have considered cylindrical symmetry for simplicity. The extension of equations to the general case is immediate. The flux "transmitted" to the output is equal to:

$$\Phi_{dir}^{out} = 2\pi\cdot L_{dir}\cdot A_{in}\cdot\int_0^{\pi/2} d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta) \quad (3)$$

The integral transmission factor, or the "direct integral transmittance" of the concentrator τ_{dir}^{int}, is given then by:

$$\tau_{dir}^{\text{int}} = \frac{\Phi_{dir}^{out}}{\Phi_{dir}^{in}} = 2\cdot\int_0^{\pi/2} d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta) = 2\cdot\eta_{dir}(0)\cdot\int_0^{\pi/2} d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}^{rel}(\theta) \quad (4)$$

When we irradiate the SC in the "inverse" mode (see Fig. 2c), the output aperture (oa), of area A_{out}, becomes a lambertian source with uniform radiance L_{inv}. The total input flux is: $\Phi_{inv}{}^{in} = \pi \cdot L_{inv} \cdot A_{out}$, with L_{inv} = *const* by definition. The inverse flux transmitted at output, the input aperture (ia) of area A_{in} of the SC, is given by:

$$\Phi_{inv}^{out} = 2\pi \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_C^{inv}(\theta) \qquad (5)$$

where (θ,ϕ) is the direction of emission of inverse rays outside the concentrator and $L_C{}^{inv}(\theta)$ is the radiance of emitted light. Now we define the "inverse integral transmittance (or emission) efficiency" $\tau_{int}{}^{inv}$ of the concentrator:

$$\tau_{inv}^{\text{int}} = \frac{\Phi_{inv}^{out}}{\Phi_{inv}^{in}} = \frac{2\pi \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_C^{inv}(\theta)}{\pi \cdot A_{out} \cdot L_{inv}} = \frac{2 \cdot C_{geo}}{L_{inv}} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_C^{inv}(\theta) = \ldots \qquad (6)$$
$$\ldots = \frac{2 \cdot C_{geo}}{L_{inv}} \cdot L_C^{inv}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_C^{inv,rel}(\theta)$$

where C_{geo} is the geometrical concentration ratio. Let us compare the two integral transmittances $\tau_{dir}{}^{int}$ and $\tau_{inv}{}^{int}$:

$$\frac{\tau_{inv}^{\text{int}}}{\tau_{dir}^{\text{int}}} = \frac{\frac{2 \cdot C_{geo}}{L_{inv}} \cdot L_C^{inv}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_C^{inv,rel}(\theta)}{2 \cdot \eta_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}^{rel}(\theta)} = \frac{\frac{C_{geo}}{L_{inv}} \cdot L_C^{inv}(0)}{\eta_{dir}(0)} = C_{geo} \cdot \frac{L_C^{inv}(0)}{\eta_{dir}(0) \cdot L_{inv}} \qquad (7)$$

In the special case $L_{dir} = L_{inv}$, the total integral flux transmitted in the "direct" and the "inverse" directions is the same: $\Phi_{dir}{}^{out} = \Phi_{inv}{}^{out}$, because the flux transmitted through the elementary coupling paths in "direct" and "inverse" directions is the same. Then we have:

$$2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) = 2\pi \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_C^{inv}(\theta) \quad \Rightarrow$$
$$L_{inv} \cdot \eta_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}^{rel}(\theta) = L_C^{inv}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_C^{inv,rel}(\theta) \quad \Rightarrow \quad \eta_{dir}(0) = \frac{L_C^{inv}(0)}{L_{inv}} \qquad (8)$$

Eq. (8) allows to calculate $\eta_{dir}(0)$ by IM measuring: $L_C{}^{inv}(0)$, the average on-axis radiance of SC, and L_{inv}, the radiance of the inverse lambertian source. To perform these measurements we need to orient the CCD towards the (ia) and to record its image. The average intensity of (ia) is proportional to $L_C{}^{inv}(0)$, while the intensity of receiver is proportional to L_{inv}. These measures are easy for CPC concentrators, where the receiver is visible. For refractive concentrators the image of receiver must be taken by removing the lens in front of the concentrator. From Eq.s (7) and (8) we obtain:

$$\frac{\tau_{inv}^{\text{int}}}{\tau_{dir}^{\text{int}}} = \frac{\Phi_{inv}^{out}}{\Phi_{inv}^{in}} \cdot \frac{\Phi_{dir}^{in}}{\Phi_{dir}^{out}} = \frac{\Phi_{dir}^{in}}{\Phi_{inv}^{in}} = C_{geo} \qquad (9)$$

that is: for equal transmitted flux in the two directions, the "inverse integral transmittance" of a SC is C_{geo} times its "direct integral transmittance", and: the "input direct integral flux" needed to sustain an equal transmitted flux in the opposite directions is C_{geo} times the "input inverse integral flux".

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