Theoretical Aspects of Light Collection in Solar Concentrators

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Abstract: The theory of light collection in solar concentrators irradiated in the "direct" mode (from input aperture) is revisited and new concepts are introduced. Application of the theory is made mainly to nonimaging (CPC) concentrators.

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1. Introduction

A solar concentrator can be regarded as a generic optical element for which we can define a reflectance, an absorbance and a transmittance. These quantities can be calculated in relation to different types of irradiations, the most natural being the collimated beam and the isotropic irradiation associated with the direct and diffuse outdoor solar radiation components, respectively. In this paper we extensively use the term "direct" meaning that the concentrator is irradiated from the input aperture. In this sense, the term "direct" is by no means to be associated to the direct component of solar radiation. We will investigate mainly concentrators derived by the nonimaging optics, in particular the Compound Parabolic Concentrators (CPC) [1], with ideal and real properties.

2. Theory of "direct" optical collection

The fundamental quantity which summarizes the optical collection properties of a solar concentrator (SC) is the transmission efficiency $\eta_{\mathrm{dir}}(\theta_{in}, \varphi_{in})$, expressed as function of the direction, that is zenithal and azimuthal angles $(\theta_{in}, \varphi_{in})$, of a collimated beam, with uniform irradiance E_{dir} at the wave front, and given by the ratio of output and input fluxes:

$$\eta_{dir}(\theta_{in},\varphi_{in}) = \frac{\Phi_{out}(\theta_{in},\varphi_{in})}{\Phi_{in}(\theta_{in},\varphi_{in})} = \frac{\Phi_{out}(\theta_{in},\varphi_{in})}{E_{in}\cdot A_{in}(\theta_{in},\varphi_{in})} \quad (1)$$

where $A_{in}(\theta_{in}, \varphi_{in})$ is the projected area of input aperture (ia) of SC, of area A_{in}, equal to $A_{in} \cdot \cos(\theta_{in})$ when the contour of (ia) is contained on a plane. Fig. 1a shows the scheme of measure of $\eta_{dir}(\theta_{in}, \varphi_{in})$, called "direct method" (DM). In general we have: $\eta_{dir}(\theta_{in}, \varphi_{in})$ = fraction of flux transmitted to output aperture (oa), $\alpha_{dir}(\theta_{in}, \varphi_{in})$ = fraction of absorbed flux, and $\rho_{dir}(\theta_{in}, \varphi_{in})$ = fraction of reflected flux, with: $\eta_{dir}(\theta_{in}, \varphi_{in}) + \alpha_{dir}(\theta_{in}, \varphi_{in}) + \rho_{dir}(\theta_{in}, \varphi_{in}) = 1$.

Let's consider now the isotropic irradiation of SC as illustrated in Fig. 1b (integral direct method or IDM). We can imagine a hemispherical screen (hs) irradiating the SC at constant radiance L_{dir}. The total incident flux is:

$$\Phi_{dir}^{in} = L_{dir}\cdot A_{in}\cdot\int_0^{2\pi} d\phi\cdot\int_0^{\pi/2} d\theta\cdot\sin\theta\cdot\cos\theta = \pi\cdot L_{dir}\cdot A_{in} \quad (2)$$

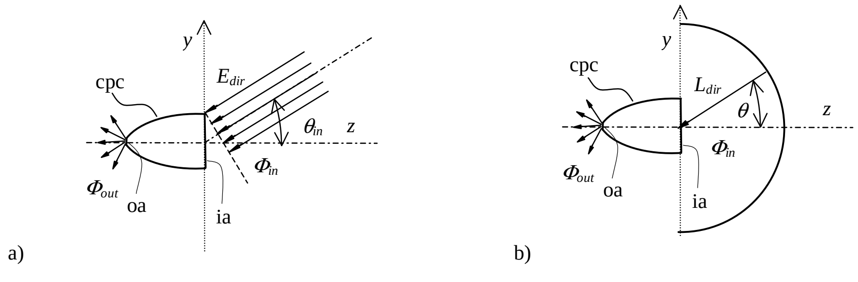

Figure 1. Schematic principle of Direct Method (DM) (a) and Integral Direct Method (IDM) (b) of irradiation.

In the following we will consider, for simplicity, only concentrators with cylindrical symmetry: $\eta_{dir}(\theta, \varphi) = \eta_{dir}(\theta) = \eta(\theta)$. The equations can be easily extended, if necessary, to the general case by reintroducing the dependence on the azimuthal angle. The flux "transmitted" to the output, becomes:

$$\Phi_{dir}^{out} = \Phi_{dir}^{\tau} = 2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta(\theta) \qquad (3)$$

The optical loss is represented by the "rejected" flux Φ_{dir}^{ρ} and by the "absorbed" flux Φ_{dir}^{α} :

$$\Phi_{dir}^{\rho} = 2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \rho(\theta) \qquad (4) \qquad\qquad \Phi_{dir}^{\alpha} = 2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \alpha(\theta) \qquad (5)$$

in such a way that: $\Phi_{dir}^{\tau} + \Phi_{dir}^{\rho} + \Phi_{dir}^{\alpha} = 1$.

In absence of loss by absorbance, the integral direct irradiation leads to a significant result: on the output aperture the irradiation distribution is uniform and the angular distribution of intensity is lambertian (constant radiance). The integral direct irradiation of an ideal (non absorbing) concentrator, therefore, produces a uniform lambertian irradiation at output. This can be demonstrated by theoretical considerations and by optical simulations. The output radiance is given by:

$$L_{dir}^{S} = \frac{\Phi_{dir}^{\tau}}{\pi \cdot A_{out}} = \frac{2 \cdot L_{dir} \cdot A_{in}}{A_{out}} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta(\theta) = 2 \cdot L_{dir} \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta(\theta) \qquad (6)$$

where we have introduced the geometrical concentration ratio C_{geo}. In the case of a non ideal SC, the output irradiation is no more uniform and lambertian and Eq. (6) gives only the average radiance at output, for a particular absorbance $\alpha(\theta)$.

$$\overline{L}_{dir}^{S(\alpha)} = \frac{\Phi_{dir}^{\tau(\alpha)}}{\pi \cdot A_{out}} = 2 \cdot L_{dir} \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{\alpha}(\theta) = 2 \cdot L_{dir} \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot [1 - \alpha(\theta) - \rho(\theta)] \qquad (7)$$

where $\Phi_{dir}{}^{\tau(\alpha)}$ is the output flux in presence of absorbance and $\eta_{\alpha}(\theta)$is the transmission efficiency in presence of absorbance. Naturally, the output radiance of a real SC is lower than the radiance of the corresponding ideal concentrator. We have simulated a 3D-CPC with $C_{geo} \approx 130$, $\theta_{acc} = 5°$, irradiating it in the integral "direct" mode (Fig. 2). The simulations confirm that the uniform spatial distribution of flux at output and the lambertian angular divergence is reached for the ideal case (wall reflectivity $R_w = 1$), not for the non ideal case (wall reflectivity $R_w = 0.8$). The spatial distribution has been tested by putting a flat absorber on the output aperture, whereas the angular distribution has been tested by putting an hemispherical absorbing globe centered on the output aperture. We now introduce a new quantity, the ratio between output and input radiances. The ratio between output and input radiances in the general, non ideal, case becomes:

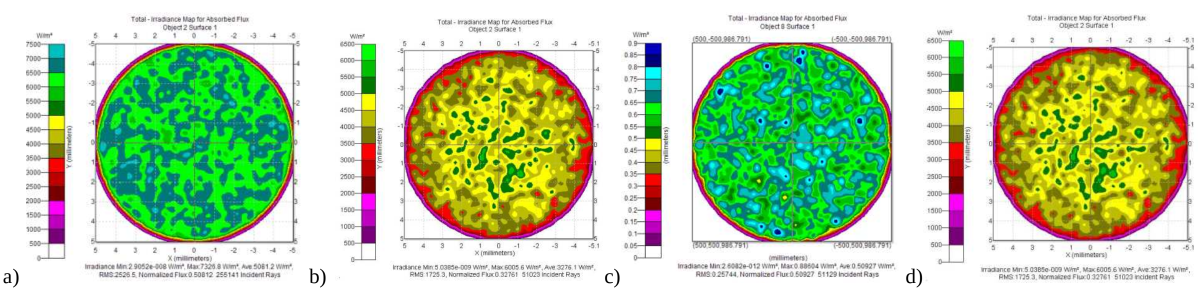

Figure 2. Map of irradiance at output aperture for R_w=1 (a) and R_w=0.8 (b). Map of irradiance on an hemispherical globe centered on the output aperture for R_w=1 (c) and R_w =0.8 (d). The irradiation distribution on the globe is a function of $\cos\theta$ for a lambertian angular distribution at output of concentrator, but this distribution becomes uniform when it is projected on a plane for representation purposes, as it is done for c) and d) maps.

$$\frac{\overline{L}_{dir}^{S(\alpha)}}{L_{dir}} = \lambda_{dir(\alpha)} = 2 \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_\alpha(\theta) = 2 \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot [1 - \alpha(\theta) - \rho(\theta)] \tag{8}$$

We introduce also the following new optical quantities: “direct integral optical efficiency $\eta_{dir}{}^{int}$, “direct integral optical absorbance” $\alpha_{dir}{}^{int}$ and “direct integral optical reflectance $\rho_{dir}{}^{int}$, given respectively by:

$$\eta_{dir}^{\text{int}} = \frac{\Phi_{dir}^{\tau}}{\Phi_{dir}^{in}} = 2 \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) = 2 \cdot \eta_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}^{rel}(\theta) \tag{9}$$

$$\alpha_{dir}^{\text{int}} = \frac{\Phi_{dir}^{\alpha}}{\Phi_{dir}^{in}} = 2 \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \alpha(\theta) = 2 \cdot \alpha(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \alpha^{rel}(\theta) \tag{10}$$

$$\rho_{dir}^{\text{int}} = \frac{\Phi_{dir}^{\rho}}{\Phi_{dir}^{in}} = 2 \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \rho(\theta) = 2 \cdot \rho(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \rho^{rel}(\theta) \tag{11}$$

where: $\eta_{dir}^{\text{int}} + \alpha_{dir}^{\text{int}} + \rho_{dir}^{\text{int}} = 1$. From Eqs. (8), (9) we find the relevant relationship:

$$\lambda_{dir(\alpha)} = \frac{\overline{L}_{dir}^{S(\alpha)}}{L_{dir}} = \eta_{dir}^{\text{int}} \cdot C_{geo} = \frac{\Phi_{dir}^{\tau}}{\Phi_{dir}^{in}} \cdot \frac{A_{in}}{A_{out}} \tag{12}$$

Eq. (12) has the same form of the relationship defining the optical concentration ratio of a SC under a collimated beam irradiation:

$$C_{opt} = \frac{E_{out}}{E_{in}} = \eta_{dir} \cdot C_{geo} = \frac{\Phi_{out}}{\Phi_{in}} \cdot \frac{A_{in}}{A_{out}} \tag{13}$$

then the quantity $\lambda_{dir(\alpha)}$ can be defined as the optical concentration ratio under integral direct irradiation, and corresponds to the concentration ratio achievable outdoors by the irradiation of diffuse solar light. Some of the above defined quantities are evaluated for the special case of a 3D-CPC concentrator, characterized by the acceptance angle θ_{acc} and the on-axis optical efficiency $\eta_{dir}(0)$ [1]. The transmission efficiency function $\eta_{dir}(\theta)$ for a 3D-CPC nonimaging concentrator can be approximated to a step function with $\eta_{dir}(\theta) \approx \eta_{dir}(0)$ for $\theta = 0 \div \theta_{acc}$, $\eta_{dir}(\theta) \approx 0$ for $\theta > \theta_{acc}$, with θ_{acc} acceptance angle measured at 50% of the 0° efficiency [1]. We obtain for the radiance ratio $\lambda_{dir(\alpha)}$, the direct integral transmission efficiency $\eta_{dir}{}^{int}$ and the direct integral absorbance $\alpha_{dir}{}^{int}$, respectively:

$$\lambda_{dir(\alpha)} = 2 \cdot C_{geo} \cdot \eta_{dir}(0) \cdot \int_0^{\theta acc} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}^{rel}(\theta) = C_{geo} \cdot \eta_{dir}(0) \cdot \sin^2\theta_{acc} \tag{14}$$

$$\eta_{dir}^{\text{int}} = 2 \cdot \eta_{dir}(0) \cdot \int_0^{\theta acc} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}^{rel}(\theta) = \eta_{dir}(0) \cdot \sin^2\theta_{acc} \tag{15}$$

$$\alpha_{dir}^{\text{int}} = 2 \cdot [1 - \eta_{dir}(0)] \cdot \int_0^{\theta acc} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}^{rel}(\theta) = [1 - \eta_{dir}(0)] \cdot \sin^2\theta_{acc} \tag{16}$$

For a CPC with $C_{geo} \approx 130$, $\theta_{acc} \approx 5°$ and $\eta_{dir}(0) \approx 0.9$, for example, we obtain: $\lambda_{dir(\alpha)} = 130 \cdot 0{,}9 \cdot 0{,}0076 = 0{,}889$; $\eta_{dir}{}^{int} = 0{,}9 \cdot 0{,}0076 = 0{,}00684 = 0{,}684\%$; $\alpha_{dir}{}^{int} = 0{,}1 \cdot 0{,}0076 = 0{,}00076 = 0{,}076\%$. This means that the 0.684% of the incident flux is transmitted to the receiver, the 0.076% is absorbed on the walls and the (100–0.684–0.076) = 99.24% is reflected backwards.

[1] R. Winston, J. C. Miñano and P. Benítez, *Nonimaging Optics* (Elsevier Academic Press, 2005).