OPTICAL MODELS OF LIGHT COLLECTION IN SOLAR CONCENTRATORS

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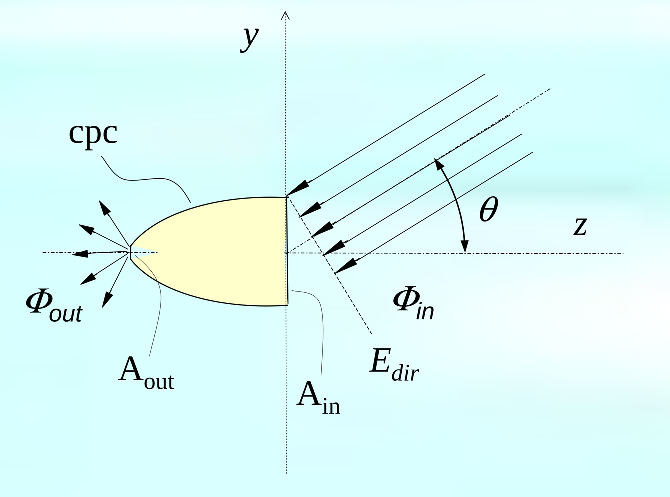
ABSTRACT

Theoretical models of light collection in solar concentrators (SC) are presented together with new advancements of the theoretical analysis which leads to the introduction of new optical concepts and the definition of new optical quantities. Solar concentrators are viewed as generic optical elements whose reflectance, absorbance and transmittance properties are expressed with respect to different irradiation conditions. They are studied under collimated or diffuse light, under local or integral irradiation, including that in which light direction is reversed. All the results were obtained applying two basic concepts: the reversibility principle and the efficiency of transmission of an elemental beam. In this paper we discuss theoretical models of irradiation, which are simplifications of the outdoor irradiation conditions. For each model we derive a specific method of characterization of the SC, that can be applied by optical simulations at a computer or by experimental measurements. A generic solar concentrator is schematised as a device confined between an entrance aperture (ia) with area A_{in} and an exit aperture (oa) with area A_{out}, where $A_{in} > A_{out}$, as required by definition of solar concentrator. A solar concentrator operates in practice under "direct" irradiation, that is under irradiation on the entrance aperture and with a receiver, the energy conversion device, at the exit aperture [1].

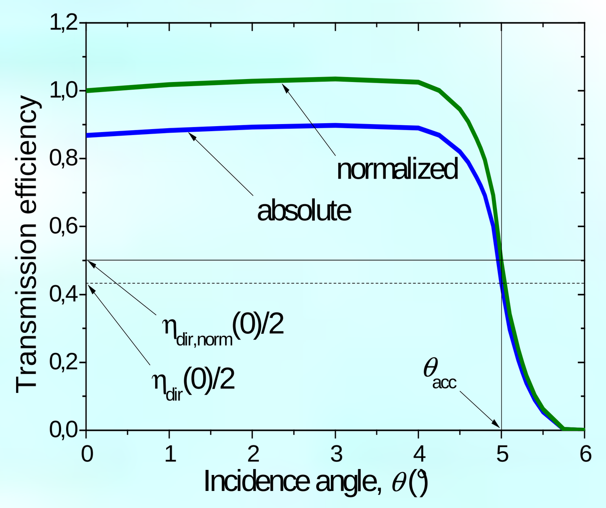
THE "DIRECT COLLIMATED METHOD" (DCM)

The optical collection properties under direct and collimated beam of a solar concentrator, with rotational symmetry, are summarized by the *angle-resolved transmission efficiency*:

$$\eta_{dir}(\theta)=\frac{\Phi_{out}(\theta)}{\Phi_{in}(\theta)}=\frac{\Phi_{out}(\theta)}{E_{dir}\cdot A_{in}\cdot\cos\theta}=\eta_{dir}(0)\cdot\eta_{dir,norm}(\theta)$$



Basic scheme of the Direct Collimated Method (DCM)



Transmission efficiency of a 3D-CPC nonimaging solar concentrator. They are shown the absolute and normalized transmission curves.
The acceptance angle θ_{acc} is the angle at which the efficiency drops to 50%. Both curves have the same θ_{acc} (5°).

In general we have:

$$\eta_{dir}(\theta)=\frac{\Phi_{out}(\theta)}{\Phi_{in}(\theta)}\qquad \alpha_{dir}(\theta)=\frac{\Phi_{\alpha}(\theta)}{\Phi_{in}(\theta)}\qquad \rho_{dir}(\theta)=\frac{\Phi_{\rho}(\theta)}{\Phi_{in}(\theta)}$$

where:

$$\eta_{dir}(\theta)+\alpha_{dir}(\theta)+\rho_{dir}(\theta)=1$$

Transmission efficiency (fraction of transmitted flux); ***Absorption efficiency*** (fraction of absorbed flux); ***Reflection efficiency*** (fraction of reflected flux); ***Conservation of energy***

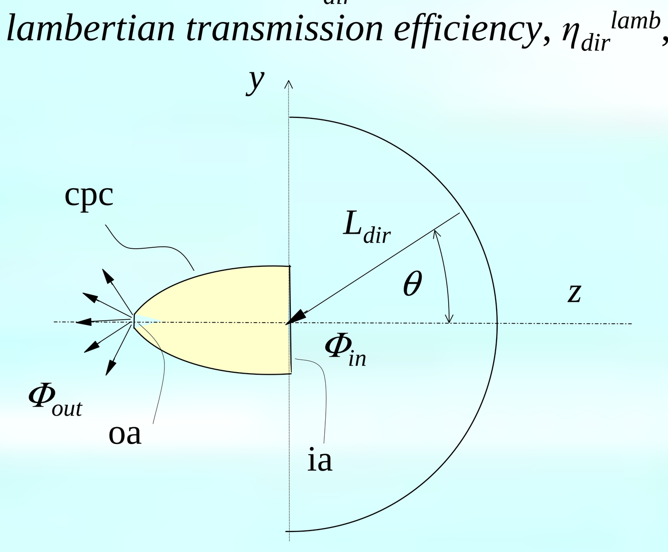
A perfect parallel beam is impossible to realize in practice, then, if $L_{dir}(\theta,\varphi)$ is the radiance of the light source from (θ,φ) direction, and $d\Omega$ is the solid angle within which light is collected, the *angle-resolved transmission efficiency* can be defined as:

$$\eta_{dir}(\theta)=\frac{d\Phi_{out}(\theta,\varphi)}{d\Phi_{in}(\theta,\varphi)}=\frac{1}{L_{dir}(\theta,\varphi)\cdot A_{in}\cdot\cos\theta}\cdot\lim(d\Omega\to 0)\frac{d\Phi_{out}(\theta,\varphi)}{d\Omega}$$

Similar expressions are used to obtain the absorption and reflection efficiencies.
The number of measurements required to apply the DCM is very high, both for simulations and for experimental measurements. This is indeed the strong limit of DCM applied to the determination of $\eta_{dir}(\theta)$. This limit can be overcome by the use of the "Inverse Lambertian Method" (ILM) of irradiation, as it will be demonstrated in the corresponding section.

THE "DIRECT LAMBERTIAN METHOD" (DLM)

The "direct lambertian method" (DLM) has been introduced to study the transmission efficiency of a concentrator integrated over all directions at input. The figure shows the scheme of DLM applied to a 3D-CPC concentrator, with L_{dir} constant radiance of an isotropic diffuse light source. From DLM we obtain the *direct lambertian transmission efficiency*, η_{dir}^{lamb}, defined as the ratio of output to input flux.



Basic scheme of the Direct Lambertian Method (DLM)

$$\Phi_{dir}^{in}=L_{dir}\cdot A_{in}\cdot\int_0^{2\pi}d\varphi\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta=\pi\cdot L_{dir}\cdot A_{in}$$

Incident flux

$$\Phi_{dir}^{out}=\Phi_{dir}^{\tau}=2\pi\cdot L_{dir}\cdot A_{in}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)$$

Transmitted flux

$$\eta_{dir}^{lamb}=\tau_{dir}^{lamb}=\frac{\Phi_{dir}^{\tau}}{\Phi_{dir}^{in}}=2\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)=\ldots$$

Direct lambertian transmission efficiency
$$\ldots=2\cdot\eta_{dir}(0)\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir,norm}(\theta)$$

Similar expressions are used to define the *direct lambertian absorptance*, α_{dir}^{lamb}, and the *direct lambertian reflectance*, ρ_{dir}^{lamb}. The output radiance, in general, is not constant like the input radiance, so we speak about an average output radiance:

$$\overline{L}_{dir}^{out}=\frac{\Phi_{dir}^{\tau}}{\pi\cdot A_{out}}=\frac{2\cdot L_{dir}\cdot A_{in}}{A_{out}}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)=2\cdot L_{dir}\cdot C_{geo}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)$$

Now we define a new quantity, C_{opt}^{lamb}, the ratio between average output and input radiance, as the *direct lambertian concentration ratio*:

$$C_{opt}^{lamb}=\frac{\overline{L}_{dir}^{out}}{L_{dir}}=2\cdot C_{geo}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)=2\cdot C_{geo}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\left[1-\alpha_{dir}(\theta)-\rho_{dir}(\theta)\right]$$

We find:
$$C_{opt}^{lamb}=\frac{\overline{L}_{dir}^{out}}{L_{dir}}=\tau_{dir}^{lamb}\cdot C_{geo}=\frac{\Phi_{dir}^{\tau}}{\Phi_{dir}^{in}}\cdot\frac{A_{in}}{A_{out}}=\frac{\overline{E}_{out}\cdot A_{out}}{E_{in}\cdot A_{in}}\cdot\frac{A_{in}}{A_{out}}=\frac{\overline{E}_{out}}{E_{in}}$$

The direct lambertian model can be applied also reducing the angular extension of the lambertian source from π/2 to a limit polar angle θ_m. We have for the input and output flux, respectively:

$$\Phi_{dir,\theta m}^{in}=\pi\cdot L_{dir}\cdot A_{in}\cdot\sin^2\theta_m\qquad \Phi_{dir,\theta m}^{out}=\Phi_{dir,\theta m}^{\tau}=2\pi\cdot L_{dir}\cdot A_{in}\cdot\int_0^{\theta m}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)$$

Direct transmission efficiency:
$$\tau_{dir,\theta m}^{lamb}=\frac{\Phi_{dir,\theta m}^{\tau}}{\Phi_{dir,\theta m}^{in}}=\frac{2}{\sin^2\theta_m}\cdot\int_0^{\theta m}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)$$

Average output radiance:
$$\overline{L}_{dir,\theta m}^{out}=\frac{\Phi_{dir,\theta m}^{\tau}}{\pi\cdot A_{out}}=2\cdot L_{dir}\cdot C_{geo}\cdot\int_0^{\theta m}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)$$

Optical concentration ratio:
$$C_{opt,\theta m}^{lamb}=\frac{\overline{L}_{dir,\theta m}^{out}}{L_{dir}}=2\cdot C_{geo}\cdot\int_0^{\theta m}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)$$

CONCLUSIONS

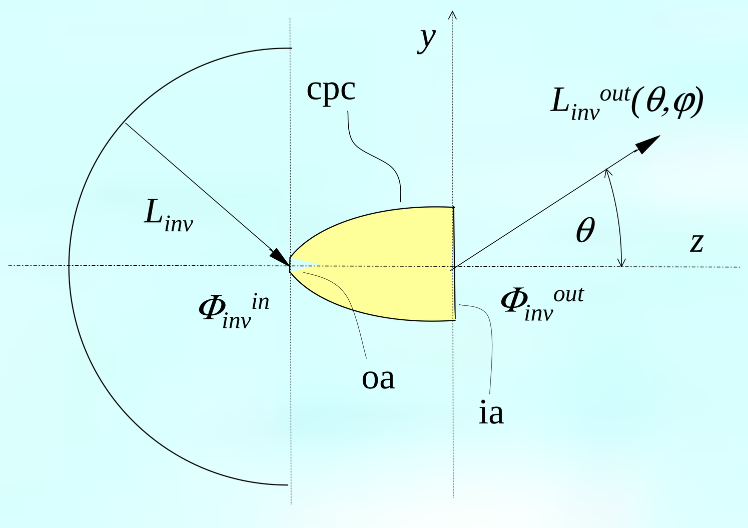
We have presented a general theoretical approach to the study of a solar concentrator looked at as a generic optical element. Irrespective of its practical use, we have considered different types of irradiation and, for each of them, reflection, absorption and transmission properties have been defined. The classical view of the concentrator fully irradiated on the front side by a collimated beam has been upset and a new way of looking to it has been introduced through the new concept of "inverse" irradiation. By inverting the irradiation on the concentrator and using a lambertian source at output, in fact, new and surprising results appear, which allow to disclose the full direct optical transmission properties of the solar concentrator by a very simple approach.

THE "INVERSE LAMBERTIAN METHOD" (ILM)

For the reversibility principle, the optical loss reported by a direct ray is the same as that shown by an inverse ray if the optical path is the same and if both starting rays are unpolarized. The attenuation factor for the radiance of the direct beam incident at point A in direction (θ,φ) represents the local direct transmission efficiency $\eta_{dir}(A,\theta,\varphi)$, while the attenuation factor for the radiance of the ray emitted by the SC from point A in the reverse direction (θ,φ) represents the local inverse transmission efficiency $\eta_{inv}(A,\theta,\varphi)$.
We extend now these concepts to all points of A_{in}. If the inverse radiance L_{inv} at output aperture is constant for all directions, that is, a Lambertian source is applied to the output aperture, then the average inverse output radiance, $L_{inv}^{out}(\theta,\varphi)$, has the same angular distribution of the average inverse transmission efficiency $\eta_{inv}(\theta,\varphi)$. But the average inverse transmission efficiency must have the same angular distribution of the average direct transmission efficiency $\eta_{dir}(\theta,\varphi)$. As a consequence, the inverse radiance of the concentrator $L_{inv}^{out}(\theta,\varphi)$, when irradiated on the output aperture with a uniform and unpolarized Lambertian source, is proportional to the efficiency of the direct transmission $\eta_{dir}(\theta,\varphi)$, of an unpolarized collimated beam, that is the two corresponding normalized quantities coincide:

$$L_{inv,norm}^{out}(\theta,\varphi)=\frac{L_{inv}^{out}(\theta,\varphi)}{L_{inv}^{out}(0)}=\eta_{dir,norm}(\theta,\varphi)=\frac{\eta_{dir}(\theta,\varphi)}{\eta_{dir}(0)}$$

The above discussion establishes therefore the suitability of the inverse lambertian method (ILM) to provide all information concerning the normalized efficiency of transmission of the concentrator under direct and collimated irradiation $\eta_{dir,norm}(\theta,\varphi)$. To perform the measurements of normalized inverse radiance, it is sufficient to project the inverse light towards a far planar screen and to record the image produced there; a simple elaboration of the image gives $L_{inv,norm}^{out}(\theta,\varphi)$.



Basic scheme of the Inverse Lambertian Method (ILM)

ILM provides also the quantity $\eta_{dir}(0)$, and so the "absolute" transmission efficiency $\eta_{dir}(\theta,\varphi)$, without recourse to any direct measure by DCM, as it is demonstrated by the forthcoming considerations.
When a 3D-CPC SC is inversely irradiated, the exit aperture (oa) becomes a Lambertian source with constant and uniform radiance L_{inv}. The total flux injected into the SC becomes:

$$\Phi_{inv}^{in}=2\pi\cdot L_{inv}\cdot A_{out}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta=\pi\cdot L_{inv}\cdot A_{out}$$

The inverse flux transmitted to output, the input aperture of the SC, is given by:

$$\Phi_{inv}^{out}=\Phi_{inv}^{\tau}=2\pi\cdot A_{in}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot L_{inv}^{out}(\theta)$$

We now define a new optical quantity, the *inverse lambertian transmittance*, τ_{inv}^{lamb}, as the ratio of output to input flux:

$$\tau_{inv}^{lamb}=\frac{\Phi_{inv}^{out}}{\Phi_{inv}^{in}}=\frac{2\pi\cdot A_{in}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot L_{inv}^{out}(\theta)}{\pi\cdot A_{out}\cdot L_{inv}}=\ldots$$

$$\ldots=\frac{2\cdot C_{geo}}{L_{inv}}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot L_{inv}^{out}(\theta)=\frac{2\cdot C_{geo}}{L_{inv}}\cdot L_{inv}^{out}(0)\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot L_{inv,norm}^{out}(\theta)$$

We obtain for the ratio between inverse and direct lambertian transmittances:

$$\frac{\tau_{inv}^{lamb}}{\tau_{dir}^{lamb}}=\frac{\frac{2\cdot C_{geo}}{L_{inv}}\cdot L_{inv}^{out}(0)\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot L_{inv,norm}^{out}(\theta)}{2\cdot\eta_{dir}(0)\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir,norm}(\theta)}=\ldots=C_{geo}\cdot\frac{L_{inv}^{out}(0)}{\eta_{dir}(0)\cdot L_{inv}}$$

This ratio is just a property of the SC and cannot depend on radiance. If we apply the condition $L_{dir}=L_{inv}$, we obtain that the total lambertian flux transmitted in "direct" and "inverse" directions is the same:

$$2\pi\cdot L_{dir}\cdot A_{in}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir}(\theta)=2\pi\cdot A_{in}\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot L_{inv}^{out}(\theta)$$

$$L_{inv}\cdot\eta_{dir}(0)\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot\eta_{dir,norm}(\theta)=L_{inv}^{out}(0)\cdot\int_0^{\pi/2}d\theta\cdot\sin\theta\cdot\cos\theta\cdot L_{inv,norm}^{out}(\theta)$$

$$\Rightarrow\quad \eta_{dir}(0)=\frac{L_{inv}^{out}(0)}{L_{inv}}$$

The last equation allows us to calculate $\eta_{dir}(0)$ by ILM measuring $L_{inv}^{out}(0)$, the average on-axis inverse radiance, and L_{inv}, the radiance of the inverse lambertian source. We obtain that the ratio between inverse and direct lambertian transmittance is independent on radiance, as foreseen, and equal to C_{geo}:

$$\Rightarrow\quad \frac{\tau_{inv}^{lamb}}{\tau_{dir}^{lamb}}=C_{geo}$$

The *"input direct lambertian flux"* needed to sustain an equal transmitted flux in the opposite direction is C_{geo} times the *"input inverse lambertian flux"*. This is a direct consequence of the geometrical asymmetry of the concentrator. Let us imagine now to irradiate both apertures of the SC by two different lambertian sources with $L_{dir}\neq L_{inv}$. If $\Delta L=L_{dir}-L_{inv}$ is the difference of incidence radiance between input and output, then we have for the net flux through SC, in the direct direction:

$$\Delta\Phi=\Phi_{dir}^{net}=\Phi_{dir}^{out}-\Phi_{inv}^{out}=\tau_{dir}^{lamb}\cdot\Phi_{dir}^{in}-\tau_{inv}^{lamb}\cdot\Phi_{inv}^{in}=\tau_{dir}^{lamb}\cdot\left[\Phi_{dir}^{in}-C_{geo}\cdot\Phi_{inv}^{in}\right]=\ldots$$

$$\ldots=\tau_{dir}^{lamb}\cdot\left[\pi\cdot L_{dir}\cdot A_{in}-C_{geo}\cdot\pi\cdot L_{inv}\cdot A_{out}\right]=(\pi\cdot A_{in}\cdot\tau_{dir}^{lamb})\cdot\Delta L$$

The equation for ΔΦ is similar to the **Ohm's law: I = G · ΔV**, where Φ_{dir}^{net} (W) has the role of current, ΔL (W/sr·m²) the role of potential difference and $(\pi\cdot A_{in}\cdot\tau_{dir}^{lamb})$ (sr·m²) the role of conductance: By inverting the sign of ΔL we obtain:

$$\Delta\Phi=\Phi_{inv}^{net}=(\pi\cdot A_{out}\cdot\tau_{inv}^{lamb})\cdot\Delta L$$

$$\Delta\Phi=\Phi_{dir}^{net}=(\pi\cdot A_{in}\cdot\tau_{dir}^{lamb})\cdot\Delta L\quad(\Delta L=L_{dir}-L_{inv})\qquad \Delta\Phi=\Phi_{inv}^{net}=(\pi\cdot A_{out}\cdot\tau_{inv}^{lamb})\cdot\Delta L\quad(\Delta L=L_{inv}-L_{dir})$$

We define the ***"direct lambertian conductance"***: $G_{dir}^{lamb}=\pi\cdot A_{in}\cdot\tau_{dir}^{lamb}$

We define the ***"inverse lambertian conductance"***: $G_{inv}^{lamb}=\pi\cdot A_{out}\cdot\tau_{inv}^{lamb}$

$$\Rightarrow\quad G_{dir}^{lamb}=G_{inv}^{lamb}$$

The optical asymmetry of the SC disappears as long as the conductance of the SC is considered. The *"lambertian optical conductance"* can be put in the form:

$$G^{lamb}=(\pi\cdot A)\cdot\tau^{lamb}$$

"optical conductance" = "étendue" x "transmittance"

Finally we define the *density of the net flux through the input aperture in direct way* and the *density of net flux through the output aperture in inverse way*:

$$J_{dir}^{net}=\frac{\Phi_{dir}^{net}}{A_{in}}=(\pi\cdot\tau_{dir}^{lamb})\cdot\Delta L\qquad J_{inv}^{net}=\frac{\Phi_{inv}^{net}}{A_{out}}=(\pi\cdot\tau_{inv}^{lamb})\cdot\Delta L$$

[1] A. Parretta, Int. Journal of Optics and Applications, **3**(4), 2013, 27-39.