OPTICAL MODELS OF LIGHT COLLECTION IN SOLAR CONCENTRATORS

Antonio Parretta[1*] and Mario Tucci[2**]
[1]ENEA Research Centre "E. Clementel", Via Martiri di Monte Sole 4, 40129 Bologna (BO), Italy
[2]ENEA Research Centre "Casaccia", Via Anguillarese 301, 00123 S. Maria di Galeria (Rome), Italy
*antonio.parretta@enea.it; **mario.tucci@enea.it

ABSTRACT: Different models of light irradiation and collection in solar concentrators are here reported, together with a theoretical analysis that leads to new optical concepts and new optical quantities. For some of them a similarity with electrical quantities is suggested. In this paper we present a general theoretical approach to the study of solar concentrators, which are seen as generic optical components with reflectance, absorbance and transmittance properties expressed with respect to different irradiation conditions. In particular, they are studied under collimated or diffuse light, under local or integral irradiation, including the irradiation in which the direction of light is reversed, i.e. oriented from the exit towards the entrance aperture. The classical view of the concentrator fully irradiated on the front side by a collimated beam has been in this way upset, and a new way of looking to it has been introduced through the concept of "inverse" irradiation. By inverting the direction of irradiation and by using a Lambertian distribution of light at output, new and surprising results appear, which allow to reveal, among other things, the transmission properties of the solar concentrator under a direct and collimated beam. All the results have been obtained by applying two basic optical concepts: the reversibility principle and the transmission efficiency of an elemental beam. This theoretical investigation on the solar concentrators is aimed at improving the knowledge of their optical properties, expanding their application field and opening new perspectives to the methods of their characterization.
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1 INTRODUCTION

Terrestrial, photovoltaic (PV) solar concentrators (SC) are optical devices designed to efficiently collect the direct solar radiation, incident at a flux density of about 800 W/m^2 (at clear sky conditions) and to transfer it to a receiver (the solar cell or module) at a flux density increased by a factor equal to the optical concentration ratio, C_{opt} , chosen according to the end use of solar energy and the specific technology used. The level of optical concentration can vary from only a few units, as for example in fixed concentrators applied to buildings, up to about 500x for the most sophisticated photovoltaic concentrators [1-8].

The class of SCs that has shown the most interesting results in terms of optical efficiency and maximum attainable concentration levels is that based on the "nonimaging optics" [9-11].

Solar concentrators are generally studied simulating or testing their behaviour under a collimated or quasi-collimated beam, which simulates the direct component of solar radiation (half angular divergence θ_{dir} ~ 0.27°), as these are the real operating conditions of an SC [12-15]. Less frequently SCs are studied under diffuse radiation, because the natural diffuse light is associated to a low radiance, and the collection of diffuse light requires a high angular aperture of the SC, more precisely a high acceptance angle θ_{acc}, which heavily influences the maximum level of concentration. We have in fact for the optical concentration ratio of a generic 3D solar concentrator:

$$C_{opt} = n_{out}^2 \frac{\sin^2 \theta_{out}}{\sin^2 \theta_{acc}} \tag{1}$$

where θ_{out} is the maximum angular divergence of output rays and n_{out} is the index of refraction of the medium embedding the receiver. When the SC is irradiated by the direct component of the sun, to collect all the direct irradiation it is necessary to have $\theta_{acc} \geq \theta_{dir}$. The maximum optical concentration ratio is reached therefore when the divergence of rays at output is θ_{out} = 90° and $\theta_{acc} = \theta_{dir}$:

$$C_{opt}^{\max} = n_{out}^2 \frac{1}{\sin^2 \theta_{dir}} \approx n_{out}^2 \cdot 46.000 \tag{2}$$

By working with diffuse light, the maximum value of C_{opt} will be given by:

$$C_{opt}^{\max} = n_{out}^2 \frac{1}{\sin^2 \theta_{diff}} \tag{3}$$

where θ_{diff} is the angular aperture of the diffuse light source. If $\theta_{diff} = 90°$, an entire hemisphere, the maximum achievable value of C_{opt} is only n_{out}^2.

The study of the optical properties of an SC are here not limited to the conditions of irradiation in practical applications; there are in fact situations in which it is useful to know its optical properties under any type of irradiation, particularly when it is subjected to an optical characterization (simulated or experimental). We have, for example, developed new methods of characterization, which have been applied inverting the usual direction of transmission of light, introducing in this way the so-called "inverse" methods of characterization [16-24]. We have demonstrated that the analysis of the output radiance of the SC when it is irradiated in a suitable way under diffuse and "reverse" light, allows to derive, in a very fast and easy way, its optical transmission properties under collimated and "direct" light, those which are directly related to its practical use.

In this paper, therefore, we go beyond the classical view of an SC irradiated by a "direct" and collimated beam, by proposing a new scenario in which the solar concentrator, regardless of its type (if 2-D or 3-D), if refractive or reflective, if imaging or nonimaging, is studied as a generic optical element, characterized by specific reflection, absorption or transmission properties, defined respect to specific models of irradiation. We distinguish, for example, between “direct” and “inverse” irradiation depending on the direction of the incoming light, or between “local” and “integral” irradiation depending if the irradiation is limited to a small area or to the whole area of entry, respectively; we finally distinguish between a quasi-collimated irradiation by a far light source, in contrast to a “diffuse” irradiation by a lambertian source. In the last case we speak of “lambertian” irradiation, understood as irradiation with constant radiance from all directions within a maximum value of solid angle.

In this paper we analyse theoretical models of irradiation, as simplifications of the real irradiation conditions found outdoors. To each model we associate a specific method of characterization, which can be applied by optical simulations or by experimental measurements. The definition of each model of irradiation is the same of the corresponding characterization method.

2 THEORETICAL MODELS OF IRRADIATION

In what follows the generic SC is schematised as a device confined between an entrance aperture (ia) with area A_{in} and an exit aperture (oa) with area A_{out}, where $A_{in}>A_{out}$. A solar concentrator operates in practice under “direct” irradiation, i.e. under irradiation on the entrance aperture and with a receiver, the energy conversion device, at the exit aperture. In our models, however, we replace the receiver by any detector suitable to measure the desired optical quantity, i.e. the total output flux, or its spatial and angular distribution (these last quantities are of relevant importance in photovoltaic solar concentrators [25,26]); we also use the exit aperture to put there any source of light for inverse irradiation. The same considerations apply when considering the input aperture of the SC; we measure the total flux exiting from it in “reverse” direction, or its spatial and angular distribution, and we also use the entrance aperture to put there any source of light for “direct” irradiation. What is there between the two apertures is specific of the particular fabrication technology used and will not be considered here, because not relevant, in principle, for a discussion on its overall optical properties.

Most of the optical properties which will be disclosed by our methods are based on two fundamental concepts: i) the transmission efficiency of an elemental, unpolarized beam impinging on the input or output aperture at point $P(x, y)$ from direction (θ, φ); ii) the principle of reversibility, applied in absence of diffusion or diffraction phenomena inside the SC, which establishes the same attenuation T_{AB} for an elemental and unpolarized beam crossing the SC from point A at input to point B at output in “direct” direction, and T_{BA} for an elemental and unpolarized beam crossing the SC from point B at output to point A at input in “reverse” direction [27].

The first and simplest irradiation method is the *“Direct Local Collimated Method”* (DLCM) [17,18], based on the transmission of an elemental beam from input to output with efficiency $\eta_{dir}(P,\theta,\varphi)$, the local optical transmission efficiency, where P is a point on the input aperture.

If the irradiation of the SC by a collimated beam is extended to the entire area of input aperture, we have the *“Direct Collimated Method”* (DCM) [19]. The application of DCM gives the curve of optical transmission efficiency, $\eta_{dir}(\theta,\varphi)$, obtained changing the polar angle of the collimated beam (see Fig. 1). The $\eta_{dir}(\theta,\varphi)$ curve is characterized by the acceptance angle $\theta_{acc}=\theta_{acc}^{50}$, corresponding to the 50% of the efficiency measured at 0°.

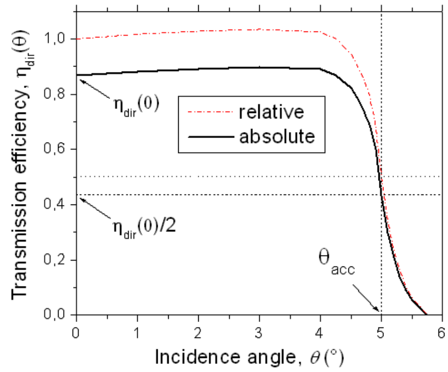

Figure 1: Typical optical transmission curve of a nonimaging solar concentrator.

The condition $T_{AB} = T_{BA}$ is the basis of the *“Inverse Lambertian Method”* (ILM), which has been conceived for deriving the absolute transmission efficiency of DCM by analysing, instead of the flux collected at the receiver (the output aperture) under “direct” irradiation, the flux emitted by the input aperture under “inverse” irradiation [20-24]. To apply this concept, the rays analysed with the “direct” irradiation must overlap those analysed with the “inverse” irradiation, that is, the respective optical paths must be identical. This is valid when the reversibility principle can be applied.

The source of the inverse rays is placed in correspondence of the receiver (the output aperture) in order to emit rays, from each point and in any direction inside the SC, at constant radiance, without discriminating any direction. In this way it is possible to produce, in the inverse mode, all the connecting paths which will overlap with those that can be produced in direct mode by a collimated beam inclined at different polar angles. In order to apply ILM in a correct way, therefore, it is needed to put a spatially uniform, lambertian and unpolarized light source at the output aperture, with L_{inv} the constant radiance. As we will see in Section 3.3, from the inverse radiance $L_{inv}^{out}(\theta,\varphi)$ we will be able to derive the collection efficiency under direct irradiation $\eta_{dir}(\theta,\varphi)$.

The irradiation of the SC at the entrance aperture by a lambertian source introduces the *“Direct Lambertian Method”* (DLM) [17-19,24].

By irradiating the concentrator simultaneously in DLM and ILM modes, all the connecting paths will overlap and, if $L_{dir}=L_{inv}$, also the elementary flux flowing through any connecting path will be the same along the two directions. Then, with $L_{dir}=L_{inv}$, also the total flux flowing through the concentrator from one aperture to the other will be the same in the two directions.

The "inverse" method applied "locally" to small regions with area ΔA_{out} and centred on point P of the exit opening is called *"Inverse Local Lambertian Method"* (ILLM) [22]. The measured inverse radiance $L_{inv}^{out}(P, \Delta A_{out}, \theta, \varphi)$ directly gives the corresponding collection efficiency under direct irradiation $\eta_{dir}(P, \Delta A_{out}, \theta, \varphi)$. The new situation is similar to that which would occur if the concentrator could be amended as follows: the new receiver is the selected area of the old receiver; the new concentrator is the old concentrator plus the excluded part of the receiver. This new way of looking at the receiver is very powerful. In this way, in fact, we can study the efficiency of collection of any portion of the optical receiver, and since the radiation on the receiver is generally not uniform when the concentrator is directly irradiated by a collimated beam, it happens often to be wonder about the direction of the input rays arriving in a certain area of the receiver. By means of the ILLM method, therefore, we can know from which direction the rays in excess in a certain area of the receiver arrive, or from which direction they are failing to arrive in a certain area of it.

In the following Section, the irradiation models briefly outlined so far will be investigated in detail in terms of transmission, reflection and absorption efficiencies.

3 THEORETICAL MODELS OF LIGHT COLLECTION

3.1 Theory of the "direct collimated methods"

In DLCM an elementary beam, incident on the point A of input aperture (ia) and flowing inside the SC in the direct mode, is transmitted to the output with an efficiency $\eta_{dir}(A, \theta, \varphi) \le 1$. By averaging $\eta_{dir}(P, \theta, \varphi)$ over a uniform distribution of points P on the input aperture, the transmission efficiency $\eta_{dir}(\theta, \varphi)$ at collimated light is defined. This corresponds to DCM, simulating the behaviour of the SC under the direct solar irradiation. The "absolute" transmission efficiency $\eta_{dir}(\theta, \varphi)$, expressed as function of polar and azimuthal angles of direction of the collimated beam, characterized by the constant irradiance E_{dir} on the wave front, is defined as:

$$\eta_{dir}(\theta,\varphi) = \frac{\Phi_{out}(\theta,\varphi)}{\Phi_{in}(\theta,\varphi)} = \frac{\Phi_{out}(\theta,\varphi)}{E_{dir} \cdot A_{in}(\theta,\varphi)} = \ldots$$
$$\ldots = \frac{\Phi_{out}(\theta,\varphi)}{E_{dir} \cdot A_{in} \cdot \cos(\theta)} \qquad (4)$$

where $A_{in}(\theta, \varphi)$ is the area of input aperture projected along direction (θ, φ). The last term in Eq. (4) applies when the contour of input aperture is contained on a plane surface.

The "absolute" transmission efficiency $\eta_{dir}(\theta, \varphi)$ can be expressed also as:

$$\eta_{dir}(\theta,\varphi) = \eta_{dir}(0) \cdot \eta_{dir,norm}(\theta,\varphi) \qquad (5)$$

where $\eta_{dir,norm}(\theta, \varphi)$ is the "normalized" transmission efficiency and $\eta_{dir}(0)$ is the transmission efficiency at 0° (see Fig. 1). It is clear that $\eta_{dir}(\theta, \varphi)$ is the average value of $\eta_{dir}(P, \theta, \varphi)$ when $\eta_{dir}(P, \theta, \varphi)$ is calculated for all the points of the entrance aperture. We have therefore for the output flux:

$$\Phi_{out}(\theta,\varphi) = \int_{Ain} dS \cdot E_{dir} \cdot \cos\theta \cdot \eta_{dir}(P,\theta,\varphi) = \ldots$$
$$= E_{dir} \cdot \cos\theta \cdot \int_{Ain} dS \cdot \eta_{dir}(P,\theta,\varphi) = \ldots \qquad (6)$$
$$= E_{dir} \cdot \cos\theta \cdot A_{in} \cdot \overline{\eta}_{dir}(P,\theta,\varphi)$$

and for the transmission efficiency:

$$\eta_{dir}(\theta,\varphi) = \frac{E_{dir} \cdot \cos\theta \cdot \int_{Ain} dS \cdot \eta_{dir}(P,\theta,\varphi)}{E_{dir} \cdot A_{in} \cdot \cos(\theta)} = \ldots \qquad (7)$$
$$= \frac{\int_{Ain} dS \cdot \eta_{dir}(P,\theta,\varphi)}{A_{in}} = \overline{\eta}_{dir}(P,\theta,\varphi)$$

To explore the light collection properties of the SC, the collimated beam must be oriented respect to the optical (z) axis of concentrator varying θ in the 0°-90° interval and φ in the 0°-360° interval. If the SC has cylindrical (rotational) symmetry, it is sufficient to fix a φ value and to vary only θ. If the SC, however, is an optical unit of a larger system, or module, it will have probably a squared or hexagonal aperture, as these geometries allow to pack them efficiently in the concentrating module, then the φ angle can be limited, in these cases, to the 0°-90° or the 0°-60° interval, respectively. Despite this limitation, however, the number of measurements required for the application of DCM is high for this type of SCs, both for simulations and for experimental measurements. This is indeed the very strong limit of DCM applied to the determination of $\eta_{dir}(\theta, \varphi)$. This limit is overcome by the application of the *"Inverse Lambertian Method"* (ILM), as will be demonstrated in Section 3.3.

Dealing with "nonimaging" SCs [9-11], whose transmission curve has a step-like profile (see Fig. 1), their characterization by DCM can be simplified: it is sufficient to vary the input angle θ from 0° to a little more than the acceptance angle at 50% of 0° efficiency, θ_{acc}^{50}. The rays incident at $\theta > \theta_{acc}^{50}$, in fact, will be rejected back by the SC before reaching the output aperture.

The quantity $\eta_{dir}(\theta, \varphi)$ (see Fig. 1) represents the fraction of flux transferred to the output, and then it represents the *"direct collimated transmittance"* of the SC, when viewed as a generic optical element. We can speak, equivalently, of a *"direct collimated reflectance"* $\rho_{dir}(\theta, \varphi)$ and of a *"direct collimated absorptance"* $\alpha_{dir}(\theta, \varphi)$ of the SC for the fraction of the input flux back reflected or absorbed, respectively:

$$\rho_{dir}(\theta,\varphi) = \frac{\Phi_{\rho}(\theta,\varphi)}{\Phi_{in}(\theta,\varphi)} \qquad (8)$$

$$\alpha_{dir}(\theta,\varphi) = \frac{\Phi_{\alpha}(\theta,\varphi)}{\Phi_{in}(\theta,\varphi)} \qquad (9)$$

We have for the conservation of energy:

$$\eta_{dir}(\theta,\varphi) + \rho_{dir}(\theta,\varphi) + \alpha_{dir}(\theta,\varphi) = 1 \qquad (10)$$

The typical $\eta_{dir}(\theta)$ curve for a 3-D CPC (Compound Parabolic Concentrator) is illustrated in Fig.

1 [9-11]; the φ angle is not represented, because the CPC has a cylindrical symmetry. We distinguish the 0° efficiency $\eta_{dir}(0)$, the acceptance angle at 50% of 0° efficiency θ_{acc}^{50} and the normalized transmission curve $\eta_{dir,norm}(\theta,\varphi)$, characterized by the same θ_{acc}^{50} angle. Respect to the CPCs, the "imaging" solar concentrators show a very different transmission curve, with a long tail and a short flat portion at small angles [10-11]. For these concentrators the DCM has to be applied by varying the polar angles from 0° to a limit θ angle, θ_m, well higher than θ_{acc}^{50}.

3.2 Theory of the "direct lambertian methods"

DLM [17-19,24] simulates the behaviour of the concentrator under diffused light, for example the diffuse solar radiation in a totally covered sky. If L_{dir} is the constant radiance of the diffused source, the total incident flux is:

$$\Phi_{dir}^{in} = L_{dir} \cdot A_{in} \cdot \int_0^{2\pi} d\phi \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta = \pi \cdot A_{in} \cdot L_{dir} \quad (11)$$

where $\pi \cdot A_{in}$ is the étendue.

In the following, we will consider, for simplicity, only concentrators with cylindrical symmetry; we will skip therefore the dependence on angle φ. The flux "transmitted" to the output aperture becomes:

$$\Phi_{dir}^{\tau} = 2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) \quad (12)$$

The "reflected" flux Φ_{dir}^{ρ} and the "absorbed" flux Φ_{dir}^{α} are expressed respectively as:

$$\Phi_{dir}^{\rho} = 2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \rho_{dir}(\theta) \quad (13)$$

$$\Phi_{dir}^{\alpha} = 2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \alpha_{dir}(\theta) \quad (14)$$

in such a way that:

$$\Phi_{dir}^{\tau} + \Phi_{dir}^{\rho} + \Phi_{dir}^{\alpha} = \Phi_{dir}^{in} \quad (15)$$

We define now the *"direct lambertian transmittance"* τ_{dir}^{lamb}, as the ratio of output to input flux:

$$\tau_{dir}^{lamb} = \frac{\Phi_{dir}^{\tau}}{\Phi_{dir}^{in}} = 2 \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) = \ldots \quad (16)$$

$$= 2 \cdot \eta_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir,norm}(\theta)$$

We define, similarly, the *"direct lambertian reflectance"* ρ_{dir}^{lamb} and the *"direct lambertian absorptance"* α_{dir}^{lamb}:

$$\rho_{dir}^{lamb} = \frac{\Phi_{dir}^{\rho}}{\Phi_{dir}^{in}} = 2 \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \rho_{dir}(\theta) = \ldots$$

$$\ldots = 2 \cdot \rho_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \rho_{dir,norm}(\theta) \quad (17)$$

$$\alpha_{dir}^{lamb} = \frac{\Phi_{dir}^{\alpha}}{\Phi_{dir}^{in}} = 2 \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \alpha_{dir}(\theta) = \ldots$$

$$\ldots = 2 \cdot \alpha_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \alpha_{dir,norm}(\theta) \quad (18)$$

where $\rho_{dir,norm}(\theta,\varphi)$ and $\alpha_{dir,norm}(\theta,\varphi)$ are the normalized efficiencies, whereas $\rho_{dir}(0)$ and $\alpha_{dir}(0)$ are the efficiencies at 0° incidence. We have:

$$\tau_{dir}^{lamb} + \rho_{dir}^{lamb} + \alpha_{dir}^{lamb} = 1 \quad (19)$$

In contrast to the incoming radiance, the output radiance is not constant, so we speak about the average output radiance:

$$\overline{L}_{dir}^{out} = \frac{\Phi_{dir}^{\tau}}{\pi \cdot A_{out}} = \frac{2 \cdot L_{dir} \cdot A_{in}}{A_{out}} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) =$$

$$= 2 \cdot L_{dir} \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) \quad (20)$$

where C_{geo} is the geometrical concentration ratio. We define the new quantity, C_{opt}^{lamb}, ratio between output and input radiance:

$$C_{opt}^{lamb} = \frac{\overline{L}_{dir}^{out}}{L_{dir}} = 2 \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) =$$

$$2 \cdot C_{geo} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \left[1 - \alpha_{dir}(\theta) - \rho_{dir}(\theta)\right] \quad (21)$$

From Eq. (16), (21) we find:

$$C_{opt}^{lamb} = \frac{\overline{L}_{dir}^{out}}{L_{dir}} = \tau_{dir}^{lamb} \cdot C_{geo} = \frac{\Phi_{dir}^{\tau}}{\Phi_{dir}^{in}} \cdot \frac{A_{in}}{A_{out}} = \ldots$$

$$\ldots = \frac{\overline{E}_{out} \cdot A_{out}}{E_{in} \cdot A_{in}} \cdot \frac{A_{in}}{A_{out}} = \frac{\overline{E}_{out}}{E_{in}} \quad (22)$$

Eq. (22) is similar to the relationship that defines the optical concentration ratio of an SC under collimated irradiation [9-11]:

$$C_{opt}^{coll} = \frac{\overline{E}_{out}}{E_{in}} = \eta_{dir} \cdot C_{geo} = \frac{\Phi_{out}}{\Phi_{in}} \cdot \frac{A_{in}}{A_{out}} \quad (23)$$

We define therefore the quantity C_{opt}^{lamb} as the *"direct lambertian concentration ratio"*.

The direct lambertian model can be applied also reducing the angular extension of the lambertian source from $\pi/2$ to a limit polar angle θ_m. The corresponding method, DLM(θ_m), is particularly useful when we analyse the behaviour of nonimaging SCs. Because of the step-like profile of their optical efficiency ($\eta_{dir}(\theta) \approx 0$ for $\theta \geq \theta_{acc}^{50}$), in fact (see Fig. 1), the characterization of these SCs under direct lambertian irradiation can be limited to angles $\theta \leq \theta_m \approx \theta_{acc}^{50}$, reducing in this way the time of computer elaboration or simplifying the experimental measurements. We have for the input and output flux:

$$\Phi^{in}_{dir,\theta m} = L_{dir} \cdot A_{in} \cdot \int_0^{2\pi} d\phi \cdot \int_0^{\theta m} d\theta \cdot \sin\theta \cdot \cos\theta = ...$$

$$... = \pi \cdot L_{dir} \cdot A_{in} \cdot \sin^2 \theta_m \qquad (24)$$

$$\Phi^{\tau}_{dir,\theta m} = 2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\theta m} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta)$$

(25)

and for the direct transmission efficiency:

$$\tau^{lamb}_{dir,\theta m} = \frac{\Phi^{\tau}_{dir,\theta m}}{\Phi^{in}_{dir,\theta m}} = \frac{2}{\sin^2 \theta_m} \cdot \int_0^{\theta m} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta)$$

(26)

The average output radiance becomes:

$$\overline{L}^{out}_{dir,\theta m} = \frac{\Phi^{\tau}_{dir,\theta m}}{\pi \cdot A_{out}} = 2 \cdot L_{dir} \cdot C_{geo} \cdot \int_0^{\theta m} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta)$$

(27)

and the optical concentration ratio $C^{lamb}_{opt,\theta m}$ becomes:

$$C^{lamb}_{opt,\theta m} = \frac{\overline{L}^{out}_{dir,\theta m}}{L_{dir}} = 2 \cdot C_{geo} \cdot \int_0^{\theta m} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta)$$

(28)

3.3 Theory of the “inverse lambertian methods”

We have seen that, for the reversibility principle, the optical loss reported by a direct ray is the same as that shown by an inverse ray if the optical path is the same and if both starting rays are unpolarized. The attenuation factor for the radiance of the direct beam incident at point *A* in direction (θ,φ) represents the local direct transmission efficiency $\eta_{dir}(A,\theta,\varphi)$, while the attenuation factor for the radiance of the ray emitted by the SC from point *A* in the reverse direction (θ,φ) represents the local inverse transmission efficiency $\eta_{inv}(A,\theta,\varphi)$. We extend now these concepts to all points of A_{in} directly irradiated in direction (θ,φ) (DCM) and to the same points of A_{in} that emit light in the reverse direction (θ,φ) (ILM). If the inverse radiance at output aperture L_{inv} is constant for all directions, that is, a Lambertian source is applied at the output aperture, then the inverse output radiance, $L^{out}_{inv}(\theta,\varphi)$, averaged over all points of A_{in}, has the same angular distribution of the inverse transmission efficiency $\eta_{inv}(\theta,\varphi)$, averaged over all points of A_{in}. But the average inverse transmission efficiency $\eta_{inv}(\theta,\varphi)$ must have the same angular distribution of the average direct transmission efficiency $\eta_{dir}(\theta,\varphi)$, because the transmission of the single connecting paths is invariant respect to the direction of travel of light. As a consequence, we deduce that the inverse radiance of the concentrator $L^{out}_{inv}(\theta,\varphi)$, irradiated on the output aperture with a uniform and unpolarized Lambertian source, is proportional to the efficiency of the direct transmission $\eta_{dir}(\theta,\varphi)$ of an unpolarized collimated beam, that is the two corresponding normalized quantities coincide. We have therefore:

$$L^{out}_{inv,norm}(\theta,\varphi) = \eta_{dir,norm}(\theta,\varphi) \qquad (29)$$

where:

$$L^{out}_{inv,norm}(\theta,\varphi) = \frac{L^{out}_{inv}(\theta,\varphi)}{L^{out}_{inv}(0)} \qquad (30)$$

and $\eta_{dir,norm}(\theta,\varphi)$ is given by Eq. (5).

Eq. (29) establishes the equivalence between the “normalized” inverse radiance and the “normalized” direct transmittance. The above discussion establishes therefore the suitability of ILM in providing all information concerning the normalized efficiency of transmission of the concentrator under direct and collimated irradiation, $\eta_{dir,norm}(\theta,\varphi)$ (see Fig. 1).

The simulated and experimental measurements of the normalized inverse radiance $L^{out}_{inv,norm}(\theta,\varphi)$ are discussed elsewhere [20-24]. They can be performed projecting the inverse light of concentrator towards a far planar screen, and recording the image produced there; a simple elaboration of the image gives $L^{out}_{inv,norm}(\theta,\varphi)$, and so $\eta_{dir,norm}(\theta,\varphi)$.

Here we emphasize another fundamental aspect of ILM, that is the fact that it provides also the quantity $\eta_{dir}(0)$, and so the “absolute” transmission efficiency $\eta_{dir}(\theta,\varphi)$ (see Eq. (5)), without recourse to any direct measure by DCM [23,24], as it will be demonstrated by the forthcoming considerations.

When the SC is irradiated in the reverse way, the exit aperture (oa) of area A_{out} becomes a Lambertian source with constant and uniform radiance L_{inv}. The total flux, injected into the SC, becomes:

$$\Phi^{in}_{inv} = L_{inv} \cdot A_{out} \cdot \int_0^{2\pi} d\phi \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta = ...$$

$$... = \pi \cdot L_{inv} \cdot A_{out} \qquad (31)$$

The inverse flux transmitted to output, the input aperture (ia) of area A_{in}, is given by:

$$\Phi^{out}_{inv} = \Phi^{\tau}_{inv} = 2\pi \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L^{out}_{inv}(\theta) \qquad (32)$$

where θ is the direction and $L^{out}_{inv}(\theta)$ is the radiance of light inversely emitted. We now define a new optical quantity, the “*inverse lambertian transmittance*”, τ^{lamb}_{inv}, as the ratio of output to input flux:

$$\tau^{lamb}_{inv} = \frac{\Phi^{out}_{inv}}{\Phi^{in}_{inv}} = \frac{2\pi \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L^{out}_{inv}(\theta)}{\pi \cdot A_{out} \cdot L_{inv}} = ...$$

$$... = \frac{2 \cdot C_{geo}}{L_{inv}} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L^{out}_{inv}(\theta) = ...$$

$$... = \frac{2 \cdot C_{geo}}{L_{inv}} \cdot L^{out}_{inv}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L^{out}_{inv,norm}(\theta)$$

(33)

By comparing the inverse lambertian transmittance τ_{inv}^{lamb} of Eq. (33) with the direct lambertian transmittance τ_{dir}^{lamb} of Eq. (16), we obtain for their ratio, after having applied Eq. (29):

$$\frac{\tau_{inv}^{lamb}}{\tau_{dir}^{lamb}} = \frac{\frac{2 \cdot C_{geo}}{L_{inv}} \cdot L_{inv}^{out}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_{inv,norm}^{out}(\theta)}{2 \cdot \eta_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir,norm}(\theta)} = \ldots$$

$$\ldots = C_{geo} \cdot \frac{L_{inv}^{out}(0)}{\eta_{dir}(0) \cdot L_{inv}} \quad (34)$$

This ratio is just a property of the SC and cannot depend on radiance quantities as it appears in Eq. (34). To demonstrate this, we calculate the ratio $\tau_{inv}^{lamb} / \tau_{dir}^{lamb}$ by applying the simple condition $L_{dir} = L_{inv}$, when the total integral flux transmitted in the "direct" and the "inverse" directions does not change: $\Phi_{dir}^{out} = \Phi_{inv}^{out}$, because such is the flux transmitted through the elementary connecting paths in the two directions. By putting $\Phi_{dir}^{out} = \Phi_{inv}^{out}$ and using Eq. (12) and (32) we find:

$$2\pi \cdot L_{dir} \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir}(\theta) = \ldots$$

$$\ldots = 2\pi \cdot A_{in} \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_{inv}^{out}(\theta) \quad (35a)$$

Putting $L_{dir} = L_{inv}$ and applying Eq. (30), (5), we have:

$$L_{inv} \cdot \eta_{dir}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot \eta_{dir,norm}(\theta) = \ldots$$

$$\ldots = L_{inv}^{out}(0) \cdot \int_0^{\pi/2} d\theta \cdot \sin\theta \cdot \cos\theta \cdot L_{inv,norm}^{out}(\theta) \quad (35b)$$

We finally find:

$$\eta_{dir}(0) = \frac{L_{inv}^{out}(0)}{L_{inv}} \quad (35c)$$

Eq. (35c) allows us to calculate $\eta_{dir}(0)$ by ILM measuring the ratio between $L_{inv}^{out}(0)$, the average on-axis inverse radiance of SC, and L_{inv}, the radiance of the inverse lambertian source [23,24]. This ratio can be measured by recording a single image of the input aperture of the SC taken by a CCD camera oriented on the optical axis of the concentrator [16-19,24,28].

From Eq. (34), (35c) we find moreover that the ratio between the inverse and direct lambertian transmittances is equal to C_{geo}, and so is independent on radiance, as foreseen. This could be also deduced by considering that, when $\Phi_{dir}^{out} = \Phi_{inv}^{out}$, we have:

$$\frac{\tau_{inv}^{lamb}}{\tau_{dir}^{lamb}} = \frac{\Phi_{inv}^{out}}{\Phi_{inv}^{in}} \cdot \frac{\Phi_{dir}^{in}}{\Phi_{dir}^{out}} = \frac{\Phi_{dir}^{in}}{\Phi_{inv}^{in}} = \frac{\pi \cdot L_{dir} \cdot A_{in}}{\pi \cdot L_{inv} \cdot A_{out}} = \ldots$$

$$\ldots = \frac{A_{in}}{A_{out}} = C_{geo} \quad (36)$$

that is: the *"inverse lambertian transmittance"* of an SC is C_{geo} times its *"direct lambertian transmittance"*, or equivalently: the *"input direct lambertian flux"* needed to sustain an equal transmitted flux in the opposite direction is C_{geo} times the *"input inverse lambertian flux"*. This result is not surprising; it is a direct consequence of the geometrical asymmetry of the concentrator and disappears when $C_{geo} = 1$, that is $A_{in} = A_{out}$. It is interesting to note that this result does not require any information about the internal features of the SC, but is only dependent on the sizes of the lateral apertures. Eq. (36) tells us that the optical "transparency" of the SC to lambertian light is not symmetric.

Let us imagine now irradiating both apertures of the SC by two different lambertian sources with $L_{dir} \neq L_{inv}$. If $\Delta L = L_{dir} - L_{inv} > 0$ is the difference of incidence radiance between input and output, then we have for the net flux through SC, in the direct direction:

$$\Delta\Phi = \Phi_{dir}^{net} = \Phi_{dir}^{out} - \Phi_{inv}^{out} = \tau_{dir}^{lamb} \cdot \Phi_{dir}^{in} - \tau_{inv}^{lamb} \cdot \Phi_{inv}^{in} = \ldots$$

$$\tau_{dir}^{lamb} \cdot \left[\Phi_{dir}^{in} - C_{geo} \cdot \Phi_{inv}^{in}\right] = \ldots$$

$$\ldots = \tau_{dir}^{lamb} \cdot \left[\pi \cdot L_{dir} \cdot A_{in} - C_{geo} \cdot \pi \cdot L_{inv} \cdot A_{out}\right] = \ldots$$

$$\ldots = (\pi \cdot A_{in} \cdot \tau_{dir}^{lamb}) \cdot \Delta L \quad (37)$$

Eq. (37) has a strong similarity with the Ohm's law: $I = G \cdot \Delta V$, where Φ_{dir}^{net} (W) has the role of current, ΔL (W/sr·m^2) the role of potential difference and $(\pi \cdot A_{in} \cdot \tau_{dir}^{lamb})$ (sr·m^2) the role of conductance. From Eq. (37) we define the new optical quantity: *"direct lambertian optical conductance"* G_{dir}^{lamb}:

$$G_{dir}^{lamb} = (\pi \cdot A_{in} \cdot \tau_{dir}^{lamb}) \quad (38)$$

If we reverse the SC keeping fix the radiance gradient, now the flux flows in the inverse direction with the same conductance. We have in fact, changing the sign to both members of Eq. (37) and using Eq. (36):

$$\Delta\Phi = \Phi_{inv}^{net} = \Phi_{inv}^{out} - \Phi_{dir}^{out} = \ldots$$

$$\ldots = \tau_{inv}^{lamb} \cdot \Phi_{inv}^{in} - \tau_{dir}^{lamb} \cdot \Phi_{dir}^{in} = \ldots$$

$$\ldots = \tau_{inv}^{lamb} \cdot \left[\pi \cdot L_{inv} \cdot A_{out} - \pi \cdot L_{dir} \cdot A_{in} / C_{geo}\right] = \ldots$$

$$\ldots = (\pi \cdot A_{out} \cdot \tau_{inv}^{lamb}) \cdot \Delta L \quad (39)$$

with $\Delta L = L_{inv} - L_{dir} > 0$.

From Eq. (39) we define the *"inverse lambertian optical conductance"* G_{inv}^{lamb}:

$$G_{inv}^{lamb} = (\pi \cdot A_{out} \cdot \tau_{inv}^{lamb}) \quad (40)$$

From Eq. (36), (38) we conclude that the two lambertian optical conductances are equal:

$$G_{dir}^{lamb} = G_{inv}^{lamb} \quad (41)$$

The result of Eq. (41) tells that the optical asymmetry of the SC disappears as long as the conductance of the SC is considered. Eq. (38) and (40) show that the "optical conductance" can be put in the form:

$$G^{lamb} = (\pi \cdot A) \cdot \tau^{lamb} \quad (42)$$

that is: *"conductance"* = *"étendue"* x *"transmittance"*. Now the equivalence between the two opposite conductances is direct consequence of the fact that the "direct" étendue is C_{geo} times the "inverse" étendue and that the "direct" transmittance is $1/C_{geo}$ times the "inverse" transmittance.

From Eq. (37) we derive the density of the net flux through the input aperture J_{dir}^{net} (the average net flux flowing through the unit area of the input aperture inside the SC in direct way):

$$J_{dir}^{net} = \frac{\Phi_{dir}^{net}}{A_{in}} = (\pi \cdot \tau_{dir}^{lamb}) \cdot \Delta L \qquad (43)$$

where $\Delta L = L_{dir} - L_{inv} > 0$, and the density of the net flux through the output aperture J_{inv}^{net} (the average net flux flowing through the unit area of the output aperture inside the SC in the reverse way) becomes:

$$J_{inv}^{net} = \frac{\Phi_{inv}^{net}}{A_{out}} = (\pi \cdot \tau_{inv}^{lamb}) \cdot \Delta L \qquad (44)$$

where $\Delta L = L_{inv} - L_{dir} > 0$.

8 CONCLUSIONS

In conclusion, we have presented in this work a general theoretical approach to the study of a solar concentrator looked at as a generic optical element. Irrespective of its practical way of use, we have considered different types of irradiation, and, for some of them, its reflection, absorption and transmission properties have been defined, both locally and on the entire surface of its apertures. But, most importantly, the classical view of the concentrator fully irradiated on the front side by a collimated beam has been upset and a new way of looking to it has been introduced through the new concept of "inverse" irradiation. By inverting the irradiation of the concentrator and by using a lambertian distribution of light at the output, new and surprising results appear, which allow us, besides other things, to disclose the full direct optical transmission properties of the solar concentrator by a very simple approach. Besides this, we have been able to introduce new optical concepts and to define new optical quantities making similarities with electrical concepts. All the results have been obtained applying two optical concepts: the reversibility principle and the efficiency of transmission through the solar concentrator of an elemental beam.

REFERENCES

[1] A. Luque and S. Hegedus, *Handbook of Photovoltaic Science and Engineering* (Wiley, 2011).

[2] A. Luque and V.M. Andreev, *Concentrator Photovoltaics* (Springer, 07)

[3] A. Martí and A. Luque, *Next Generation Photovoltaics* (Institute of Physics, 2004).

[4] V.M. Andreev, V.A. Grilikhes and V.D. Rumyantsev, *Photovoltaic Conversion of Concentrated Sunlight* (Wiley, 1997).

[5] A. Luque, *Solar Cells and Optics for Photovoltaic Concentrators* (Institute of Physics, 1989).

[6] D. Barlev, R. Vidu, P. Stroeve, *Innovations in concentrated solar power*, Solar Energy Materials and Solar Cells 95 (2011) 2703-2725.

[7] T.M. Razikov, C.S. Ferekides, D. Morel, E. Stefanakos, H.S. Hullal, H.M. Upadhyaya, *Solar photovoltaic electricity: Current status and future prospects*, Solar Energy 85 (2011) 1580-1608.

[8] R.M. Swanson, *The Promise of Concentrators*, Progress in Photovoltaics 8 (2000) 93-111.

[9] J. Chaves, *Introduction to Nonimaging Optics* (CRC Press, 2008).

[10] R. Winston, J.C. Miñano and P. Benítez, *Nonimaging Optics* (Elsevier, 2005).

[11] W.T. Welford and R. Winston, *High Collection Nonimaging Optics* (Academic, 1989).

[12] P. Sansoni, D. Fontani, F. Francini, L. Mercatelli, D. Jafrancesco, *CPV Optics: Optical Design and Tests, in Solar Collectors: Energy Conservation, Design and Applications* (Nova Science Publishers Inc., 2009) Chap. 10, 253-277.

[13] D. Fontani, P. Sansoni, F. Francini, L. Mercatelli, D. Jafrancesco, *Optical tests for sunlight concentrators*, Proceedings ISES Solar World Congress, (2009) 1213-1221.

[14] F. Francini, D. Fontani, P. Sansoni, L. Mercatelli, D. Jafrancesco, *Tools and methods to test solar collectors*, Proceedings FOTONICA 2009, Pisa, Italy, 27-29 May 2009.

[15] P. Sansoni, F. Francini and D. Fontani, *Optical characterization of solar concentrator*, Optics and Lasers in Engineering 45 (2007) 351-9.

[16] A. Parretta, G. Martinelli, E. Bonfiglioli, A. Antonini, M. Butturi, P. Di Benedetto, D. Uderzo, P. Zurru, E. Milan and D. Roncati, *Indoor optical characterization of the nonimaging 'Rondine' PV solar concentrator*, Proceedings 24th European Photovoltaic Solar Energy Conference, European Commission JRC (2009) 747-52.

[17] A. Parretta, G. Martinelli, A. Antonini and D. Vincenzi, *Direct and inverse methods of characterization of solar concentrators*, Proceedings Optics for Solar Energy (SOLAR), Optical Society of America (2010) STuA1.

[18] A. Parretta, A. Antonini, M. Butturi, E. Milan E, P. Di Benedetto, D. Uderzo and P. Zurru, *Optical methods for indoor characterization of small-size concentrators prototypes*, Advances in Science and Technology 74 (2010) 196-204.

[19] A. Parretta, L. Zampierolo, D. Roncati, *Theoretical aspects of light collection in solar concentrators*, Proceedings Optics for Solar Energy (SOLAR), Optical Society of America (2010) STuE1.

[20] A. Parretta, A. Antonini, M. Butturi, P. Di Benedetto, E. Milan, M. Stefancich, D. Uderzo, P. Zurru, D. Roncati, G. Martinelli, M. Armani, *How to 'display' the angle-resolved transmission efficiency of a solar concentrator reversing the light path*, Proceedings 23rd European Photovoltaic Solar Energy Conference, European Commission JRC (2008) 95-8.

[21] A. Parretta, A. Antonini, M. Stefancich, G. Martinelli and M. Armani, *Optical characterization of CPC concentrator by an inverse illumination method*, Proceedings 22th European Photovoltaic Solar Energy Conference, European Commission JRC (2007) 740-4.

[22] A. Parretta, A. Antonini, M. Stefancich, G. Martinelli and M. Armani, *Inverse illumination method for characterization of CPC concentrators*, Proceedings SPIE, Optics and Photonics Conference, ed. D.R. Myers, Vol. 6652 (2007) 665205.

[23] A. Parretta, A. Antonini, E. Milan, M. Stefancich, G. Martinelli and M. Armani, *Optical efficiency of solar concentrators by a reverse optical path method*, Optics Letters 33 (2008) 2044-6.

[24] A. Parretta and D. Roncati, 2010 *Theory of the 'inverse method' for characterization of solar concentrators*, Proceedings Optics for Solar Energy (SOLAR), Optical Society of America (2010) StuE2.

[25] D. Vincenzi, M. Stefancich, S. Baricordi, M. Gualdi, G. Martinelli, A. Parretta and A. Antonini, *Effects of irradiance distribution uneveness on the ohmic losses of CPV receivers*, Proceedings 24th European Photovoltaic Solar Energy Conference, European Commission JRC (2009) 725-8.

[26] I. Antón, R. Solar, G. Sala and D. Pachón, *IV Testing of Concentration Modules and Cells with Non-Uniform Light Patterns*, Proceedings 17th European Photovoltaic Solar Energy Conference, European Commission JRC (2001) 611-614.

[27] R.C. Jones, *On Reversibility and Irreversibility in Optics*, Journal of the Optical Society of America 43 (1953) 138-144.

[28] A. Parretta, A. Antonini, E. Bonfiglioli, M. Campa, D. Vincenzi, G. Martinelli, *Il metodo inverso svela le proprietà dei concentratori solari*, PV Technology 3 (2009) 58-64.