Coordinate Systems

Lecture 02

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Astrophysical Measurements

Outline

1. Coordinate Systems
   - The Horizontal System
   - The Equatorial System
   - The Eclyptic System
   - The Galactic System

Outline

The Horizontal System

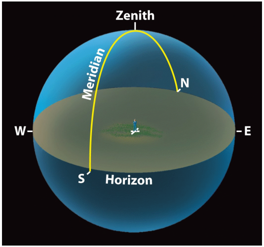

Figure 2-21
Universe, Eighth Edition

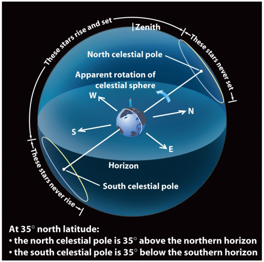

Figure 2-10
Universe, Eighth Edition

- *Horizon*: tangent plane of the Earth passing through the observer
- *Zenith*: point above the observer
- *Nadir*: anti-podal point of the zenith
- *Verticals*: great circles through zenith
- Stars culminate on the NZS vertical
- *Meridian*: the NZS vertical

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600 YEARS OF LOOKING FORWARD.

The Horizontal System

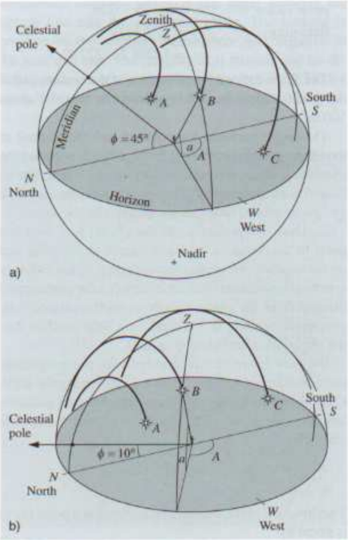
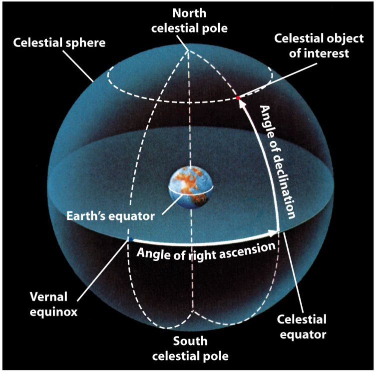


Fig. 2.9. (a) The apparent motions of stars during a night as seen from latitude $\phi = 45°$. (b) The same stars seen from latitude $\phi = 10°$

- *Altitude or Elevation a*: angle from the horizon along the vertical of the object
- *Zenith distance*: $z = 90° - a$
- *Azimuth, A*: *clockwise* angle of the vertical of the object from the South

Outline

The Equatorial System



Box 2-1
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- *Celestial Equator*: intersection of celestial sphere and equatorial plane
- *North/South celestial poles*: poles of the celestial equator
- *Declination* δ: angular distance from equator.
- *Right Ascension (R.A.)* α: angle from the *vernal equinox* measured along the equator *counterclockwise*
- The meridian is split by the north pole into north and south meridians

Hour Angle and Local Sidereal Time

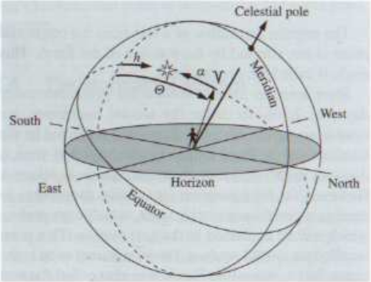

Fig. 2.11. The sidereal time Θ (the hour angle of the vernal equinox) equals the hour angle plus right ascension of any object

- *Hour angle*: measured *clockwise* from the meridian
- α: R.A.
- *sidereal time* Θ: hour angle of vernal equinox
  - $\Theta = h + \alpha$ (for any object)
- All measured in units of time:
  - 1 hour = 15°
  - 1 minute = $15'$ (arcminutes)
  - 1 second = $15''$ (arcseconds)

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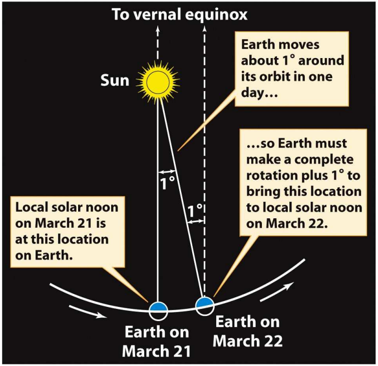
Solar vs. Sidereal Time



A month's motion of the Earth along its orbit

Box 2-2
Universe, Eighth Edition

24^h solar time = $24^h\ 3^m\ 56.56^s$ sidereal time

Orbital motion of the Earth makes stars move faster than the Sun

Horizontal–Equatorial Transformations

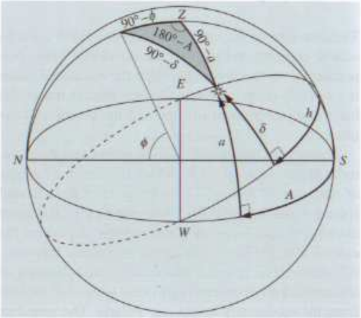


Fig. 2.12. The nautical triangle for deriving transformations between the horizontal and equatorial frames

$$\psi = 90^\circ - A, \quad \theta = a$$
$$\psi' = 90^\circ - h, \quad \theta' = \delta, \quad \chi = 90^\circ - \phi$$

From the rotation transformations:

$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$
$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$
$$\sin\theta' = -\sin\psi\cos\theta\sin\chi + \sin\theta\cos\chi$$

we finally get:

$$\sin h\cos\delta = \sin A\cos a$$
$$\cos h\cos\delta = \cos A\cos a\sin\phi + \sin a\cos\phi$$
$$\sin\delta = -\cos A\cos a\cos\phi + \sin a\sin\phi$$

…and their inverse transformations

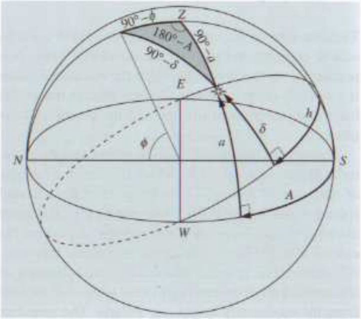


Fig. 2.12. The nautical triangle for deriving transformations between the horizontal and equatorial frames

$$\psi = 90^\circ - h, \quad \theta = \delta$$
$$\psi' = 90^\circ - A, \quad \theta' = a, \quad \chi = -(90^\circ - \phi)$$

From the rotation transformations:

$$\sin A \cos a = \sin h \cos \delta$$
$$\cos A \cos a = \cos h \cos \delta \sin \phi - \sin \delta \cos \phi$$
$$\sin a = \cos h \cos \delta \cos \phi + \sin \delta \sin \phi$$

Transit or *upper culmination* of a star

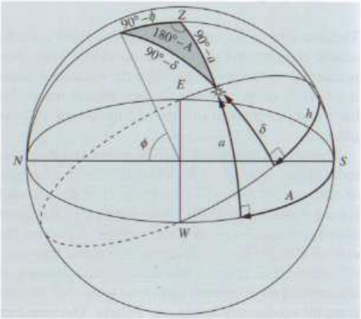


Fig. 2.12. The nautical triangle for deriving transformations between the horizontal and equatorial frames

The altitude a of a star is greatest when it is on the south meridian.
Its hour angle is zero: $h = 0$ and the star is at transit:
$\sin a_{\max} = \cos\delta\cos\phi + \sin\delta\sin\phi = \cos(\phi - \delta) = \sin(90^\circ - \phi + \delta)$

Culmination altitude:

$$a_{\max} = \begin{cases} 90^\circ - \phi + \delta, & \delta < \phi \quad \text{culm.S} \\ 90^\circ + \phi - \delta, & \delta > \phi \quad \text{culm.N} \end{cases}$$

Altitude of a star at Transit

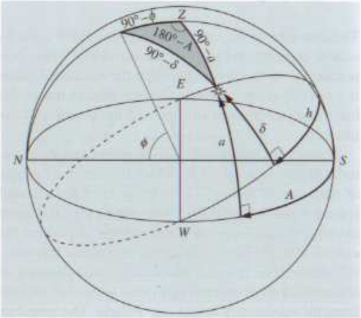


Fig. 2.12. The nautical triangle for deriving transformations between the horizontal and equatorial frames

- $\delta < \phi - 90^\circ$, never visible at lat. ϕ
- When $h = 12^{\rm h}$:
  $\sin a_{\rm min} = -\cos\delta\cos\phi + \sin\delta\sin\phi =$
  $-\cos(\delta + \phi) = \sin(\delta + \phi - 90^\circ)$
  $a_{\rm min} = \delta + \phi - 90^\circ$
- *Circumpolar stars*:
  $\delta > 90^\circ - \phi$, never set at latitude ϕ

Easy way to estimate δ

$\delta = (a_{\rm min} + a_{\rm max})/2 \quad ;\ \phi > 45^\circ, 90^\circ - \phi < \delta < \phi$

$\delta = 90^\circ - (a_{\rm max} - a_{\rm min})/2 \quad ;\ \delta > \max(90^\circ - \phi, \phi)$

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Rising and Setting Times

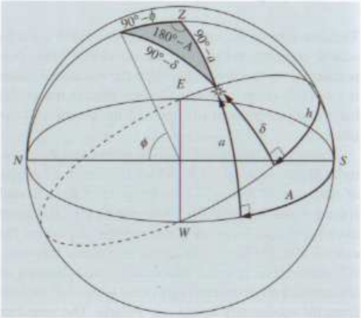
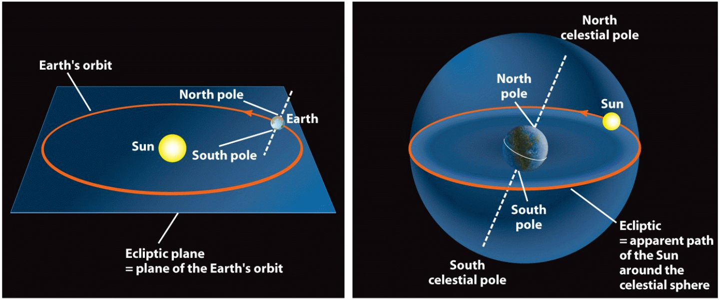


Fig. 2.12. The nautical triangle for deriving transformations between the horizontal and equatorial frames

- We alread know:
  $\sin a = \cos h \cos\delta\cos\phi + \sin\delta\sin\phi$
- rising and setting times occur when $a = 0$:
  $\cos h = -\tan\delta\tan\phi$
- If α is known, the sidereal time can be found:
  $\cos(\Theta - \alpha) = -\tan\delta\tan\phi$

Outline

Ecliptic Plane



(a) In reality the Earth orbits the Sun once a year (b) It appears to us that the Sun travels around the celestial sphere once a year

Figure 2-14
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- *Ecliptic*: orbital plane of the Earth.
- Also: the great circle on the celestial sphere described by the Sun in the course of one year.

Vernal Equinox

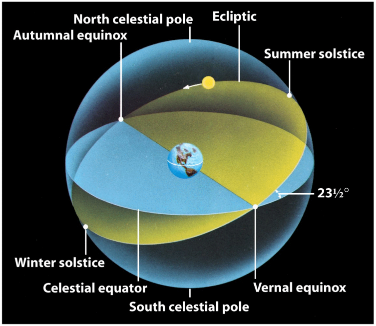

Figure 2-15
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Vernal Equinox

- *Time at which the Sun crosses the celestial equator from Southern to Northern hemisphere.*
- The Sun coordinates are $(\alpha, \delta) = (0, 0)$.
- The equatorial and ecliptic planes intersect along a straight line directed towards the vernal equinox.
- Also known as γ point, used as origin for measuring α.

Ecliptic Coordinates

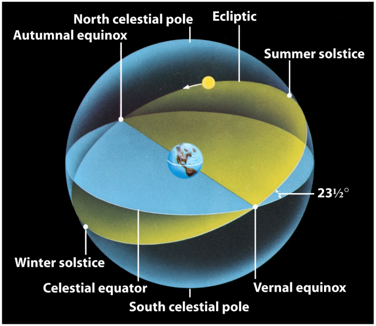

Figure 2-15
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$\varepsilon = 23°26'$ is the obliquity of the ecliptic

Ecliptic longitude λ, and latitude β

- λ: measured counterclockwise from vernal equinox
- β: angular distance from ecliptic

Equatorial–Ecliptic Transformations

$$\sin\lambda\cos\beta = \sin\delta\sin\varepsilon + \cos\delta\cos\varepsilon\sin\alpha$$
$$\cos\lambda\,\cos\beta = \cos\delta\cos\alpha$$
$$\sin\beta = \sin\delta\cos\varepsilon - \cos\delta\sin\varepsilon\sin\alpha$$

... and inverse Transformations

$$\sin\alpha\cos\delta = -\sin\beta\sin\varepsilon + \cos\beta\cos\varepsilon\sin\lambda$$
$$\cos\alpha\,\cos\delta = \cos\lambda\cos\beta$$
$$\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\varepsilon\sin\lambda$$

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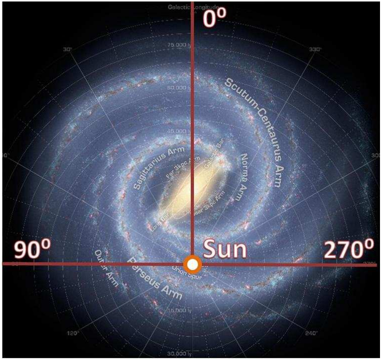
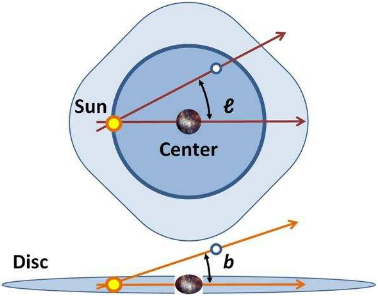
Galactic Coordinates



The galactic longitude l is measured counterclockwise from the centre of the Milky Way ($\alpha = 17^{\mathrm{h}}45.7^{\mathrm{m}}$, $\delta = -29^{\circ}00'$).



The galactic latitude b is measured from the galactic plane.

Galactic Coordinates

Equatorial–Galactic Transformations

Consider the spherical triangle with the following vertices:

- the galactic North pole P ($\alpha_P = 12^{\mathrm{h}}51.4^{\mathrm{m}}, \delta_P = 27^\circ 08'$)
- the North pole ($l_N = 123.0^\circ$)
- the generic star.

$$\sin(l_N - l)\cos b = \cos\delta \sin(\alpha - \alpha_P)$$

$$cos(l_N - l)\cos b = -\cos\delta \sin\delta_P \cos(\alpha - \alpha_P) + \sin\delta \cos\delta_P$$

$$\sin b = \cos\delta \cos\delta_P \cos(\alpha - \alpha_P) + \sin\delta \sin\delta_P$$

References

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- H. Karttunen et al., “Fundamental Astronomy”, 5th ed., Springer.
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