Spherical Astronomy

Lecture 01

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Astrophysical Measurements

Outline

university of ferrara
600 YEARS OF LOOKING FORWARD.

Outline

Great Circles and Triangles

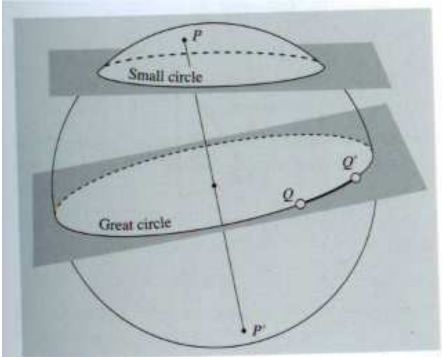

Fig. 2.1. A great circle is the intersection of a sphere and a plane passing through its centre. P and P' are the poles of the great circle. The shortest path from Q to Q' follows the great circle

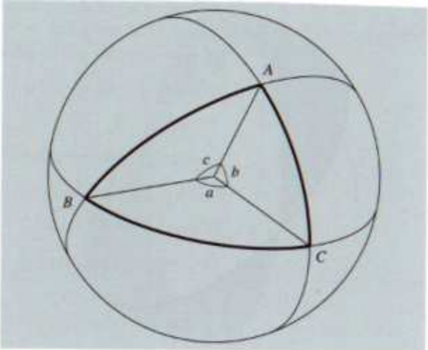

Fig. 2.2. A spherical triangle is bounded by three arcs of great circles, AB, BC and CA. The corresponding central angles are c, a, and b

- Small and Great Circles.
- Spherical triangle: arcs of great circles

Spherical angles and sides

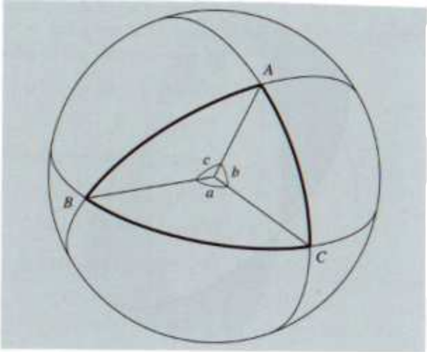

Fig. 2.2. A spherical triangle is bounded by three arcs of great circles, AB, BC and CA. The corresponding central angles are c, a, and b

Angles & sides

Unique correspondence
$a \leftrightarrow BC, b \leftrightarrow AC, c \leftrightarrow AB$

Spherical Excess

- $E = A + B + C - 180°$
- it's not constant

Spherical Excess and triangle area

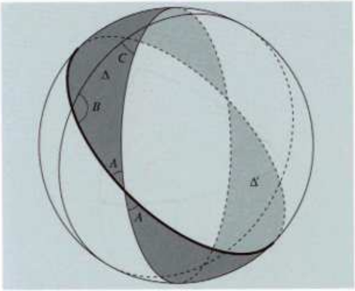

Fig. 2.3. If the sides of a spherical triangle are extended all the way around the sphere, they form another triangle Δ′, antipodal and equal to the original triangle Δ. The shaded area is the slice $S(A)$

Spherical Excess

- $\mathscr{A}(\Delta)$: triangle Δ area
- $S(A)/4\pi r^2 = A/\pi$
- $\frac{A+B+C}{\pi} 4\pi r^2 = 4\pi r^2 + 4\mathscr{A}(\Delta)$
- $\mathscr{A}(\Delta) = (A+B+C-\pi) r^2 = E r^2$

Outline

1. Spherical Trigonometry
   - Basics
   - Spherical Triangle: angles and sides relations
2. The Earth
   - Coordinates
   - Geodetic/geocentric latitude

Spherical Triangle: angles and sides relations

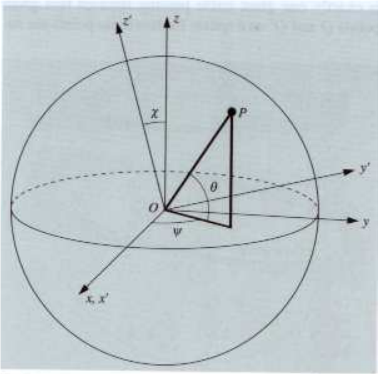

Fig. 2.4. The location of a point P on the surface of a unit sphere can be expressed by rectangular xyz coordinates or by two angles, ψ and θ. The $x'y'z'$ frame is obtained by rotating the xyz frame around its x axis by an angle χ

Rotation around x axis by angle χ

$$x = \cos\psi\ \cos\theta \qquad x' = \cos\psi'\ \cos\theta'$$
$$y = \sin\psi\ \cos\theta \qquad y' = \sin\psi'\ \cos\theta'$$
$$z = \sin\theta \qquad z' = \sin\theta'$$

$$x' = x$$
$$y' = y\cos\chi + z\sin\chi$$
$$z' = -y\sin\chi + z\cos\chi$$

Spherical Triangle: angles and sides relations

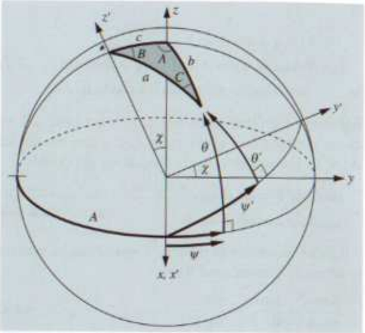

Fig. 2.6. To derive triangulation formulas for the spherical triangle ABC, the spherical coordinates ψ, θ, ψ' and θ' of the vertex C are expressed in terms of the sides and angles of the triangle

Replacing, we get:

$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$
$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$
$$\sin\theta' = -\sin\psi\cos\theta\sin\chi + \sin\theta\cos\chi$$

with spherical angles (P is now C)

$$\psi = A - 90^\circ, \qquad \theta = 90^\circ - b$$
$$\psi' = 90^\circ - B, \qquad \theta' = 90^\circ - a, \qquad \chi = c$$

Spherical Triangle: angles and sides relations

Replacing, we get:

$$\cos(90^\circ - B)\cos(90^\circ - a) = \cos(A - 90^\circ)\cos(90^\circ - b)$$
$$\sin(90^\circ - B)\cos(90^\circ - a) =$$
$$\sin(A - 90^\circ)\cos(90^\circ - b)\cos c + \sin(90^\circ - b)\sin c$$
$$\sin(90^\circ - a) = -\sin(A - 90^\circ)\cos(90^\circ - b)\sin c + \sin(90^\circ - b)\cos c$$

or

$$\sin B \sin a = \sin A \sin b$$
$$\cos B \sin a = -\cos A \sin b \cos c + \cos b \sin c$$
$$\cos a = \cos A \sin b \sin c + \cos b \cos c$$

Sine and Cosine Formulas

Sine Formula

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Sine-Cosine Formula

$$\sin c \cos b = \sin b \cos c \cos A + \sin a \cos B$$

Cosine Formula

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Outline

Coordinates

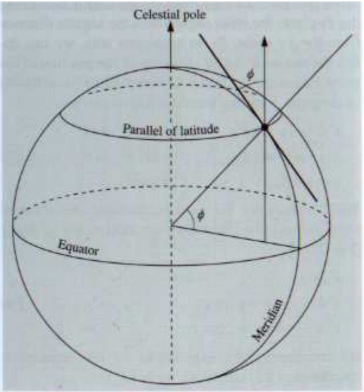

Fig. 2.7. The latitude ϕ is obtained by measuring the altitude of the celestial pole. The celestial pole can be imagined as a point at an infinite distance in the direction of the Earth's rotation axis

Coordinates

- Longitude: angle between the meridian and that of Greenwich
- Latitude
  - geodetic (geographical) ϕ (plumb line)
  - geocentric ϕ'

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Geodetic/geocentric latitude

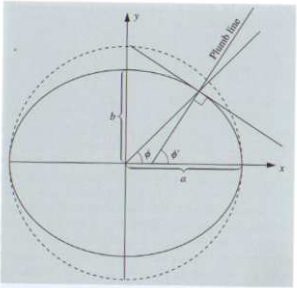

Fig. 2.8. Due to the flattening of the Earth, the geographic latitude ϕ and geocentric latitude ϕ' are different

Geodetic Reference System 1980 (GRS-80) Ellipsoid:

- equatorial radius: $a = 6,378$ Km
- polar radius: $b = 6,357$ Km
- flattening: $f = (a - b)/b = 1/298$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \tan\phi = -\frac{dx}{dy} = \frac{a^2}{b^2}\,\frac{y}{x}$$

$$\tan\phi' = \frac{y}{x} \Rightarrow \tan\phi' = \frac{b^2}{a^2}\tan\phi = (1 - e^2)\tan\phi$$

Max difference $\Delta\phi = \phi - \phi' = 11.5'$ (@ 45°)