

Chapter Seven

- 7.1** The probability distribution of the population data is called the population distribution. Tables 7.1 and 7.2 on page 291 of the text provide an example of such a distribution. The probability distribution of a sample statistic is called its sampling distribution. Tables 7.3 to 7.5 on page 293 of the text provide an example of the sampling distribution of the sample mean.
- 7.2** Sampling error is the difference between the value of the sample statistic and the value of the corresponding population parameter, assuming that the sample is random and no nonsampling error has been made. Example 7-1 on page 295 of the text exhibits sampling error. Sampling errors occur only in sample surveys.
- 7.3** Nonsampling errors are errors that may occur during collection, recording, and tabulation of data. The second part of Example 7-1 on pages 295 and 296 of the text exhibits nonsampling error. Nonsampling errors occur both in sample surveys and censuses.
- 7.4**
- a. $\mu = (15 + 13 + 8 + 17 + 9 + 12)/6 = 74/6 = 12.33$
 - b. $\bar{x} = (13 + 8 + 9 + 12)/4 = 42/4 = 10.50$
Sampling error $= \bar{x} - \mu = 10.50 - 12.33 = -1.83$
 - c. Liza's incorrect $\bar{x} = (13 + 8 + 6 + 12)/4 = 39/4 = 9.75$
 $\bar{x} - \mu = 9.75 - 12.33 = -2.58$
Sampling error(from part b) $= -1.83$
Nonsampling error $= -2.58 - (-1.83) = -.75$

d.

Sample	\bar{x}	$\bar{x} - \mu$
15, 13, 8, 17	13.25	.92
15, 13, 8, 9	11.25	-1.08
15, 13, 17, 9	13.50	1.17
15, 8, 17, 9	12.25	-.08
13, 8, 17, 9	11.75	-.58
15, 13, 8, 12	12.00	-.33
15, 13, 17, 12	14.25	1.92
15, 8, 17, 12	13.00	.67
13, 8, 17, 12	12.50	.17
15, 13, 9, 12	12.25	-.08
15, 8, 9, 12	11.00	-1.33
13, 8, 9, 12	10.50	-1.83
15, 17, 9, 12	13.25	.92
13, 17, 9, 12	12.75	.42
8, 17, 9, 12	11.50	-.83

- 7.5**
- a. $\mu = (20 + 25 + 13 + 19 + 9 + 15 + 11 + 7 + 17 + 30)/10 = 166/10 = 16.60$
- b. $\bar{x} = (20 + 25 + 13 + 9 + 15 + 11 + 7 + 17 + 30)/9 = 147/9 = 16.33$
 Sampling error $= \bar{x} - \mu = 16.33 - 16.60 = -.27$
- c. Rich's incorrect $\bar{x} = (20 + 25 + 13 + 9 + 15 + 11 + 17 + 17 + 30)/9 = 157/9 = 17.44$
 $\bar{x} - \mu = 17.44 - 16.60 = .84$
 Sampling error (from part b) $= -.27$
 Nonsampling error $= .84 - (-.27) = 1.11$

d.

Sample	\bar{x}	$\bar{x} - \mu$
25, 13, 19, 9, 15, 11, 7, 17, 30	16.22	-.38
20, 13, 19, 9, 15, 11, 7, 17, 30	15.67	-.93
20, 25, 19, 9, 15, 11, 7, 17, 30	17.00	.40
20, 25, 13, 9, 15, 11, 7, 17, 30	16.33	-.27
20, 25, 13, 19, 15, 11, 7, 17, 30	17.44	.84
20, 25, 13, 19, 9, 11, 7, 17, 30	16.78	.18
20, 25, 13, 19, 9, 15, 7, 17, 30	17.22	.62
20, 25, 13, 19, 9, 15, 11, 17, 30	17.67	1.07
20, 25, 13, 19, 9, 15, 11, 7, 30	16.56	-.04
20, 25, 13, 19, 9, 15, 11, 7, 17	15.11	-1.49

7.6

x	$P(x)$	$xP(x)$	$x^2P(x)$
70	.20	14.00	980.00
78	.20	15.60	1216.80
80	.40	32.00	2560.00
95	.20	19.00	1805.00
		$\sum xP(x) = 80.60$	$\sum x^2P(x) = 6561.80$

$$\mu = \sum xP(x) = 80.60$$

$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{6561.80 - (80.60)^2} = 8.09$$

7.7

a.

x	$P(x)$
55	$1/6 = .167$
53	$1/6 = .167$
28	$1/6 = .167$
25	$1/6 = .167$
21	$1/6 = .167$
15	$1/6 = .167$

b.

Sample	\bar{x}
55, 53, 28, 25, 21	36.4
55, 53, 28, 25, 15	35.2
55, 53, 28, 21, 15	34.4
55, 53, 25, 21, 15	33.8
55, 28, 25, 21, 15	28.8
53, 28, 25, 21, 15	28.4

\bar{x}	$P(x)$
36.4	$1/6 = .167$
35.2	$1/6 = .167$
34.4	$1/6 = .167$
33.8	$1/6 = .167$
28.8	$1/6 = .167$
28.4	$1/6 = .167$

- c. The mean for the population data is:

$$\mu = \frac{55 + 53 + 28 + 25 + 21 + 15}{6} = \frac{197}{6} = 32.83$$

Suppose the random sample of five family members includes the observations: 55, 28, 25, 21, and 15. The mean for this sample is:

$$\bar{x} = \frac{55 + 28 + 25 + 21 + 15}{5} = \frac{144}{5} = 28.80$$

Then the sampling error is: $\bar{x} - \mu = 28.80 - 32.83 = -4.03$

7.8

a.

x	$P(x)$
14	$1/5 = .20$
8	$1/5 = .20$
7	$2/5 = .40$
20	$1/5 = .20$

b.

Sample	\bar{x}
14, 8, 7, 7	9.00
14, 8, 7, 20	12.25
14, 8, 7, 20	12.25
14, 7, 7, 20	12.00
8, 7, 7, 20	10.50

\bar{x}	$P(\bar{x})$
9.00	$1/5 = .20$
12.25	$2/5 = .40$
12.00	$1/5 = .20$
10.50	$1/5 = .20$

- c. The mean for the population data is:

$$\mu = \frac{14 + 8 + 7 + 7 + 20}{5} = \frac{56}{5} = 11.20$$

Suppose the random sample of four faculty members includes the observations: 14, 8, 7, and 20. The mean for the sample is:

$$\mu = \frac{14 + 8 + 7 + 20}{4} = \frac{49}{4} = 12.25$$

Then the sampling error is: $\bar{x} - \mu = 12.25 - 11.20 = 1.05$

7.9

- a. Mean of $\bar{x} = \mu_{\bar{x}} = \mu$

- b. Standard deviation of $\bar{x} = \sigma_{\bar{x}} = \sigma/\sqrt{n}$ where σ = population standard deviation and n = sample size.

7.10 A sample statistic used to estimate a population parameter is called an estimator. An estimator is unbiased when its mean is equal to the corresponding population parameter. The sample mean \bar{x} is an unbiased estimator of μ , because the mean of \bar{x} is equal to μ .

7.11 An estimator is consistent when its standard deviation decreases as the sample size is increased. The sample mean \bar{x} is a consistent estimator of μ because its standard deviation decreases as the sample size increases. This is obvious from the formula $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.

7.12 Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, increasing n decreases $\sigma_{\bar{x}}$. Hence, as n increases $\sigma_{\bar{x}}$ decreases.

7.13 $\mu = 60$ and $\sigma = 10$

- a. $\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{18} = 2.357$
 b. $\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{90} = 1.054$

7.14 $\mu = 90$ and $\sigma = 18$

- a. $\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{10} = 5.692$
 b. $\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{35} = 3.043$

7.15 a. $n/N = 300/5000 = .06 > .05$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{25}{\sqrt{300}} \sqrt{\frac{5000-300}{5000-1}} = 1.400$$

- b. $n/N = 100/5000 = .02 < .05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 25/\sqrt{100} = 2.500$

Calculator screen showing:
 $25/\sqrt{(300)} * \sqrt{(5000-300)/(5000-1)}$
 1.399544597
 $25/\sqrt{(100)}$
 2.5

Part a. $\sigma_{\bar{x}} = 1.3995$ (rounded)

Part b. $\sigma_{\bar{x}} = 2.5$

7.16 a. $n/N = 2,500/100,000 = .025 < .05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 40/\sqrt{2500} = .800$

- b. $n/N = 7,000/100,000 = .07 > .05$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{40}{\sqrt{7,000}} \sqrt{\frac{100,000-7,000}{100,000-1}} = .461$$

7.17 $\mu = 125$ and $\sigma = 36$

- a. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 3.6$, so $n = (\sigma/\sigma_{\bar{x}})^2 = (36/3.6)^2 = 100$
 b. $n = (\sigma/\sigma_{\bar{x}})^2 = (36/2.25)^2 = 256$

7.18 $\mu = 46$ and $\sigma = 10$

- a. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.0$, so $n = (\sigma/\sigma_{\bar{x}})^2 = (10/2.0)^2 = 25$
 b. $n = (\sigma/\sigma_{\bar{x}})^2 = (10/1.6)^2 = 39$ approximately

A calculator screen showing the following calculations:
 $(9/2)^2 = 20.25$
 $(9/1.6)^2 = 31.640625$

Part a. $n = 25$ Part b. $n = 39$

- 7.19** $\mu = \$3.12$, $\sigma = \$.75$, and $n = 400$

$$\mu_{\bar{x}} = \mu = \$3.12 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = .75/\sqrt{400} = \$.038$$

- 7.20** $\mu = 41.8$ hours, $\sigma = 4$ hours, and $n = 36$

$$\mu_{\bar{x}} = \mu = 41.8 \text{ hours and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 4/\sqrt{36} = .667 \text{ hours}$$

- 7.21** $\mu = \$51,400$, $\sigma = \$7400$, and $N = 1050$

$$n = 16, \text{ and } n/N = 16/1050 = .015 < .05$$

$$\mu_{\bar{x}} = \mu = \$51,400 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 7400/\sqrt{16} = \$1850$$

- 7.22** $\mu = 62.2$ years, $\sigma = 4$ years, and $n = 700$

$$\mu_{\bar{x}} = \mu = 62.2 \text{ years and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 4/\sqrt{700} = .151 \text{ years}$$

A calculator screen showing the following calculations:
 62.2
 $4/\sqrt{700} = .1511857892$

 $\mu_{\bar{x}} = 62.2$ years $\sigma_{\bar{x}} = .1512$ years

- 7.23** $\sigma = \$3600$ and $\sigma_{\bar{x}} = \$180$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n}, \text{ so } n = (\sigma/\sigma_{\bar{x}})^2 = (3600/180)^2 = 400$$

- 7.24** $\sigma = \$139.50$ million and $\sigma_{\bar{x}} = \$15.50$ million

$$\sigma_{\bar{x}} = \sigma/\sqrt{n}, \text{ so } n = (\sigma/\sigma_{\bar{x}})^2 = (139.50/15.50)^2 = 81$$

7.25 a.

\bar{x}	$P(\bar{x})$	$\bar{x}P(\bar{x})$	$\bar{x}^2P(\bar{x})$
76.00	.20	15.200	1155.200
76.67	.10	7.667	587.829
79.33	.10	7.933	629.325
81.00	.10	8.100	656.100
81.67	.20	16.334	1333.998
84.33	.20	16.866	1422.310
85.00	.10	8.500	722.500
		$\sum \bar{x}P(\bar{x}) = 80.600$	$\sum \bar{x}^2P(\bar{x}) = 6507.262$

$\mu_{\bar{x}} = \sum \bar{x}P(\bar{x}) = 80.60 = \text{same value found in Exercise 7.6 for } \mu$

- b. $\sigma_{\bar{x}} = \sqrt{\sum x^2P(x) - (\mu_{\bar{x}})^2} = \sqrt{6507.262 - (80.60)^2} = 3.302$
- c. $\sigma/\sqrt{n} = 8.09/\sqrt{3} = 4.67$ is not equal to $\sigma_{\bar{x}} = 3.30$ in this case because $n/N = 3/5 = .60$, which is greater than .05.
- d. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{8.09}{\sqrt{3}} \sqrt{\frac{5-3}{5-1}} = 3.302$

7.26 The population from which the sample is drawn must be normally distributed.

7.27 The central limit theorem states that for a large sample, the sampling distribution of the sample mean is approximately normal, irrespective of the shape of the population distribution. Furthermore, $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, where μ and σ are the population mean and standard deviation, respectively. A sample size of 30 or more is considered large enough to apply the central limit theorem to \bar{x} .

7.28 The central limit theorem will apply in cases a and c since $n \geq 30$. It will not apply in case b because $n < 30$.

- 7.29** a. Slightly skewed to the right
- b. Approximately normal because $n > 30$ and central limit theorem applies
- c. Close to normal with perhaps a slight skew to the right

7.30 In both cases the sampling distribution of \bar{x} would be normal because the population distribution is normal.

7.31 In both cases the sampling distribution of \bar{x} would be normal because the population distribution is normal.

7.32 $\mu = 120$ minutes, $\sigma = 12$ minutes, and $n = 16$
 $\mu_{\bar{x}} = \mu = 120$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 12/\sqrt{16} = 3.00$ minutes
 The sampling distribution of \bar{x} is normal because the population is normally distributed.

7.33 $\mu = 71$ miles per hour, $\sigma = 3.2$ miles per hour, and $n = 20$
 $\mu_{\bar{x}} = \mu = 71$ miles per hour and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 3.2/\sqrt{20} = .716$ miles per hour
 The sampling distribution of \bar{x} is normal because the population is normally distributed.

- 7.34** $\mu = \$45$, $\sigma = \$8$, and $n = 25$

$$\mu_{\bar{x}} = \mu = \$45 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 8/\sqrt{25} = \$1.60$$

The sampling distribution of \bar{x} is approximately normal because the population is approximately normally distributed.

- 7.35** $\mu = 3.02$, $\sigma = .29$, $N = 5540$, and $n = 48$

$$\mu_{\bar{x}} = \mu = 3.02$$

Since $n/N = 48/5540 = .009$, which is less than .05,

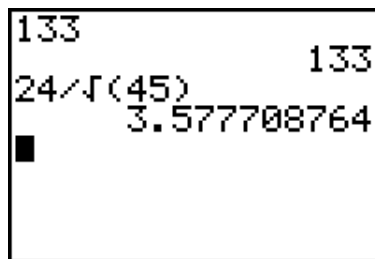
$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = .29/\sqrt{48} = .042$$

The sampling distribution of \bar{x} is approximately normal because the population is approximately normally distributed.

- 7.36** $\mu = 133$ pounds, $\sigma = 24$ pounds, and $n = 45$

$$\mu_{\bar{x}} = \mu = 133 \text{ pounds and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 24/\sqrt{45} = 3.578 \text{ pounds}$$

The sampling distribution of \bar{x} is approximately normal because the sample size is large ($n > 30$).



$$\mu_{\bar{x}} = 133 \text{ pounds}$$

$$\sigma_{\bar{x}} = 3.5777 \text{ pounds}$$

Sampling distribution of \bar{x} is approx. normal.

- 7.37** $\mu = \$75$, $\sigma = \$27$, and $n = 90$

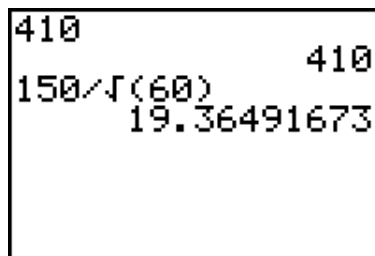
$$\mu_{\bar{x}} = \mu = \$75 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 27/\sqrt{90} = \$2.846$$

The sampling distribution of \bar{x} is approximately normal because the sample size is large ($n > 30$).

- 7.38** $\mu = \$410$, $\sigma = \$150$, and $n = 60$

$$\mu_{\bar{x}} = \mu = \$410 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 150/\sqrt{60} = \$19.365$$

The sampling distribution of \bar{x} is approximately normal because the sample size is large ($n > 30$).



$$\mu_{\bar{x}} = \$410$$

$$\sigma_{\bar{x}} = \$19.36 \text{ (rounded)}$$

Sampling distribution of \bar{x} is approx. normal.

- 7.39** $\mu = \$33,702$, $\sigma = \$3900$, and $n = 700$

$$\mu_{\bar{x}} = \mu = \$33,702 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 3900/\sqrt{700} = \$147.406$$

The sampling distribution of \bar{x} is approximately normal because the sample size is large ($n > 30$).

7.40 $P(\mu - 2.50\sigma_{\bar{x}} \leq \bar{x} \leq \mu + 2.50\sigma_{\bar{x}}) = P(-2.50 \leq z \leq 2.50) = .4938 + .4938 = .9876$ or 98.76%

7.41 $P(\mu - 1.50\sigma_{\bar{x}} \leq \bar{x} \leq \mu + 1.50\sigma_{\bar{x}}) = P(-1.50 \leq z \leq 1.50) = .4332 + .4332 = .8664$ or 86.64%

7.42 $\mu = 124$, $\sigma = 18$, and $n = 36$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{36} = 3.00$$

a. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (128.60 - 124)/3.00 = 1.53$

b. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (119.30 - 124)/3.00 = -1.57$

A TI-84 calculator screen showing the calculation of z-scores. The first calculation is (128.60-124)/(18/√(36)) resulting in 1.533333333. The second calculation is (119.30-124)/(18/√(36)) resulting in -1.566666667. A cursor is visible at the bottom left.

Part a. $z = 1.533$

Part b. $z = -1.567$

c. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (116.88 - 124)/3.00 = -2.37$

d. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (132.05 - 124)/3.00 = 2.68$

A TI-84 calculator screen showing the calculation of z-scores. The first calculation is (116.88-124)/(18/√(36)) resulting in -2.373333333. The second calculation is (132.05-124)/(18/√(36)) resulting in 2.683333333. A cursor is visible at the bottom left.

Part c. $z = -2.373$

Part d. $z = 2.683$

7.43 $\mu = 66$, $\sigma = 7$, and $n = 49$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 7/\sqrt{49} = 1$$

a. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (68.44 - 66)/1.00 = 2.44$

b. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (58.75 - 66)/1.00 = -7.25$

c. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (62.35 - 66)/1.00 = -3.65$

d. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (71.82 - 66)/1.00 = 5.82$

7.44 $\mu = 75$, $\sigma = 14$, and $n = 20$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 14/\sqrt{20} = 3.13049517$$

a. For $\bar{x} = 68.5$: $z = (68.5 - 75)/3.13049517 = -2.08$

For $\bar{x} = 77.3$: $z = (77.3 - 75)/3.13049517 = .73$

$$\begin{aligned} P(68.5 < \bar{x} < 77.3) &= P(-2.08 < z < .73) = P(-2.08 < z < 0) + P(0 < z < .73) \\ &= .4812 + .2673 = .7485 \end{aligned}$$


```
normalcdf(68.5,7
7.3,75,14/√(20))

.7498106314
```

Prob. = .7498

- b. For $\bar{x} = 72.4$: $z = (72.4 - 75)/3.13049517 = -.83$
 $P(\bar{x} < 72.4) = P(z < -.83) = .5 - P(-.83 \leq z \leq 0) = .5 - .2967 = .2033$

```
normalcdf(-10^99
,72.4,75,14/√(20
))

.2031168454
```

Prob. = .2031

7.45 $\mu = 48$, $\sigma = 8$, and $n = 16$

$$\mu_{\bar{x}} = \mu = 48 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 8/\sqrt{16} = 2.0$$

- a. For $\bar{x} = 49.6$: $z = (49.6 - 48)/2.0 = .80$
 For $\bar{x} = 52.2$: $z = (52.2 - 48)/2.0 = 2.10$
 $P(49.6 < \bar{x} < 52.2) = P(.80 < z < 2.10) = .4821 - .2881 = .1940$
- b. For $\bar{x} = 45.7$: $z = (45.7 - 48)/2.0 = -2.3/2.0 = -1.15$
 $P(\bar{x} > 45.7) = P(z > -1.15) = P(-1.15 < z < 0) + .5 = .3749 + .5 = .8749$

7.46 $\mu = 60$, $\sigma = 10$, and $n = 40$

$$\mu_{\bar{x}} = \mu = 60 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{40} = 1.58113883$$

- a. For $\bar{x} = 62.20$: $z = (62.20 - 60)/1.58113883 = 2.2/1.58113883 = 1.39$
 $P(\bar{x} < 62.20) = P(z < 1.39) = .5 + P(0 < z < 1.39) = .5 + .4177 = .9177$
- b. For $\bar{x} = 61.4$: $z = (61.4 - 60)/1.58113883 = 1.4/1.58113883 = .89$
 for $\bar{x} = 64.2$: $z = (64.2 - 60)/1.58113883 = 4.2/1.58113883 = 2.66$
 $P(61.4 < \bar{x} < 64.2) = P(.89 < z < 2.66) = P(0 < z < 2.66) - P(0 < z < .89)$
 $= .4961 - .3133 = .1828$

7.47 $\mu = 90$, $\sigma = 18$, and $n = 64$

$$\mu_{\bar{x}} = \mu = 90 \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{64} = 2.25$$

- a. For $\bar{x} = 82.3$: $z = (82.3 - 90)/2.25 = -3.42$
 $P(\bar{x} < 82.3) = P(z < -3.42) = .5 - P(-3.42 \leq z \leq 0) = .5 - .5 = .0000$ approximately
- b. For $\bar{x} = 86.7$: $z = (86.7 - 90)/2.25 = -1.47$
 $P(\bar{x} > 86.7) = P(z > -1.47) = P(-1.47 < z < 0) + .5 = .4292 + .5 = .9292$

7.48 $\mu = 71$ miles per hour, $\sigma = 3.2$ miles per hour, and $n = 16$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 3.2/\sqrt{16} = .80 \text{ miles per hour}$$

a. For $\bar{x} = 69$: $z = (69 - 71)/.80 = -2.50$

$$P(\bar{x} < 69) = P(z < -2.50) = .5 - P(-2.50 \leq z \leq 0) = .5 - .4938 = .0062$$

```
normalcdf(-10^99,
69,71,.80)
.0062096799
```

Prob. = .0062

b. For $\bar{x} = 72$: $z = (72 - 71)/.80 = 1.25$

$$P(\bar{x} > 72) = P(z > 1.25) = .5 - P(0 \leq z \leq 1.25) = .5 - .3944 = .1056$$

```
normalcdf(72,10^
99,71,.80)
.105649839
```

Prob. = .1056

c. For $\bar{x} = 70$: $z = (70 - 71)/.80 = -1.25$

for $\bar{x} = 72$: $z = (72 - 71)/.80 = 1.25$

$$\begin{aligned} P(70 \leq \bar{x} \leq 72) &= P(-1.25 \leq z \leq 1.25) = P(-1.25 \leq z \leq 0) + P(0 \leq z \leq 1.25) \\ &= .3944 + .3944 = .7888 \end{aligned}$$

```
normalcdf(70,72,
71,.80)
.7887003221
```

Prob. = .7887

7.49 $\mu = 3.02$, $\sigma = .29$, and $n = 20$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = .29/\sqrt{20} = .06484597$$

a. For $\bar{x} = 3.10$: $z = (3.10 - 3.02)/.06484597 = 1.23$

$$P(\bar{x} \geq 3.10) = P(z \geq 1.23) = .5 - P(0 \leq z \leq 1.23) = .5 - .3907 = .1093$$

b. For $\bar{x} = 2.90$: $z = (2.90 - 3.02)/.06484597 = -1.85$

$$P(\bar{x} \leq 2.90) = P(z \leq -1.85) = .5 - P(-1.85 \leq z \leq 0) = .5 - .4678 = .0322$$

c. For $\bar{x} = 2.95$: $z = (2.95 - 3.02)/.06484597 = -1.08$

For $\bar{x} = 3.11$: $z = (3.11 - 3.02)/.06484597 = 1.39$

$$\begin{aligned}
 P(2.95 \leq \bar{x} \leq 3.11) &= P(-1.08 \leq z \leq 1.39) = P(-1.08 \leq z \leq 0) + P(0 \leq z \leq 1.39) \\
 &= .3599 + .4177 = .7776
 \end{aligned}$$

7.50 $\mu = 120$ minutes, $\sigma = 12$ minutes, and $n = 16$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 12/\sqrt{16} = 3.00$$

a. For $\bar{x} = 122$: $z = (122 - 120)/3.00 = .67$

For $\bar{x} = 125$: $z = (125 - 120)/3.00 = 1.67$

$$\begin{aligned}
 P(122 < \bar{x} < 125) &= P(.67 < z < 1.67) = P(0 < z < 1.67) - P(0 < z < .67) \\
 &= .4525 - .2486 = .2039
 \end{aligned}$$

b. $P(\bar{x} \text{ within 4 minutes of } \mu) = P(116 \leq \bar{x} \leq 124)$

For $\bar{x} = 116$: $z = (116 - 120)/3.00 = -1.33$

For $\bar{x} = 124$: $z = (124 - 120)/3.00 = 1.33$

$$\begin{aligned}
 P(116 \leq \bar{x} \leq 124) &= P(-1.33 \leq z \leq 1.33) = P(-1.33 \leq z \leq 0) + P(0 \leq z \leq 1.33) \\
 &= .4082 + .4082 = .8164
 \end{aligned}$$

c. $P(\bar{x} \text{ lower than } \mu \text{ by 3 minutes or more}) = P(\bar{x} \leq 117)$

For $\bar{x} = 117$: $z = (117 - 120)/3.00 = -1.00$

$$P(\bar{x} \leq 117) = P(z \leq -1.00) = .5 - P(-1.00 < z < 0) = .5 - .3413 = .1587$$

7.51 $\mu = 41.8$ hours, $\sigma = 4$ hours, and $n = 25$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 4/\sqrt{25} = .80$$

a. For $\bar{x} = 42$: $z = (42 - 41.8)/.80 = .25$

For $\bar{x} = 44$: $z = (44 - 41.8)/.80 = 2.75$

$$\begin{aligned}
 P(42 < \bar{x} < 44) &= P(.25 < z < 2.75) = P(0 \leq z \leq 2.75) - P(0 \leq z \leq .25) \\
 &= .4970 - .0987 = .3983
 \end{aligned}$$

b. $P(\bar{x} \text{ within 1.5 hours of } \mu) = P(40.3 \leq \bar{x} \leq 43.3)$

For $\bar{x} = 40.3$: $z = (40.3 - 41.8)/.80 = -1.88$

For $\bar{x} = 43.3$: $z = (43.3 - 41.8)/.80 = 1.88$

$$\begin{aligned}
 P(40.3 \leq \bar{x} \leq 43.3) &= P(-1.88 \leq z \leq 1.88) = P(-1.88 \leq z \leq 0) + P(0 \leq z \leq 1.88) \\
 &= .4699 + .4699 = .9398
 \end{aligned}$$

c. $P(\bar{x} \text{ lower than } \mu \text{ by one hour or more}) = P(\bar{x} \leq 40.8)$

For $\bar{x} = 40.8$: $z = (40.8 - 41.8)/.80 = -1.25$

$$P(\bar{x} \leq 40.8) = P(z \leq -1.25) = .5 - P(-1.25 < z < 0) = .5 - .3944 = .1056$$

7.52 $\mu = 8.4$ hours, $\sigma = 2.7$ hours, and $n = 45$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.7/\sqrt{45} = .40249224 \text{ hours}$$

a. For $\bar{x} = 8$: $z = (8 - 8.4)/.40249224 = -.99$

For $\bar{x} = 9$: $z = (9 - 8.4)/.40249224 = 1.49$

$$\begin{aligned}
 P(8 < \bar{x} < 9) &= P(-.99 < z < 1.49) = P(-.99 < z < 0) + P(0 < z < 1.49) \\
 &= .3389 + .4319 = .7708
 \end{aligned}$$

b. For $\bar{x} = 8$: $z = (8 - 8.4)/.40249224 = -.99$

$$P(\bar{x} < 8) = P(z < -.99) = .5 - P(-.99 \leq z \leq 0) = .5 - .3389 = .1611$$

7.53 $\mu = 27$ years, $\sigma = 4.4$ years, and $n = 36$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 4.4/\sqrt{36} = .73333333 \text{ years}$$

a. For $\bar{x} = 25.5$: $z = (25.5 - 27)/.73333333 = -2.05$

$$\text{For } \bar{x} = 28: z = (28 - 27)/.73333333 = 1.36$$

$$\begin{aligned} P(25.5 < \bar{x} < 28) &= P(-2.05 < z < 1.36) = P(-2.05 < z < 0) + P(0 < z < 1.36) \\ &= .4798 + .4131 = .8929 \end{aligned}$$

b. For $\bar{x} = 25.5$: $z = (25.5 - 27)/.73333333 = -2.05$

$$P(\bar{x} < 25.5) = P(z < -2.05) = .5 - P(-2.05 \leq z \leq 0) = .5 - .4798 = .0202$$

7.54 $\mu = \$33,702$, $\sigma = \$3900$, and $n = 200$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 3900/\sqrt{200} = \$275.77164466$$

a. For $\bar{x} = 34,000$: $z = (34,000 - 33,702)/275.77164466 = 1.08$

$$\text{For } \bar{x} = 34,500: z = (34,500 - 33,702)/275.77164466 = 2.89$$

$$\begin{aligned} P(34,000 \leq \bar{x} \leq 34,500) &= P(1.08 \leq z \leq 2.89) = P(0 \leq z \leq 2.89) - P(0 < z < 1.08) \\ &= .4981 - .3599 = .1382 \end{aligned}$$

b. $P(\bar{x} \text{ within } \$400 \text{ of } \mu) = P(33,302 \leq \bar{x} \leq 34,102)$

$$\text{For } \bar{x} = 33,302: z = (33,302 - 33,702)/275.77164466 = -1.45$$

$$\text{For } \bar{x} = 34,102: z = (34,102 - 33,702)/275.77164466 = 1.45$$

$$\begin{aligned} P(33,302 \leq \bar{x} \leq 34,102) &= P(-1.45 \leq z \leq 1.45) = P(-1.45 \leq z \leq 0) + P(0 \leq z \leq 1.45) \\ &= .4265 + .4265 = .8530 \end{aligned}$$

c. $P(\bar{x} \text{ below } \mu \text{ by } \$500 \text{ or more}) = P(\bar{x} \leq 33,202)$

$$\text{For } \bar{x} = 33,202: z = (33,202 - 33,702)/275.77164466 = -1.81$$

$$P(\bar{x} \leq 33,202) = P(z \leq -1.81) = .5 - P(-1.81 < z < 0) = .5 - .4649 = .0351$$

7.55 $\mu = \$65$, $\sigma = \$25$, and $n = 75$

$$\sigma_{\bar{x}} = 25/\sqrt{75} = 2.88675135$$

a. For $\bar{x} = 58$: $z = (58 - 65)/2.88675135 = -2.42$

$$\text{For } \bar{x} = 63: z = (63 - 65)/2.88675135 = -.69$$

$$\begin{aligned} P(58 \leq \bar{x} \leq 63) &= P(-2.42 \leq z \leq -.69) = P(-2.42 \leq z \leq 0) - P(-.69 \leq z \leq 0) \\ &= .4922 - .2549 = .2373 \end{aligned}$$

b. $P(\mu - 6 \leq \bar{x} \leq \mu + 6) = P(59 \leq \bar{x} \leq 71)$

$$\text{For } \bar{x} = 59: z = (59 - 65)/2.88675135 = -2.08$$

$$\text{For } \bar{x} = 71: z = (71 - 65)/2.88675135 = 2.08$$

$$P(59 \leq \bar{x} \leq 71) = P(-2.08 \leq z \leq 2.08) = .4812 + .4812 = .9624$$

c. $P(\bar{x} \geq \mu + 5) = P(\bar{x} \geq 70)$

$$\text{For } \bar{x} = 70: z = (70 - 65)/2.88675135 = 1.73$$

$$P(\bar{x} \geq 70) = P(z \geq 1.73) = .5 - P(0 < z < 1.73) = .5 - .4582 = .0418$$

7.56 $\mu = \$12,450$, $\sigma = \$4300$, and $n = 50$

$$\sigma_{\bar{x}} = 4300/\sqrt{50} = \$608.11183182$$

- a. For $\bar{x} = \$11,500$: $z = (11,500 - 12,450)/608.11183182 = -1.56$
 $P(\bar{x} > 11,500) = P(z > -1.56) = P(-1.56 < z < 0) + .5 = .4406 + .5 = .9406$

```
normalcdf(11500,
10^99,12450,4160
/√(50))
.9468216051
```

Prob. = .9468

- b. For $\bar{x} = 12,000$: $z = (12,000 - 12,450)/608.11183182 = -.74$
 For $\bar{x} = 13,800$: $z = (13,800 - 12,450)/608.11183182 = 2.22$
 $P(12,000 \leq \bar{x} \leq 13,800) = P(-.74 \leq z \leq 2.22) = P(-.74 \leq z \leq 0) + P(0 \leq z \leq 2.22)$
 $= .2704 + .4868 = .7632$

```
normalcdf(12000,
13800,12450,4160
/√(50))
.7669590715
```

Prob. = .7670

- c. $P(\mu - 1500 \leq \bar{x} \leq \mu + 1500) = P(10,950 \leq \bar{x} \leq 13,950)$
 For $\bar{x} = 10,950$: $z = (10,950 - 12,450)/608.11183182 = -2.47$
 For $\bar{x} = 13,950$: $z = (13,950 - 12,450)/608.11183182 = 2.47$
 $P(10,950 \leq \bar{x} \leq 13,950) = P(-2.47 \leq z \leq 2.47) = .4932 + .4932 = .9864$

```
normalcdf(12450-
1500,12450+1500,
12450,4160/√(50)
)
.9892172689
```

Prob. = .9892

- d. $P(\bar{x} > \mu + 1000) = P(\bar{x} > 13,450)$
 For $\bar{x} = 13,450$: $z = (13,450 - 12,450)/608.11183182 = 1.64$
 $P(\bar{x} > 13,450) = P(z > 1.64) = .5 - P(0 \leq z \leq 1.64) = .5 - .4495 = .0505$

```
normalcdf(12450+
1000,10^99,12450
,4160/√(50))
.0445865107
```

Prob. = .0446

7.57 $\mu = 68$ inches, $\sigma = 4$ inches, and $n = 100$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 4/\sqrt{100} = .4 \text{ inches}$$

a. For $\bar{x} = 67.8$: $z = (67.8 - 68)/.4 = -.50$

$$P(\bar{x} < 67.8) = P(z < -.50) = .5 - P(-.50 \leq z \leq 0) = .5 - .1915 = .3085$$

b. For $\bar{x} = 67.5$: $z = (67.5 - 68)/.4 = -1.25$

$$\text{For } \bar{x} = 68.7: z = (68.7 - 68)/.4 = 1.75$$

$$\begin{aligned} P(67.5 \leq \bar{x} \leq 68.7) &= P(-1.25 \leq z \leq 1.75) = P(-1.25 \leq z \leq 0) + P(0 \leq z \leq 1.75) \\ &= .3944 + .4599 = .8543 \end{aligned}$$

c. $P(\bar{x} \text{ within } .6 \text{ inches of } \mu) = P(67.4 \leq \bar{x} \leq 68.6)$

$$\text{For } \bar{x} = 67.4: z = (67.4 - 68)/.4 = -1.50$$

$$\text{For } \bar{x} = 68.6: z = (68.6 - 68)/.4 = 1.50$$

$$P(67.4 \leq \bar{x} \leq 68.6) = P(-1.50 \leq z \leq 1.50) = .4332 + .4332 = .8664$$

d. $P(\bar{x} \text{ lower than } \mu \text{ by } .5 \text{ inches or more}) = P(\bar{x} \leq 67.5)$

$$\text{For } \bar{x} = 67.5: z = (67.5 - 68)/.4 = -1.25$$

$$P(\bar{x} \leq 67.5) = P(z \leq -1.25) = .5 - P(-1.25 \leq z \leq 0) = .5 - .3944 = .1056$$

7.58 $\bar{x} = 2250$ hours, $\sigma = 150$ hours, and $n = 100$

$$\sigma_{\bar{x}} = 150/\sqrt{100} = 15.00 \text{ hours}$$

We are to find $P(\mu - 25 \leq \bar{x} \leq \mu + 25)$.

$$\text{For } \bar{x} = \mu - 25: z = (\mu - 25 - \mu)/15.00 = -1.67$$

$$\text{For } \bar{x} = \mu + 25: z = (\mu + 25 - \mu)/15.00 = 1.67$$

$$P(\mu - 25 \leq \bar{x} \leq \mu + 25) = P(-1.67 \leq z \leq 1.67) = .4525 + .4525 = .9050$$

7.59 $\bar{x} = 3$ inches, $\sigma = .1$ inches, and $n = 25$

$$\sigma_{\bar{x}} = .1/\sqrt{25} = .02 \text{ inches}$$

$$\text{For } \bar{x} = 2.95: z = (2.95 - 3)/.02 = -2.50$$

$$\text{For } \bar{x} = 3.05: z = (3.05 - 3)/.02 = 2.50$$

$$\begin{aligned} P(\bar{x} < 2.95) + P(\bar{x} > 3.05) &= P(z < -2.50) + P(z > 2.50) = (.5 - .4938) + (.5 - .4938) \\ &= .0062 + .0062 = .0124 \end{aligned}$$

7.60 $p = 640/1000 = .64$ and $\hat{p} = 24/40 = .60$

7.61 $p = 600/5000 = .12$ and $\hat{p} = 18/120 = .15$

7.62 Number with characteristic in population = $18,700 \times .30 = 5610$

Number with characteristic in sample = $250 \times .25 = 62.5 \approx 62$ or 63

7.63 Number in population with characteristic = $9500 \times .75 = 7125$

Number in sample with characteristic = $400 \times .78 = 312$

7.64 a. The mean of \hat{p} is: $\mu_{\hat{p}} = p$

b. The standard deviation of \hat{p} is: $\sigma_{\hat{p}} = \sqrt{pq/n}$

c. The sampling distribution of \hat{p} is approximately normal if np and nq are both greater than 5.

7.65 Sampling error = $\hat{p} - p = .66 - .71 = -.05$

7.66 Sampling error = $\hat{p} - p = .33 - .29 = .04$

7.67 The estimator of p is the sample proportion \hat{p} .

The sample proportion \hat{p} is an unbiased estimator of p , since the mean of \hat{p} is equal to p .

7.68 The sample proportion \hat{p} is a consistent estimator of p , since $\sigma_{\hat{p}}$ decreases as the sample size is increased.

7.69 $\sigma_{\hat{p}} = \sqrt{pq/n}$, hence $\sigma_{\hat{p}}$ decreases as n increases.

7.70 $p = .63$, $q = 1 - p = 1 - .63 = .37$

a. $n = 100$, $\mu_{\hat{p}} = p = .63$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.63(.37)/100} = .048$

Calculator screen showing the calculation of the standard deviation of the sample proportion for $n = 100$. The display shows $.63$, then the formula $\sqrt{(.63 \cdot .37)/100}$, and the result $.0482804308$.

$$\mu_{\hat{p}} = .63$$

$$\sigma_{\hat{p}} = .0483$$

b. $n = 900$, $\mu_{\hat{p}} = p = .63$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.63(.37)/900} = .016$

Calculator screen showing the calculation of the standard deviation of the sample proportion for $n = 900$. The display shows $.63$, then the formula $\sqrt{(.63 \cdot .37)/900}$, and the result $.0160934769$.

$$\mu_{\hat{p}} = .63$$

$$\sigma_{\hat{p}} = .0161$$

7.71 $p = .21$, $q = 1 - p = 1 - .21 = .79$

a. $n = 400$, $\mu_{\hat{p}} = p = .21$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.21(.79)/400} = .020$

b. $n = 750$, $\mu_{\hat{p}} = p = .21$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.21(.79)/750} = .015$

7.72 $N = 4000$, $p = .12$, and $q = 1 - p = 1 - .12 = .88$

a. $n/N = 800/4000 = .20 > .05$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n} \frac{N-n}{N-1}} = \sqrt{\frac{.12(.88)}{800} \frac{4000-800}{4000-1}} = .010$$

b. $n/N = 30/4000 = .0075 < .05$, $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.12(.88)/30} = .059$

7.73 $N = 1400$, $p = .47$, and $q = 1 - p = 1 - .47 = .53$

a. $n/N = 90/1400 = .064 > .05$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n} \frac{N-n}{N-1}} = \sqrt{\frac{.47(.53)}{90} \frac{1400-90}{1400-1}} = .051$$

b. $n/N = 50/1400 = .036 < .05$, $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.47(.53)/50} = .071$

7.74 A sample is considered large enough to apply the central limit theorem if np and nq are both greater than 5.

7.75 a. $np = 400(.28) = 112$ and $nq = 400(.72) = 288$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

b. $np = 80(.05) = 4$; since $np < 5$, the central limit theorem does not apply.

c. $np = 60(.12) = 7.2$ and $nq = 60(.88) = 52.8$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

d. $np = 100(.035) = 3.5$; since $np < 5$, the central limit theorem does not apply.

7.76 a. $np = 20(.45) = 9$ and $nq = 20(.55) = 11$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

b. $np = 75(.22) = 16.5$ and $nq = 75(.78) = 58.5$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

c. $np = 350(.01) = 3.5$; since $np < 5$, the central limit theorem does not apply.

d. $np = 200(.022) = 4.4$; since $np < 5$, the central limit theorem does not apply.

7.77 a. The proportion of these TV sets that are good is $4/6 = .667$

b. Total number of samples of size 5 is: $\binom{6}{5} = 6$

c & d. Let: G = good TV set and D = defective TV set

Let the six TV sets be denoted as: 1 = G , 2 = G , 3 = D , 4 = D , 5 = G , and 6 = G . The six possible samples, their sample proportions, and the sampling errors are given in the table below.

Sample	TV sets	\hat{p}	Sampling error
1, 2, 3, 4, 5	G, G, D, D, G	$3/5 = .60$	$.60 - .667 = -.067$
1, 2, 3, 4, 6	G, G, D, D, G	$3/5 = .60$	$.60 - .667 = -.067$
1, 2, 3, 5, 6	G, G, D, G, G	$4/5 = .80$	$.80 - .667 = .133$
1, 2, 4, 5, 6	G, G, D, G, G	$4/5 = .80$	$.80 - .667 = .133$
1, 3, 4, 5, 6	G, D, D, G, G	$3/5 = .60$	$.60 - .667 = -.067$
2, 3, 4, 5, 6	G, D, D, G, G	$3/5 = .60$	$.60 - .667 = -.067$

\hat{p}	f	Relative Frequency
.60	4	$4/6 = .667$
.80	2	$2/6 = .333$
$\sum f = 6$		

\hat{p}	$P(\hat{p})$
.60	.667
.80	.333

- 7.78** a. Three of the five employees of this company are female. Hence, the proportion of employees who are female is:

$$p = 3/5 = .60$$

- b. The total number of samples of size 3 that can be selected is:

$$\binom{5}{3} = 10$$

- c & d. Let the five employees of the company be denoted as:

A = male, B = female, C = female, D = male, and E = female

The following tables list all the possible samples of size 3, the sample proportions, the sampling errors and the sampling distribution of the sample proportion.

Sample	Employees	\hat{p}	Sampling error
A, B, C	male, female, female	$2/3 = .667$	$.667 - .60 = .067$
A, B, D	male, female, male	$1/3 = .333$	$.333 - .60 = -.267$
A, B, E	male, female, female	$2/3 = .667$	$.667 - .60 = .067$
A, C, D	male, female, male	$1/3 = .333$	$.333 - .60 = -.267$
A, C, E	male, female, female	$2/3 = .667$	$.667 - .60 = .067$
A, D, E	male, male, female	$1/3 = .333$	$.333 - .60 = -.267$
B, C, D	female, female, male	$2/3 = .667$	$.667 - .60 = .067$
B, C, E	female, female, female	$3/3 = 1.000$	$1.000 - .60 = .400$
B, D, E	female, male, female	$2/3 = .667$	$.667 - .60 = .067$
C, D, E	female, male, female	$2/3 = .667$	$.667 - .60 = .067$

\hat{p}	f	Relative Frequency
.333	3	$3/10 = .30$
.667	6	$6/10 = .60$
1.000	1	$1/10 = .10$

\hat{p}	$P(\hat{p})$
.333	.30
.667	.60
1.000	.10

7.79 $n = 300$, $p = .66$ and $q = 1 - p = 1 - .66 = .34$

$$\mu_{\hat{p}} = p = .66 \text{ and } \sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.66(.34)/300} = .027$$

$$np = 300(.66) = 198 \text{ and } nq = 300(.34) = 102$$

Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.

7.80 $n = 250$, $p = .70$, and $q = 1 - p = 1 - .70 = .30$

$$\mu_{\hat{p}} = p = .70 \text{ and } \sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.70)(.30)/250} = .029$$

$$np = 250(.70) = 175 \text{ and } nq = 250(.30) = 75$$

Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.

7.81 $n = 50$, $p = .20$ and $q = 1 - p = 1 - .20 = .80$

$$\mu_{\hat{p}} = p = .20 \text{ and } \sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.20(.80)/50} = .057$$

$$np = 50(.20) = 10 \text{ and } nq = 50(.80) = 40$$

Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.

7.82 $n = 100$, $p = .12$, and $q = 1 - p = 1 - .12 = .88$

$$\mu_{\hat{p}} = p = .12 \text{ and } \sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.12(.88)/100} = .032$$

$$np = 100(.12) = 12 \text{ and } nq = 100(.88) = 88$$

Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.

7.83 $P(p - 2.0\sigma_{\hat{p}} \leq \hat{p} \leq p + 2.0\sigma_{\hat{p}}) = P(-2.00 \leq z \leq 2.00) = .4772 + .4772 = .9544$

Thus, 95.44% of the sample proportions will be within 2 standard deviations of the population proportion.

7.84 $P(p - 3.0\sigma_{\hat{p}} \leq \hat{p} \leq p + 3.0\sigma_{\hat{p}}) = P(-3.00 \leq z \leq 3.00) = .4987 + .4987 = .9974$

Thus, 99.74% of the sample proportions will be within 3 standard deviations of the population proportion.

7.85 $n = 100$, $p = .59$, and $q = 1 - .59 = .41$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.59(.41)/100} = .04918333$$

a. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.56 - .59)/.04918333 = -.61$

b. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.68 - .59)/.04918333 = 1.83$

c. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.53 - .59)/.04918333 = -1.22$

d. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.65 - .59)/.04918333 = 1.22$

7.86 $n = 70$, $p = .25$, and $q = 1 - .25 = .75$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.25(.75)/70} = .05175492$$

a. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.26 - .25)/.05175492 = .19$

b. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.32 - .25)/.05175492 = 1.35$

```
(.26-.25)/√(.25*
.75/70)
.1932183566
(.32-.25)/√(.25*
.75/70)
1.352528496
```

Part a. $z = .193$

Part b. $z = 1.353$

c. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.17 - .25)/.05175492 = -1.55$

d. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.20 - .25)/.05175492 = -.97$

```
(.17-.25)/√(.25*
.75/70)
-1.545746853
(.20-.25)/√(.25*
.75/70)
-.9660917831
```

Part c. $z = -1.546$

Part d. $z = -.966$

7.87 $p = .21$, $q = 1 - p = 1 - .21 = .79$, and $n = 400$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.21(.79)/400} = .02036541$$

a. For $\hat{p} = .15$: $z = (.15 - .21)/.02036541 = -2.95$

For $\hat{p} = .25$: $z = (.25 - .21)/.02036541 = 1.96$

$$P(.15 < \hat{p} < .25) = P(-2.95 < z < 1.96) = P(-2.95 < z \leq 0) + P(0 \leq z < 1.96) \\ = .4984 + .4750 = .9734$$

```
normalcdf(.15,.2
5,.21,√(.21*.79/
400))
.9736329233
```

Prob. = .9736

b. For $\hat{p} = .20$: $z = (.20 - .21)/.02036541 = -.49$

$$P(\hat{p} < .20) = P(z < -.49) = .5 - P(-.49 \leq z \leq 0) = .5 - .1879 = .3121$$

```
normalcdf(-10^99
,.20,.21,1/400))
.3117031024
```

Prob. = .3117

7.88 $p = .64$, $q = 1 - p = 1 - .64 = .36$, and $n = 50$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.64(.36)/50} = .06788225$$

a. For $\hat{p} = .54$: $z = (.54 - .64)/.06788225 = -1.47$

For $\hat{p} = .61$: $z = (.61 - .64)/.06788225 = -.44$

$$\begin{aligned} P(.54 < \hat{p} < .61) &= P(-1.47 < z < -.44) = P(-1.47 < z < 0) - P(-.44 \leq z \leq 0) \\ &= .4292 - .1700 = .2592 \end{aligned}$$

b. For $\hat{p} = .71$: $z = (.71 - .64)/.06788225 = 1.03$

$$P(\hat{p} > .71) = P(z > 1.03) = .5 - P(0 \leq z \leq 1.03) = .5 - .3485 = .1515$$

7.89 $p = .338$, $q = 1 - .338 = .662$, and $n = 100$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.338(.662)/100} = .04730285$$

a. For $\hat{p} = .30$: $z = (.30 - .338)/.04730285 = -.80$

For $\hat{p} = .35$: $z = (.35 - .338)/.04730285 = .25$

$$\begin{aligned} P(.30 < \hat{p} < .35) &= P(-.80 < z < .25) = P(-.80 < z < 0) + P(0 \leq z < .25) \\ &= .2881 + .0987 = .3868 \end{aligned}$$

b. For $\hat{p} = .32$: $z = (.32 - .338)/.04730285 = -.38$

$$P(\hat{p} > .32) = P(z > -.38) = P(-.38 < z < 0) + .5 = .1480 + .5 = .6480$$

7.90 $p = .85$, $q = 1 - p = 1 - .85 = .15$, and $n = 100$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.85(.15)/100} = .03570714$$

a. For $\hat{p} = .81$: $z = (.81 - .85)/.03570714 = -1.12$

For $\hat{p} = .88$: $z = (.88 - .85)/.03570714 = .84$

$$\begin{aligned} P(.81 < \hat{p} < .88) &= P(-1.12 < z < .84) = P(-1.12 < z < 0) + P(0 < z < .84) \\ &= .3686 + .2995 = .6681 \end{aligned}$$

b. For $\hat{p} = .87$: $z = (.87 - .85)/.03570714 = .56$

$$P(\hat{p} < .87) = P(z < .56) = .5 + P(0 < z < .56) = .5 + .2123 = .7123$$

7.91 $p = .06$, $q = 1 - p = 1 - .06 = .94$, and $n = 100$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.06(.94)/100} = .02374868$$

For $\hat{p} = .08$: $z = (.08 - .06)/.02374868 = .84$

$$P(\hat{p} \geq .08) = P(z \geq .84) = .5 - P(0 < z < .84) = .5 - .2995 = .2005$$

7.92 $p = .80$, $q = 1 - .80 = .20$, and $n = 100$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.80(.20)/100} = .040$$

- a. The required probability is: $P(.80 - .05 \leq \hat{p} \leq .80 + .05) = P(.75 \leq \hat{p} \leq .85)$
 For $\hat{p} = .75$: $z = (.75 - .80)/.040 = -1.25$
 For $\hat{p} = .85$: $z = (.85 - .80)/.040 = 1.25$
 $P(.75 \leq \hat{p} \leq .85) = P(-1.25 \leq z \leq 1.25) = P(-1.25 \leq z \leq 0) + P(0 \leq z \leq 1.25)$
 $= .3944 + .3944 = .7888$

```
normalcdf(.80-.05,.80+.05,.80,√(.80*.20/100))
.7887003221
```

Prob. = .7887

- b. We are to find the probability: $P(\hat{p} \leq p - .06) = P(\hat{p} \leq .80 - .06) = P(\hat{p} \leq .74)$
 For $\hat{p} = .74$: $z = (.74 - .80)/.040 = -1.50$
 $P(\hat{p} \leq .74) = P(z \leq -1.50) = .5 - P(-1.50 < z < 0) = .5 - .4332 = .0668$

```
normalcdf(-10^99,.80-.06,.80,√(.80*.20/100))
.0668072287
```

Prob. = .0668

- c. $P(\hat{p} \geq p + .07) = P(\hat{p} \geq .80 + .07) = P(\hat{p} \geq .87)$
 For $\hat{p} = .87$: $z = (.87 - .80)/.040 = 1.75$
 $P(\hat{p} \geq .87) = P(z \geq 1.75) = .5 - P(0 < z < 1.75) = .5 - .4599 = .0401$

```
normalcdf(.80+.07,10^99,.80,√(.80*.20/100))
.0400591135
```

Prob. = .0401

7.93 $\mu = 750$ hours, $\sigma = 55$ hours, and $n = 25$

$\mu_{\bar{x}} = \mu = 750$ hours and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 55/\sqrt{25} = 11$ hours

The sampling distribution of \bar{x} is normal because the population is normally distributed.

7.94 $\mu = \$438$, $\sigma = \$45$, $n = 100$, $n/N = 100/2480 = .04 < .05$

$\mu_{\bar{x}} = \mu = \$438$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 45/\sqrt{100} = \4.50

The sampling distribution of \bar{x} is approximately normal because $n > 30$.

7.95 $\mu = 750$ hours, $\sigma = 55$ hours, and $n = 25$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 55/\sqrt{25} = 11 \text{ hours}$$

- For $\bar{x} = 735$: $z = (735 - 750)/11 = -1.36$
 $P(\bar{x} > 735) = P(z > -1.36) = P(-1.36 < z < 0) + .5 = .4131 + .5 = .9131$
- For $\bar{x} = 725$: $z = (725 - 750)/11 = -2.27$
 For $\bar{x} = 740$: $z = (740 - 750)/11 = -.91$
 $P(725 < \bar{x} < 740) = P(-2.27 < z < -.91) = P(-2.27 < z < 0) - P(-.91 \leq z \leq 0)$
 $= .4884 - .3186 = .1698$
- $P(\bar{x} \text{ within 15 hours of } \mu) = P(735 \leq \bar{x} \leq 765)$
 For $\bar{x} = 735$: $z = (735 - 750)/11 = -1.36$
 For $\bar{x} = 765$: $z = (765 - 750)/11 = 1.36$
 $P(735 \leq \bar{x} \leq 765) = P(-1.36 \leq z \leq 1.36) = .4131 + .4131 = .8262$
- $P(\bar{x} \text{ lower than } \mu \text{ by 20 hours or more}) = P(\bar{x} \leq 730)$
 For $\bar{x} = 730$: $z = (730 - 750)/11 = -1.82$
 $P(\bar{x} \leq 730) = P(z \leq -1.82) = .5 - P(-1.82 < z < 0) = .5 - .4656 = .0344$

7.96 $\mu = \$438$, $\sigma = \$45$, and $n = 100$; $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 45/\sqrt{100} = \4.50

- For $\bar{x} = 433$: $z = (433 - 438)/4.50 = -1.11$
 $P(\bar{x} < 433) = P(z < -1.11) = .5 - P(-1.11 \leq z \leq 0) = .5 - .3665 = .1335$
- For $\bar{x} = 435$: $z = (435 - 438)/4.50 = -.67$
 For $\bar{x} = 440$: $z = (440 - 438)/4.50 = .44$
 $P(435 < \bar{x} < 440) = P(-.67 < z < .44) = P(-.67 < z < 0) + P(0 < z < .44)$
 $= .2486 + .1700 = .4186$
- $P(\bar{x} \text{ within } \$7 \text{ of } \mu) = P(\mu - 7 \leq \bar{x} \leq \mu + 7) = P(431 \leq \bar{x} \leq 445)$
 For $\bar{x} = 431$: $z = (431 - 438)/4.50 = -1.56$
 For $\bar{x} = 445$: $z = (445 - 438)/4.50 = 1.56$
 $P(431 \leq \bar{x} \leq 445) = P(-1.56 \leq z \leq 1.56) = P(-1.56 \leq z \leq 0) + P(0 \leq z \leq 1.56)$
 $= .4406 + .4406 = .8812$
- $P(\bar{x} \text{ greater than } \mu \text{ by } \$2.50 \text{ or more}) = P(\bar{x} \geq \mu + 2.50) = P(\bar{x} \geq 440.50)$
 For $\bar{x} = 440.50$: $z = (440.50 - 438)/4.50 = .56$
 $P(\bar{x} \geq 440.50) = P(z \geq .56) = .5 - P(0 < z < .56) = .5 - .2123 = .2877$

7.97 $\mu = 10$ hours, $\sigma = 2.1$ hours, and $n = 80$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.1/\sqrt{80} = .23478714 \text{ hours}$$

- For $\bar{x} = 10.45$: $z = (10.45 - 10)/.23478714 = 1.92$
 $P(\bar{x} > 10.45) = P(z > 1.92) = .5 - P(0 \leq z \leq 1.92) = .5 - .4726 = .0274$

```
normalcdf(10.45,
10^99,10,2.1/√(80))
.0276424282
```

Prob. = .0276

- b. For $\bar{x} = 9.75$: $z = (9.75 - 10)/.23478714 = -1.06$
 For $\bar{x} = 10.5$: $z = (10.50 - 10)/.23478714 = 2.13$
 $P(9.75 < \bar{x} < 10.50) = P(-1.06 < z < 2.13) = P(-1.06 < z < 0) + P(0 < z < 2.13)$
 $= .3554 + .4834 = .8388$

```
normalcdf(9.75,10.50,10,2.13/√(80))
)
)
.8399127134
```

Prob. = .8399

- c. $P(\mu - .25 \leq \bar{x} \leq \mu + .25) = P(9.75 \leq \bar{x} \leq 10.25)$
 For $\bar{x} = 9.75$: $z = (9.75 - 10)/.23478714 = -1.06$
 For $\bar{x} = 10.25$: $z = (10.25 - 10)/.23478714 = 1.06$
 $P(9.75 \leq \bar{x} \leq 10.25) = P(-1.06 \leq z \leq 1.06) = P(-1.06 \leq z \leq 0) + P(0 \leq z \leq 1.06)$
 $= .3554 + .3554 = .7108$

```
normalcdf(10-.25,10+.25,10,2.13/√(80))
)
)
.7130309048
```

Prob. = .7130

- d. $P(\bar{x} \leq \mu - .50) = P(\bar{x} \leq 9.50)$
 For $\bar{x} = 9.50$: $z = (9.50 - 10)/.23478714 = -2.13$
 $P(\bar{x} \leq 9.50) = P(z \leq -2.13) = .5 - P(-2.13 < z < 0) = .5 - .4834 = .0166$

```
normalcdf(-10^99,10-.50,10,2.13/√(80))
)
)
.016602739
```

Prob. = .0166

7.98 $\mu = 64$ ounces, $\sigma = .4$ ounces, and $n = 16$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = .4/\sqrt{16} = .10$$

$$\text{For } \bar{x} = 63.75: z = (63.75 - 64)/.10 = -2.50$$

$$\text{For } \bar{x} = 64.25: z = (64.25 - 64)/.10 = 2.50$$

$$P(\bar{x} < 63.75) + P(\bar{x} > 64.25) = P(z < -2.50) + P(z > 2.50) = (.5 - .4938) + (.5 - .4938)$$

$$= .0062 + .0062 = .0124$$

7.99 $p = .10$, $q = 1 - .10 = .90$, and $n = 80$

$$\mu_{\hat{p}} = p = .10 \text{ and } \sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.10(.90)/80} = .034$$

$$np = 80(.10) = 8 \text{ and } nq = 80(.90) = 72$$

Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.

7.100 $p = .70$, $q = 1 - .70 = .30$, and $n = 400$

$$\mu_{\hat{p}} = p = .70 \text{ and } \sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.70(.30)/400} = .023$$

$$np = 400(.70) = 280 \text{ and } nq = 400(.30) = 120$$

Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.

7.101 $p = .70$, $q = 1 - p = 1 - .70$, and $n = 400$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.70(.30)/400} = .02291288$$

- a.
 - i. For $\hat{p} = .65$: $z = (.65 - .70)/.02291288 = -2.18$
 $P(\hat{p} < .65) = P(z < -2.18) = P(-2.18 \leq z \leq 0) = .5 - .4854 = .0146$
 - ii. For $\hat{p} = .73$: $z = (.73 - .70)/.02291288 = 1.31$
 For $\hat{p} = .76$: $z = (.76 - .70)/.02291288 = 2.62$
 $P(.73 < \hat{p} < .76) = P(1.31 < z < 2.62) = P(0 < z < 2.62) - P(0 < z < 1.31)$
 $= .4956 - .4049 = .0907$
- b. $P(p - .06 \leq \hat{p} \leq p + .06) = P(.70 - .06 \leq \hat{p} \leq .70 + .06) = P(.64 \leq \hat{p} \leq .76)$
 For $\hat{p} = .64$: $z = (.64 - .70)/.02291288 = -2.62$
 For $\hat{p} = .76$: $z = (.76 - .70)/.02291288 = 2.62$
 $P(.64 \leq \hat{p} \leq .76) = P(-2.62 \leq z \leq 2.62) = .4956 + .4956 = .9912$
- c. $P(\hat{p} \geq p + .05) = P(\hat{p} \geq .70 + .05) = P(\hat{p} \geq .75)$
 For $\hat{p} = .75$: $z = (.75 - .70)/.02291288 = 2.18$
 $P(\hat{p} \geq .75) = P(z \geq 2.18) = .5 - P(0 < z < 2.18) = .5 - .4854 = .0146$

7.102 $p = .58$, $q = 1 - .58 = .42$, and $n = 100$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.58(.42)/100} = .04935585$$

- a. $P(\hat{p} \text{ is within } .08 \text{ of } p) = P(.50 \leq \hat{p} \leq .66)$
 For $\hat{p} = .50$: $z = (.50 - .58)/.04935585 = -1.62$
 For $\hat{p} = .66$: $z = (.66 - .58)/.04935585 = 1.62$
 $P(.50 \leq \hat{p} \leq .66) = P(-1.62 \leq z \leq 1.62) = .4474 + .4474 = .8948$
- b. $P(\hat{p} \text{ is not within } .08 \text{ of } p) = 1 - P(\hat{p} \text{ is within } .08 \text{ of } p) = 1 - .8948 \text{ (from part a)} = .1052$

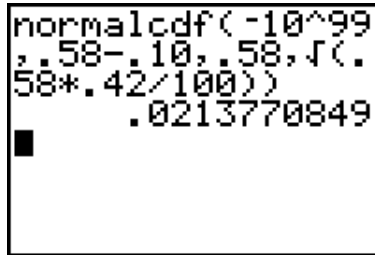
```
normalcdf(.58-.08, .58+.08, .58, sqrt(
.58*.42/100))
.8949570257
1-Ans
.1050429743
```

Part a. Prob. = .895

Part b. Prob. = $1 - .895 = .105$

- c. $P(\hat{p} \text{ is lower than } p \text{ by } .10 \text{ or more}) = P(\hat{p} \leq .48)$
 For $\hat{p} = .48$: $z = (.48 - .58)/.04935585 = -2.03$

$$P(\hat{p} \leq .48) = P(z \leq -2.03) = .5 - P(-2.03 < z < 0) = .5 - .4788 = .0212$$



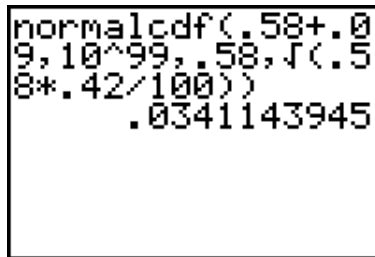
```
normalcdf(-10^99, .58, .58, sqrt(.58*.42/100))
.0213770849
```

Prob. = .0214

d. $P(\hat{p} \text{ is greater than } p \text{ by } .09 \text{ or more}) = P(\hat{p} \geq .67)$

For $\hat{p} = .67$: $z = (.67 - .58)/.04935585 = 1.82$

$$P(\hat{p} \geq .67) = P(z \geq 1.82) = .5 - P(0 < z < 1.82) = .5 - .4656 = .0344$$



```
normalcdf(.58+.09, 10^99, .58, sqrt(.58*.42/100))
.0341143945
```

Prob. = .0341

7.103 $\sigma = \$65,000$ and $n = 100$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{65,000}{\sqrt{100}} = \$6,500$$

The required probability is: $P(\mu - 10,000 \leq \bar{x} \leq \mu + 10,000)$

For $\bar{x} = \mu - 10,000$: $z = [(\mu - 10,000) - \mu]/6500 = -1.54$

For $\bar{x} = \mu + 10,000$: $z = [(\mu + 10,000) - \mu]/6500 = 1.54$

$$P(\mu - 10,000 \leq \bar{x} \leq \mu + 10,000) = P(-1.54 \leq z \leq 1.54) = .4382 + .4382 = .8764$$

7.104 Given $P(x > 90) = .15$ and $P(x < 65) = .30$

The corresponding z values are approximately $z = 1.04$ and $z = -.52$ respectively.

First we use $x = \mu + z\sigma$ to find σ .

We have $90 = \mu + 1.04\sigma$ and $65 = \mu - .52\sigma$

Subtracting, gives $25 = 1.56\sigma$, so $\sigma = 16.0256$

Since $65 = \mu - .52\sigma$, we have $65 = \mu - .52(16.0256)$ so $\mu = 65 + .52(16.0256) \approx 73.33$.

7.105 $\mu = c$, $\sigma = .8$ ppm

We want $P(\mu - .5 \leq \bar{x} \leq \mu + .5) = .95$

$1.96\sigma_{\bar{x}} = .5$ or $\sigma_{\bar{x}} = .255$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}, \text{ so } n \geq \frac{\sigma^2}{(\sigma_{\bar{x}})^2} = \frac{(.8)^2}{(.255)^2} = 9.84$$

Ten measurements are necessary.

- 7.106** a. $p = .6$, $np = 25 \cdot (.6) = 15$, $nq = 25 \cdot (.4) = 10$, so we can use the normal approximation to the binomial distribution.

$$\mu = np = 15, \sigma = \sqrt{npq} = \sqrt{25(.6)(.4)} = 2.44948974$$

$$z = \frac{12.5 - 15}{2.44948974} = -1.02$$

$$P(x > 12.5) = P(z > -1.02) = .8461$$

- b. $P(z > -1.65) = .9505$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}}, \text{ so } \sigma_{\hat{p}} = \frac{\hat{p} - p}{z} = \frac{.5 - .6}{-1.65} = .0606$$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}, \text{ hence } n = \frac{pq}{(\sigma_{\hat{p}})^2} = \frac{.6(.4)}{(.0606)^2} = 65.34$$

The reporter should take a sample of 66 voters or more.

- 7.107** a. $p = .53$, $n = 200$, and we assume that $\frac{n}{N} \leq .05$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.53(.47)}{200}} = .03529164$$

The shape of the sampling distribution is approximately normal.

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.50 - .53}{.03529164} = -.85$$

$$P(\hat{p} \geq .50) = P(z \geq -.85) = .5 + .3023 = .8023$$

- b. $P(z > -1.65) = .9505$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}}, \text{ so } \sigma_{\hat{p}} = \frac{\hat{p} - p}{z} = \frac{.5 - .53}{-1.65} = .01818182$$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}, \text{ hence } n = \frac{pq}{(\sigma_{\hat{p}})^2} = \frac{(.53)(.47)}{(.01818182)^2} = 753.53$$

The sample should include at least 754 voters.

- 7.108** $\mu = 290$ feet, $\sigma = 10$ feet, and $n = 3$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{3} = 5.77350269 \text{ feet}$$

$$P(\text{total length of three throws exceeds } 885) = P(\text{mean length of three throws exceeds } 885/3) \\ = P(\bar{x} > 295)$$

$$\text{For } \bar{x} = 295: z = (295 - 290)/5.77350269 = .87$$

$$P(\bar{x} > 295) = P(z > .87) = .5 - P(0 \leq z \leq .87) = .5 - .3078 = .1922$$

- 7.109** $\mu = 160$ pounds, $\sigma = 25$ pounds, $n = 35$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 25/\sqrt{35} = 4.22577127$$

Since $n > 30$, \bar{x} is approximately normally distributed.

$$P(\text{sum of 35 weights exceeds } 6000 \text{ pounds}) = P(\text{mean weight exceeds } 6000/35) = P(\bar{x} > 171.43)$$

$$\text{For } \bar{x} = 171.43: z = (171.43 - 160)/4.22577127 = 2.70$$

$$P(\bar{x} > 171.43) = P(z > 2.70) = .5 - P(0 \leq z \leq 2.70) = .5 - .4965 = .0035$$

Self-Review Test for Chapter Seven

1. b 2. b 3. a 4. a 5. b 6. b
 7. c 8. a 9. a 10. a 11. c 12. a

13. According to the central limit theorem, for a large sample size, the sampling distribution of the sample mean is approximately normal irrespective of the shape of the population distribution. The mean and standard deviation of the sampling distribution of the sample mean are:

$$\mu_{\bar{x}} = \mu \text{ and } \sigma_{\bar{x}} = \sigma/\sqrt{n}$$

The sample size is usually considered to be large if $n \geq 30$.

From the same theorem, the sampling distribution of \hat{p} is approximately normal for large samples. In the case of proportion, the sample is large if $np > 5$ and $nq > 5$.

14. $\mu = 145$ pounds and $\sigma = 18$ pounds

a. $\mu_{\bar{x}} = \mu = 145$ pounds and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{25} = 3.60$ pounds

b. $\mu_{\bar{x}} = \mu = 145$ pounds and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{100} = 1.80$ pounds

In both cases the sampling distribution of \bar{x} is approximately normal because the population has an approximate normal distribution.

15. $\mu = 47$ minutes and $\sigma = 8.4$ minutes

a. $\mu_{\bar{x}} = \mu = 47$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 8.4/\sqrt{20} = 1.878$ minutes

Since the population has an unknown distribution and $n < 30$, we can draw no conclusion about the shape of the sampling distribution of \bar{x} .

b. $\mu_{\bar{x}} = \mu = 47$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 8.4/\sqrt{70} = 1.004$ minutes

Since $n > 30$, the sampling distribution of \bar{x} is approximately normal.

16. $\mu = 694$ minutes, $\sigma = 140$ minutes, and $n = 60$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 140/\sqrt{60} = 18.07392228 \text{ minutes}$$

a. For $\bar{x} = 660$: $z = (660 - 694)/18.07392228 = -1.88$

For $\bar{x} = 680$: $z = (680 - 694)/18.07392228 = -.77$

$$P(660 < x < 680) = P(-1.88 < z < -.77) = P(-1.88 < z < 0) - P(-.77 \leq z \leq 0) \\ = .4699 - .2794 = .1905$$

b. For $\bar{x} = 730$: $z = (730 - 694)/18.07392228 = 1.99$

$$P(x > 730) = P(z > 1.99) = 5 - P(0 \leq z \leq 1.99) = .5 - .4767 = .0233$$

c. For $\bar{x} = 715$: $z = (715 - 694)/18.07392228 = -1.16$

$$P(\bar{x} < 715) = P(z < -1.16) = 5 + P(0 < z < 1.16) = .5 + .3770 = .8770$$

d. For $\bar{x} = 675$: $z = (675 - 694)/18.07392228 = -1.05$

For $\bar{x} = 725$: $z = (725 - 694)/18.07392228 = 1.72$

$$P(675 < x < 725) = P(-1.05 < z < 1.72) = P(-1.05 < z < 0) + P(0 < z < 1.72) \\ = .3531 + .4573 = .8104$$

17. $\mu = 16$ ounces, $\sigma = .18$ ounces, and $n = 16$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = .18/\sqrt{16} = .045$$

- a. i. For $\bar{x} = 15.90$: $z = (15.90 - 16)/.045 = -2.22$
 For $\bar{x} = 15.95$: $z = (15.95 - 16)/.045 = -1.11$
 $P(15.90 < \bar{x} < 15.95) = P(-2.22 < z < -1.11) = P(-2.22 < z < 0) - P(-1.11 < z < 0)$
 $= .4868 - .3665 = .1203$
- ii. For $\bar{x} = 15.95$: $z = (15.95 - 16)/.045 = -1.11$
 $P(\bar{x} < 15.95) = P(z < -1.11) = .5 - P(-1.11 \leq z \leq 0) = .5 - .3665 = .1335$
- iii. For $\bar{x} = 15.97$: $z = (15.97 - 16)/.045 = -.67$
 $P(\bar{x} > 15.97) = P(z > -.67) = P(-.67 < z < 0) + .5 = .2486 + .5 = .7486$
- b. $P(16 - .10 \leq \bar{x} \leq 16 + .10) = P(15.90 \leq \bar{x} \leq 16.10)$
 For $\bar{x} = 15.90$: $z = (15.90 - 16)/.045 = -2.22$
 For $\bar{x} = 16.10$: $z = (16.10 - 16)/.045 = 2.22$
 $P(15.90 \leq \bar{x} \leq 16.10) = P(-2.22 \leq z \leq 2.22) = .4868 + .4868 = .9736$
- c. $P(\bar{x} \leq 16 - .135) = P(\bar{x} \leq 15.865)$
 For $\bar{x} = 15.865$: $z = (15.865 - 16)/.045 = -3.00$
 $P(\bar{x} \leq 15.865) = P(z \leq -3.00) = .5 - P(-3.00 \leq z \leq 0) = .5 - .4987 = .0013$

18. $p = .08$ and $q = 1 - .08 = .92$

- a. $n = 50$, $\mu_{\hat{p}} = p = .08$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.08(.92)/50} = .038$
 $np = 50(.08) = 4$
 Since $np < 5$, the central limit theorem does not apply and we can draw no conclusion about the shape of the sampling distribution of \hat{p} .
- b. $n = 200$, $\mu_{\hat{p}} = p = .08$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.08(.92)/200} = .019$
 $np = 200(.08) = 16$ and $nq = 200(.92) = 184$
 Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.
- c. $n = 1800$, $\mu_{\hat{p}} = p = .08$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.08(.92)/1800} = .006$
 $np = 1800(.08) = 144$ and $nq = 1800(.92) = 1656$
 Since np and nq are both greater than 5, the sampling distribution of \hat{p} is approximately normal.

19. $p = .70$, $q = 1 - .70 = .30$, and $n = 400$

$$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{.70(.30)/400} = .02291288$$

- a. i. For $\hat{p} = .73$: $z = (.73 - .70)/.02291288 = 1.31$
 $P(\hat{p} > .73) = P(z > 1.31) = .5 - P(0 \leq z \leq 1.31) = .5 - .4049 = .0951$
- ii. For $\hat{p} = .66$: $z = (.66 - .70)/.02291288 = -1.75$
 For $\hat{p} = .74$: $z = (.74 - .70)/.02291288 = 1.75$
 $P(.66 < \hat{p} < .74) = P(-1.75 < z < 1.75) = P(-1.75 < z < 0) + P(0 < z < 1.75)$
 $= .4599 + .4599 = .9198$
- iii. For $\hat{p} = .73$: $z = (.73 - .70)/.02291288 = 1.31$
 $P(\hat{p} < .73) = P(z < 1.31) = .5 + P(0 < z < 1.31) = .5 + .4049 = .9049$
- iv. For $\hat{p} = .65$: $z = (.65 - .70)/.02291288 = -2.18$
 For $\hat{p} = .68$: $z = (.68 - .70)/.02291288 = -.87$
 $P(.65 < \hat{p} < .68) = P(-2.18 < z < -.87) = P(-2.18 < z < 0) - P(-.87 \leq z \leq 0)$
 $= .4854 - .3078 = .1776$
- b. $P(.70 - .05 \leq \hat{p} \leq .70 + .05) = P(.65 \leq \hat{p} \leq .75)$

For $\hat{p} = .65$: $z = (.65 - .70)/.02291288 = -2.18$

For $\hat{p} = .75$: $z = (.75 - .70)/.02291288 = 2.18$

$$\begin{aligned} P(.65 \leq \hat{p} \leq .75) &= P(-2.18 \leq z \leq 2.18) = P(-2.18 \leq z \leq 0) + P(0 \leq z \leq 2.18) \\ &= .4854 + .4854 = .9708 \end{aligned}$$

c. $P(\hat{p} \leq .70 - .04) = P(\hat{p} \leq .66)$

For $\hat{p} = .66$: $z = (.66 - .70)/.02291288 = -1.75$

$$P(\hat{p} \leq .66) = P(z \leq -1.75) = .5 - P(-1.75 < z < 0) = .5 - .4599 = .0401$$

d. $P(\hat{p} \geq .70 + .03) = P(\hat{p} \geq .73)$

For $\hat{p} = .73$: $z = (.73 - .70)/.02291288 = 1.31$

$$P(\hat{p} \geq .73) = P(z \geq 1.31) = .5 - P(0 < z < 1.31) = .5 - .4049 = .0951$$