10

ESTIMATION AND HYPOTHESIS TESTING: TWO POPULATIONS

Chapters 8 and 9 discussed the estimation and hypothesis-testing procedures for μ and p involving a single population. This chapter extends the discussion of estimation and hypothesis-testing procedures to the difference between two population means and the difference between two population proportions. For example, we may want to make a confidence interval for the difference between mean prices of houses in California and in New York. Or we may want to test the hypothesis that the mean price of houses in California is different from that in New York. As another example, we may want to make a confidence interval for the difference between the proportions of all male and female adults who abstain from drinking. Or we may want to test the hypothesis that the proportion of all adult men who abstain from drinking is different from the proportion of all adult women who abstain from drinking. Constructing confidence intervals and testing hypotheses about population parameters are referred to as *making inferences*.

10.1 INFERENCES ABOUT THE DIFFERENCE BETWEEN TWO POPULATION MEANS FOR LARGE AND INDEPENDENT SAMPLES

Let μ_1 be the mean of the first population and μ_2 be the mean of the second population. Suppose we want to make a confidence interval and test a hypothesis about the difference between these two population means, that is, $\mu_1 - \mu_2$. Let $\bar{x}_1$ be the mean of a sample taken from the first population and $\bar{x}_2$ be the mean of a sample taken from the second population. Then, $\bar{x}_1 - \bar{x}_2$ is the sample statistic that is used to make an interval estimate and to test a hypothesis about $\mu_1 - \mu_2$. This section discusses how to make confidence intervals and test hypotheses about $\mu_1 - \mu_2$ when the two samples are large and independent. As discussed in earlier chapters, in the case of μ, a sample is considered to be large if it contains 30 or more observations. The concept of independent and dependent samples is explained next.

10.1.1 INDEPENDENT VERSUS DEPENDENT SAMPLES

Two samples are **independent** if they are drawn from two different populations and the elements of one sample have no relationship to the elements of the second sample. If the elements of the two samples are somehow related, then the samples are said to be **dependent**. Thus, in two independent samples, the selection of one sample has no effect on the selection of the second sample.

> **INDEPENDENT VERSUS DEPENDENT SAMPLES** Two samples drawn from two populations are *independent* if the selection of one sample from one population does not affect the selection of the second sample from the second population. Otherwise, the samples are *dependent.*

Examples 10–1 and 10–2 illustrate independent and dependent samples, respectively.

Illustrating two independent samples.

EXAMPLE 10–1 Suppose we want to estimate the difference between the mean salaries of all male and all female executives. To do so, we draw two samples, one from the population of male executives and another from the population of female executives. These two samples are *independent* because they are drawn from two different populations, and the samples have no effect on each other. ■

Illustrating two dependent samples.

EXAMPLE 10–2 Suppose we want to estimate the difference between the mean weights of all participants before and after a weight loss program. To accomplish this, suppose we take a sample of 40 participants and measure their weights before and after the completion of this program. Note that these two samples include the same 40 participants. This is an example of two *dependent* samples. Such samples are also called *paired* or *matched samples.* ■

This section and Sections 10.2, 10.3, and 10.5 discuss how to make confidence intervals and test hypotheses about the difference between two population parameters when samples are independent. Section 10.4 discusses how to make confidence intervals and test hypotheses about the difference between two population means when samples are dependent.

10.1.2 MEAN, STANDARD DEVIATION, AND SAMPLING DISTRIBUTION OF $\bar{x}_1 - \bar{x}_2$

Suppose we draw two (independent) large samples from two different populations that are referred to as population 1 and population 2. Let

μ_1 = the mean of population 1

μ_2 = the mean of population 2

σ_1 = the standard deviation of population 1

σ_2 = the standard deviation of population 2

n_1 = the size of the sample drawn from population 1 ($n_1 \geq 30$)

n_2 = the size of the sample drawn from population 2 ($n_2 \geq 30$)

$\bar{x}_1$ = the mean of the sample drawn from population 1

$\bar{x}_2$ = the mean of the sample drawn from population 2

Then, from the central limit theorem, $\bar{x}_1$ is approximately normally distributed with mean μ_1 and standard deviation $\sigma_1/\sqrt{n_1}$, and $\bar{x}_2$ is approximately normally distributed with mean μ_2 and standard deviation $\sigma_2/\sqrt{n_2}$.

Using these results, we can make the following statements about the mean, the standard deviation, and the shape of the sampling distribution of $\bar{x}_1 - \bar{x}_2$. Figure 10.1 shows the sampling distribution of $\bar{x}_1 - \bar{x}_2$.

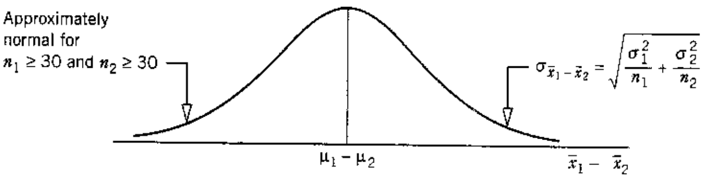

Figure 10.1

1. The mean of $\bar{x}_1 - \bar{x}_2$, denoted by $\mu_{\bar{x}_1 - \bar{x}_2}$, is

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$$

2. The standard deviation of $\bar{x}_1 - \bar{x}_2$, denoted by $\sigma_{\bar{x}_1 - \bar{x}_2}$, is[1]

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

3. Regardless of the shapes of the two populations, the shape of the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal. This is so because the difference between two normally distributed random variables is also normally distributed. Note again that for this to hold true, both samples must be large.

[1]The formula for the standard deviation of $\bar{x}_1 - \bar{x}_2$ can also be written as

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2}$$

where $\sigma_{\bar{x}_1} = \sigma_1/\sqrt{n_1}$ and $\sigma_{\bar{x}_2} = \sigma_2/\sqrt{n_2}$.

SAMPLING DISTRIBUTION, MEAN, AND STANDARD DEVIATION OF $\bar{x}_1 - \bar{x}_2$ For two large and independent samples selected from two different populations, the *sampling distribution* of $\bar{x}_1 - \bar{x}_2$ is (approximately) normal with its *mean* and *standard deviation* as follows:

$$\mu_{\bar{x}_1-\bar{x}_2} = \mu_1 - \mu_2 \quad \text{and} \quad \sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

However, we usually do not know the standard deviations σ_1 and σ_2 of the two populations. In such cases, we replace $\sigma_{\bar{x}_1-\bar{x}_2}$ by its point estimator $s_{\bar{x}_1-\bar{x}_2}$, which is calculated as follows.

ESTIMATE OF THE STANDARD DEVIATION OF $\bar{x}_1 - \bar{x}_2$ The value of $s_{\bar{x}_1-\bar{x}_2}$, which gives an *estimate* of $\sigma_{\bar{x}_1-\bar{x}_2}$, is calculated as

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where s_1 and s_2 are the standard deviations of the two samples selected from the two populations.

Thus, when both samples are large, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal. Consequently, in such cases, we use the normal distribution to make a confidence interval and to test a hypothesis about $\mu_1 - \mu_2$.

10.1.3 INTERVAL ESTIMATION OF $\mu_1 - \mu_2$

By constructing a confidence interval for $\mu_1 - \mu_2$, we find the difference between the means of two populations. For example, we may want to find the difference between the mean heights of male and female adults. The difference between the two sample means, $\bar{x}_1 - \bar{x}_2$, is the point estimator of the difference between the two population means, $\mu_1 - \mu_2$. Again, in this section we assume that the two samples are large and independent. When these assumptions hold true, we use the normal distribution to make a confidence interval for the difference between the two population means. The following formula gives the interval estimation for $\mu_1 - \mu_2$.

CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$

The $(1 - \alpha)100\%$ *confidence interval for* $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm z\sigma_{\bar{x}_1-\bar{x}_2} \quad \text{if } \sigma_1 \text{ and } \sigma_2 \text{ are known}$$

$$(\bar{x}_1 - \bar{x}_2) \pm z s_{\bar{x}_1-\bar{x}_2} \quad \text{if } \sigma_1 \text{ and } \sigma_2 \text{ are not known}$$

The value of z is obtained from the normal distribution table for the given confidence level. The values of $\sigma_{\bar{x}_1-\bar{x}_2}$ and $s_{\bar{x}_1-\bar{x}_2}$ are calculated as explained earlier.

Examples 10–3 and 10–4 illustrate the procedure to construct a confidence interval for $\mu_1 - \mu_2$ for large samples. In Example 10–3 the population standard deviations are known, and in Example 10–4 they are not known.

Constructing a confidence interval for $\mu_1 - \mu_2$: σ_1 and σ_2 known.

EXAMPLE 10–3 According to the American Medical Association, the average annual earnings of radiologists in the United States are \$230,000 and those of surgeons are \$225,000 (*Fortune*, September 27, 1999). Suppose that these means are based on random samples of 300 radiologists and 400 surgeons and that the population standard deviations of the annual earnings of radiologists and surgeons are \$28,000 and \$32,000, respectively.

(a) What is the point estimate of $\mu_1 - \mu_2$? What is the margin of error?

(b) Construct a 97% confidence interval for the difference between the mean annual earnings of radiologists and surgeons.

Solution Let us refer to all radiologists as population 1 and all surgeons as population 2. The respective samples, then, are samples 1 and 2. Let μ_1 and μ_2 be the mean annual earnings of populations 1 and 2, and let $\bar{x}_1$ and $\bar{x}_2$ be the means of the respective samples. From the given information,

For radiologists: $n_1 = 300$, $\bar{x}_1 = \$230{,}000$, $\sigma_1 = \$28{,}000$

For surgeons: $n_2 = 400$, $\bar{x}_2 = \$225{,}000$, $\sigma_2 = \$32{,}000$

(a) The point estimate of $\mu_1 - \mu_2$ is given by the value of $\bar{x}_1 - \bar{x}_2$. Thus,

$$\text{Point estimate of } \mu_1 - \mu_2 = \$230{,}000 - \$225{,}000 = \mathbf{\$5000}$$

To find the margin of error, first we calculate $\sigma_{\bar{x}_1 - \bar{x}_2}$ as follows:

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{(28{,}000)^2}{300} + \frac{(32{,}000)^2}{400}} = \$2274.496281$$

Then,

$$\text{Margin of error} = \pm 1.96\sigma_{\bar{x}_1 - \bar{x}_2} = \pm 1.96(2274.496281) = \mathbf{\pm\$4458.01}$$

(b) The confidence level is $1 - \alpha = .97$. In part a we calculated the standard deviation of $\bar{x}_1 - \bar{x}_2$ as

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \$2274.496281$$

Next, we find the z value for the 97% confidence level. From the normal distribution table, this value of z is 2.17. Finally, substituting all the values in the confidence interval formula, we obtain the 97% confidence interval for $\mu_1 - \mu_2$:

$$(\bar{x}_1 - \bar{x}_2) \pm z\sigma_{\bar{x}_1 - \bar{x}_2} = (230{,}000 - 225{,}000) \pm 2.17(2274.496281)$$
$$= 5000 \pm 4935.66 = \mathbf{\$64.34 \text{ to } \$9935.66}$$

Thus, with 97% confidence we can state that the difference between the mean annual earnings of radiologists and surgeons is \$64.34 to \$9935.66. ■

Constructing a confidence interval for $\mu_1 - \mu_2$: σ_1 and σ_2 not known.

EXAMPLE 10–4 According to the most recent estimates by the U.S. Department of Labor, the average retirement age in the United States is 62.7 years for women and 62.2 years for men. Assume that these means are based on samples of 800 women and 1000 men and that the sample standard deviations for the two samples are 4.5 years and 3 years, respectively.

Find a 99% confidence interval for the difference between the corresponding population means.

Solution Let μ_1 be the mean retirement age for women and μ_2 the mean retirement age for men. Let $\bar{x}_1$ and $\bar{x}_2$ be the means of the respective samples. From the given information,

For women:	$n_1 = 800$,	$\bar{x}_1 = 62.7$ years,	$s_1 = 4.5$ years
For men:	$n_2 = 1000$,	$\bar{x}_2 = 62.2$ years,	$s_2 = 3$ years

The confidence level is $1 - \alpha = .99$.

Because σ_1 and σ_2 are not known, we use $s_{\bar{x}_1 - \bar{x}_2}$ in the confidence interval formula. The value of $s_{\bar{x}_1 - \bar{x}_2}$ is

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(4.5)^2}{800} + \frac{(3)^2}{1000}} = .18523634$$

From the normal distribution table, the z value for a 99% confidence level is (approximately) 2.58. The 99% confidence interval for $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm z s_{\bar{x}_1 - \bar{x}_2} = (62.7 - 62.2) \pm 2.58(.18523634)$$
$$= .5 \pm .48 = \mathbf{.02 \text{ to } .98 \text{ year}}$$

Thus, with 99% confidence we can state that the difference between the two population means is .02 to .98 year. ∎

10.1.4 HYPOTHESIS TESTING ABOUT $\mu_1 - \mu_2$

It is often necessary to compare the means of two populations. For example, we may want to know if the mean price of houses in Chicago is the same as that in Los Angeles. Similarly, we may be interested in knowing if, on average, American children spend fewer hours in school than Japanese children do. In both these cases, we will perform a test of hypothesis about $\mu_1 - \mu_2$. The alternative hypothesis in a test of hypothesis may be that the means of the two populations are different, or that the mean of the first population is greater than the mean of the second population, or that the mean of the first population is less than the mean of the second population. These three situations are described next.

1. Testing an alternative hypothesis that the means of two populations are different is equivalent to $\mu_1 \neq \mu_2$, which is the same as $\mu_1 - \mu_2 \neq 0$.
2. Testing an alternative hypothesis that the mean of the first population is greater than the mean of the second population is equivalent to $\mu_1 > \mu_2$, which is the same as $\mu_1 - \mu_2 > 0$.
3. Testing an alternative hypothesis that the mean of the first population is less than the mean of the second population is equivalent to $\mu_1 < \mu_2$, which is the same as $\mu_1 - \mu_2 < 0$.

The procedure followed to perform a test of hypothesis about the difference between two population means is similar to the one used to test hypotheses about single population parameters in Chapter 9. The procedure involves the same five steps that were used in Chapter 9 to test hypotheses about μ and p. Because we are dealing with large (and independent) samples in this section, we will use the normal distribution to conduct a test of hypothesis about $\mu_1 - \mu_2$.

TEST STATISTIC z FOR $\bar{x}_1 - \bar{x}_2$ The value of the *test statistic z for $\bar{x}_1 - \bar{x}_2$* is computed as

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

The value of $\mu_1 - \mu_2$ is substituted from H_0. If the values of σ_1 and σ_2 are not known, we replace $\sigma_{\bar{x}_1 - \bar{x}_2}$ by $s_{\bar{x}_1 - \bar{x}_2}$ in the formula.

Making a two-tailed test of hypothesis about $\mu_1 - \mu_2$: large samples.

EXAMPLE 10-5 Refer to Example 10–3 about the mean annual earnings of radiologists and surgeons. Test at the 1% significance level whether the mean annual earnings of radiologists and surgeons are different.

Solution From the information given in Example 10–3,

For radiologists: $n_1 = 300$, $\bar{x}_1 = \$230{,}000$, $\sigma_1 = \$28{,}000$

For surgeons: $n_2 = 400$, $\bar{x}_2 = \$225{,}000$, $\sigma_2 = \$32{,}000$

Let μ_1 and μ_2 be the mean annual earnings of radiologists and surgeons, respectively.

Step 1. *State the null and alternative hypotheses.*

We are to test whether the two population means are different. The two possibilities are:

(i) The mean annual earnings of radiologists and surgeons are not different. In other words, $\mu_1 = \mu_2$, which can be written as $\mu_1 - \mu_2 = 0$.

(ii) The mean annual earnings of radiologists and surgeons are different. That is, $\mu_1 \neq \mu_2$, which can be written as $\mu_1 - \mu_2 \neq 0$.

From these two possibilities, the null and alternative hypotheses are

H_0: $\mu_1 - \mu_2 = 0$ (The two population means are not different)

H_1: $\mu_1 - \mu_2 \neq 0$ (The two population means are different)

Step 2. *Select the distribution to use.*

Because $n_1 > 30$ and $n_2 > 30$, both samples are large. Therefore, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal, and we use the normal distribution to perform the hypothesis test.

Step 3. *Determine the rejection and nonrejection regions.*

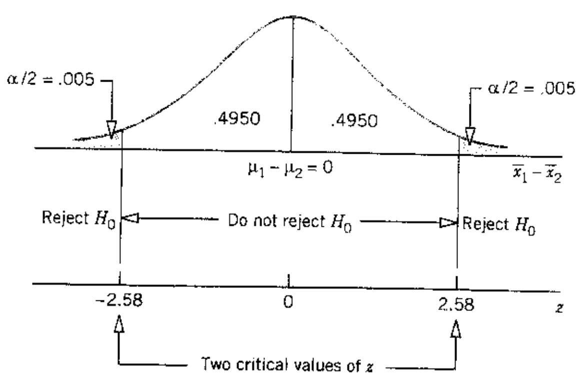

Figure 10.2

The significance level is given to be .01. The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed. The area in each tail of the normal distribution curve is $\alpha/2 = .01/2 = .005$. The critical values of z for .005 area in each tail of the normal distribution curve are (approximately) 2.58 and -2.58. These values are shown in Figure 10.2.

Step 4. *Calculate the value of the test statistic.*

The value of the test statistic z for $\bar{x}_1 - \bar{x}_2$ is computed as follows:

$$\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{(28{,}000)^2}{300} + \frac{(32{,}000)^2}{400}} = \$2274.496281$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1-\bar{x}_2}} = \frac{(230{,}000 - 225{,}000) - \overset{\text{From } H_0}{0}}{2274.496281} = 2.20$$

Step 5. *Make a decision.*

Because the value of the test statistic $z = 2.20$ falls in the nonrejection region, we fail to reject the null hypothesis H_0. Therefore, we conclude that the mean annual earnings of radiologists and surgeons are not different. Note that we cannot be sure that the two population means are not different. All we can say is that the evidence from the two samples indicates that the corresponding population means are not different. ■

Making a right-tailed test of hypothesis about $\mu_1 - \mu_2$: large samples.

EXAMPLE 10-6 Refer to Example 10–4 about the mean retirement ages for women and men. Test at the 2.5% significance level whether the mean retirement age for all women is higher than the mean retirement age for all men.

Solution From the information given in Example 10–4,

For women:	$n_1 = 800,$	$\bar{x}_1 = 62.7$ years,	$s_1 = 4.5$ years
For men:	$n_2 = 1000,$	$\bar{x}_2 = 62.2$ years,	$s_2 = 3$ years

Let μ_1 and μ_2 be the mean retirement ages for women and men, respectively.

Step 1. *State the null and alternative hypotheses.*

The two possibilities are:

(i) The mean retirement age for all women is not higher than that for all men, which can be written as $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$.

(ii) The mean retirement age for all women is higher than that for all men, which can be written as $\mu_1 > \mu_2$ or $\mu_1 - \mu_2 > 0$.

The null and alternative hypotheses are

H_0: $\mu_1 - \mu_2 = 0$ (μ_1 is equal to μ_2)

H_1: $\mu_1 - \mu_2 > 0$ (μ_1 is greater than μ_2)

Step 2. *Select the distribution to use.*

Because $n_1 > 30$ and $n_2 > 30$, both samples are large. Therefore, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal, and we use the normal distribution to perform the hypothesis test.

Step 3. Determine the rejection and nonrejection regions.

The significance level is given to be .025. The > sign in the alternative hypothesis indicates that the test is right-tailed. Consequently, the critical value of z is 1.96, as shown in Figure 10.3.

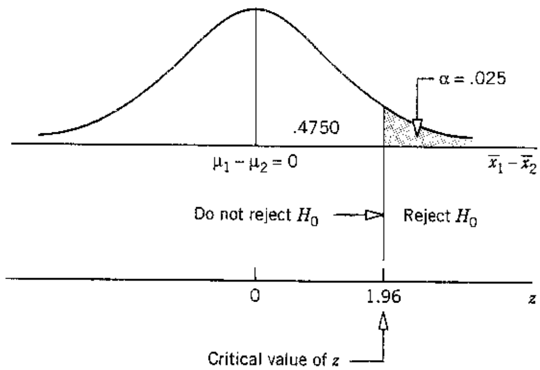

Figure 10.3

Step 4. Calculate the value of the test statistic.

The value of the test statistic z for $\bar{x}_1 - \bar{x}_2$ is computed as follows:

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(4.5)^2}{800} + \frac{(3)^2}{1000}} = .18523634$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}} = \frac{(62.7 - 62.2) - \overset{\text{From } H_0}{0}}{.18523634} = 2.70$$

Step 5. Make a decision.

Because the value of the test statistic $z = 2.70$ falls in the rejection region, we reject the null hypothesis H_0. Therefore, we conclude that the mean retirement age for women is higher than the mean retirement age for men. ∎

EXERCISES

■ Concepts and Procedures

10.1 Briefly explain the meaning of independent and dependent samples. Give one example of each.

10.2 Describe the sampling distribution of $\bar{x}_1 - \bar{x}_2$ for large and independent samples. What are the mean and standard deviation of this sampling distribution?

10.3 The following information is obtained from two independent samples selected from two populations.

$$n_1 = 240 \qquad \bar{x}_1 = 5.56 \qquad s_1 = 1.65$$
$$n_2 = 270 \qquad \bar{x}_2 = 4.80 \qquad s_2 = 1.58$$

a. What is the point estimate of $\mu_1 - \mu_2$?

b. Construct a 99% confidence interval for $\mu_1 - \mu_2$.

10.4 The following information is obtained from two independent samples selected from two populations.

$$n_1 = 300 \qquad \bar{x}_1 = 22.0 \qquad s_1 = 4.9$$
$$n_2 = 250 \qquad \bar{x}_2 = 27.6 \qquad s_2 = 4.5$$

a. What is the point estimate of $\mu_1 - \mu_2$?
b. Construct a 95% confidence interval for $\mu_1 - \mu_2$.

10.5 Refer to the information given in Exercise 10.3. Test at the 5% significance level if the two population means are different.

10.6 Refer to the information given in Exercise 10.4. Test at the 1% significance level if the two population means are different.

10.7 Refer to the information given in Exercise 10.3. Test at the 1% significance level if μ_1 is greater than μ_2.

10.8 Refer to the information given in Exercise 10.4. Test at the 5% significance level if μ_1 is less than μ_2.

Applications

10.9 According to the U.S. Bureau of the Census, the average annual salaries of men and women over 25 years of age who hold full-time year-round jobs but do not have a high school diploma are \$24,604 and \$17,129, respectively (*USA TODAY*, September 11, 1998). Suppose that these mean salaries are based on random samples of 200 men and 210 women, and further assume that the sample standard deviation is \$3350 for the annual salaries of such men and \$3270 for the annual salaries of such women.

a. Let μ_1 and μ_2 be the population means of annual salaries for such men and women, respectively. Find the point estimate of $\mu_1 - \mu_2$ and its margin of error.
b. Find a 99% confidence interval for $\mu_1 - \mu_2$.
c. Using the 1% level of significance, can you conclude that the mean annual salaries for all such men and for all such women are different?

10.10 According to the U.S. Bureau of the Census, the average annual salaries of men and women over 25 years of age who hold full-time year-round jobs and have a bachelor's degree are \$53,102 and \$37,304, respectively (*USA TODAY*, September 11, 1998). Suppose that these means are based on random samples of 108 men and 102 women, and further assume that the sample standard deviation is \$5040 for the annual salaries of such men and \$4910 for the annual salaries of such women.

a. Let μ_1 and μ_2 be the population means of annual salaries for such men and women, respectively. Find the point estimate of $\mu_1 - \mu_2$ and its margin of error.
b. Find a 97% confidence interval for $\mu_1 - \mu_2$.
c. Test at the 2% significance level whether the mean annual salary for all such men is different from the mean annual salary for all such women.

10.11 According to data published in *Newsweek* magazine, in 1998 the average starting salary for history majors was \$26,820 and the starting salary for psychology majors was \$25,689 (*Newsweek*, February 1, 1999). Suppose that these mean starting salaries are based on random samples of 105 history majors and 111 psychology majors, and further assume that the standard deviations for the starting salaries of these majors were \$3850 and \$3710, respectively, in 1998.

a. Let μ_1 and μ_2 be the 1998 mean starting salaries for all history and all psychology majors, respectively. What are the point estimate of $\mu_1 - \mu_2$ and its margin of error?
b. Find a 90% confidence interval for $\mu_1 - \mu_2$.
c. Test at the 1% significance level whether the 1998 mean starting salary for all history majors exceeded that for all psychology majors.

10.12 According to the U.S. Department of Labor, the average hourly earnings of workers in the manufacturing sector were \$13.40 in July 1998, and the corresponding average for workers in wholesale trade was \$13.98 (*Bureau of Labor Statistics News*, August 18, 1998). Suppose that these means are based on random samples of 1100 workers selected from the manufacturing sector and 1200 workers selected from wholesale trade. Assume that the sample standard deviations for the hourly earnings of workers in the manufacturing sector and in wholesale trade were \$1.10 and \$1.06, respectively, in July 1998.

a. Let μ_1 and μ_2 be the population means of hourly earnings for workers in the manufacturing sector and in wholesale trade, respectively. What are the point estimate of $\mu_1 - \mu_2$ and its margin of error?
b. Construct a 97% confidence interval for $\mu_1 - \mu_2$.
c. Using the 2% significance level, can you conclude that the July 1998 mean hourly earnings of all workers in the manufacturing sector were less than those of all workers in wholesale trade?

10.13 A business consultant wanted to investigate if providing day-care facilities on premises by companies reduces the absentee rate of working mothers with 6-year-old or younger children. She took a sample of 45 such mothers from companies that provide day-care facilities on premises. These mothers missed an average of 6.4 days from work last year with a standard deviation of 1.20 days. Another sample of 50 such mothers taken from companies that do not provide day-care facilities on premises showed that these mothers missed an average of 9.3 days last year with a standard deviation of 1.85 days.

a. Construct a 98% confidence interval for the difference between the two population means.
b. Using the 2.5% significance level, can you conclude that the mean number of days missed per year by mothers working for companies that provide day-care facilities on premises is less than the mean number of days missed per year by mothers working for companies that do not provide day-care facilities on premises?
c. What are the Type I error and its probability for the test of hypothesis in part b? Explain.

10.14 According to the Centers for Disease Control and Prevention/National Center for Health Statistics, the average length of stay in short-stay hospitals was 5.8 days for men and 4.9 days for women (*Advance Data*, August 31, 1998). Suppose that these means were based on random samples of 110 such stays for men and 124 such stays for women. Assume that the population standard deviations were 2.1 days for men and 2.0 days for women.

a. Construct a 99% confidence interval for the difference between the two population means.
b. Using the 1% significance level, can you conclude that the mean stay in such hospitals for all men is longer than that for all women?
c. What are the Type I error and its probability for the hypothesis test in part b? Explain.

10.15 Psychologists Claudia Mueller and Carol Dweck studied the motivational effects of praising students for work well done. In six studies involving 412 fifth-graders, these researchers found that telling students they were smart when they performed a task well tended to lower their motivation for future assignments. When students were told that their success was due to their efforts, however, they were more likely to work hard in the future (*The Hartford Courant*, August 22, 1998). Suppose that 100 fifth-graders who passed a test are randomly divided into two groups of 50 each. Students in the first group are told that they did well because they are intelligent, whereas students in the second group are told that their success was the result of hard work. Subsequently, these 100 students are given another test. The first group's mean score is 72 with a standard deviation of 12 and the second group's mean score is 80 with a standard deviation of 10. Assume that these 100 students are representative of all fifth-graders.

a. Construct a 95% confidence interval for the difference in the mean scores for the populations of the two groups.
b. Using the 2.5% level of significance, can you conclude that the mean score on such a test for all fifth-graders who are praised for their intelligence is lower than that of all fifth-graders who are praised for their efforts?
c. What would your decision in part b be if the probability of making a Type I error were zero? Explain.

10.16 According to IRS data, taxpayers with adjusted annual gross incomes of \$60,000 to \$75,000 averaged \$6486 in deductions for medical expenses, whereas those with adjusted annual gross incomes of \$75,001 to \$100,000 averaged \$6291 in such deductions (*USA TODAY*, March 5, 1999). Suppose that these means were obtained from random samples of 1000 taxpayers in each income group. Further assume that the sample standard deviations for such deductions were \$1600 for the taxpayers in the first income group and \$1550 for the taxpayers in the second income group.

a. Construct a 99% confidence interval for the difference between the two population means.
b. Using the 1% significance level, can you conclude that the mean deductions for medical expenses are different for the two income groups?
c. What would your decision in part b be if the probability of making a Type I error were zero? Explain.

10.17 The management at the New Century Bank claims that the mean waiting time for all customers at its branches is less than that at the Public Bank, which is its main competitor. A business consulting firm took a sample of 200 customers from the New Century Bank and found that they waited an average of 4.5 minutes with a standard deviation of 1.2 minutes before being served. Another sample of 300 customers taken from the Public Bank showed that these customers waited an average of 4.75 minutes with a standard deviation of 1.5 minutes before being served.

a. Make a 97% confidence interval for the difference between the two population means.
b. Test at the 2.5% significance level whether the claim of the management of the New Century Bank is true.
*c. Calculate the p-value for the test of part b. Based on this p-value, would you reject the null hypothesis if $\alpha = .01$? What if $\alpha = .05$?

10.18 Maine Mountain Dairy claims that its 8-ounce low-fat yogurt cups contain, on average, fewer calories than the 8-ounce low-fat yogurt cups produced by a competitor. A consumer agency wanted to check this claim. A sample of 50 such yogurt cups produced by this company showed that they contained an average of 141 calories per cup with a standard deviation of 5.5 calories. A sample of 40 such yogurt cups produced by its competitor showed that they contained an average of 144 calories per cup with a standard deviation of 6.4 calories.

a. Make a 98% confidence interval for the difference between the mean number of calories in the 8-ounce low-fat yogurt cups produced by the two companies.
b. Test at the 1% significance level whether Maine Mountain Dairy's claim is true.
*c. Calculate the p-value for the test of part b. Based on this p-value, would you reject the null hypothesis if $\alpha = .005$? What if $\alpha = .025$?

10.2 INFERENCES ABOUT THE DIFFERENCE BETWEEN TWO POPULATION MEANS FOR SMALL AND INDEPENDENT SAMPLES: EQUAL STANDARD DEVIATIONS

Many times, due to either budget constraints or the nature of the populations, it may not be possible to take large samples to make inferences about the difference between two population means. This section discusses how to make a confidence interval and test a hypothesis about the difference between two population means when the samples are small ($n_1 < 30$ and $n_2 < 30$) and independent. Our main assumption in this case is that the two populations from which the two samples are drawn are (approximately) normally distributed. If this assumption is true and we know the population standard deviations, we can still use the normal distribution to make inferences about $\mu_1 - \mu_2$ when samples are small and independent. However, we usually do not know the population standard deviations σ_1 and σ_2. In such cases, we replace the normal distribution by the t distribution to make inferences about $\mu_1 - \mu_2$ for small and independent samples. We will make one more assumption in this section that the standard deviations of the two populations are equal. In other words, we assume that although σ_1 and σ_2 are unknown, they are equal. The case when σ_1 and σ_2 are not equal will be discussed in Section 10.3.

WHEN TO USE THE t DISTRIBUTION TO MAKE INFERENCES ABOUT $\mu_1 - \mu_2$ The t distribution is used to make inferences about $\mu_1 - \mu_2$ when the following assumptions hold true:

1. The two populations from which the two samples are drawn are (approximately) normally distributed.
2. The samples are small ($n_1 < 30$ and $n_2 < 30$) and independent.
3. The standard deviations σ_1 and σ_2 of the two populations are unknown but they are assumed to be equal; that is, $\sigma_1 = \sigma_2$.

When the standard deviations of the two populations are equal, we can use σ for both σ_1 and σ_2. Since σ is unknown, we replace it by its point estimator s_p, which is called the **pooled sample standard deviation** (hence, the subscript p). The value of s_p is computed by using the information from the two samples as follows.

POOLED STANDARD DEVIATION FOR TWO SAMPLES The *pooled standard deviation for two samples* is computed as

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

where n_1 and n_2 are the sizes of the two samples and s_1^2 and s_2^2 are the variances of the two samples.

In this formula, $n_1 - 1$ are the degrees of freedom for sample 1, $n_2 - 1$ are the degrees of freedom for sample 2, and $n_1 + n_2 - 2$ are the *degrees of freedom for the two samples taken together.*

When s_p is used as an estimator of σ, the standard deviation $\sigma_{\bar{x}_1-\bar{x}_2}$ of $\bar{x}_1 - \bar{x}_2$ is estimated by $s_{\bar{x}_1-\bar{x}_2}$. The value of $s_{\bar{x}_1-\bar{x}_2}$ is calculated by using the following formula.

ESTIMATOR OF THE STANDARD DEVIATION OF $\bar{x}_1 - \bar{x}_2$ The *estimator of the standard deviation of $\bar{x}_1 - \bar{x}_2$* is

$$s_{\bar{x}_1-\bar{x}_2} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Now we are ready to discuss the procedures that are used to make confidence intervals and test hypotheses about $\mu_1 - \mu_2$ for small and independent samples selected from two populations with unknown but equal standard deviations.

10.2.1 INTERVAL ESTIMATION OF $\mu_1 - \mu_2$

As was mentioned earlier, the difference between the two sample means, $\bar{x}_1 - \bar{x}_2$, is the point estimator of the difference between the two population means, $\mu_1 - \mu_2$. The following formula gives the confidence interval for $\mu_1 - \mu_2$ when the t distribution is used.

CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$

The $(1-\alpha)100\%$ *confidence interval for* $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t s_{\bar{x}_1 - \bar{x}_2}$$

where the value of t is obtained from the t distribution table for the given confidence level and $n_1 + n_2 - 2$ degrees of freedom, and $s_{\bar{x}_1 - \bar{x}_2}$ is calculated as explained on page 441.

Example 10–7 describes the procedure to make a confidence interval for $\mu_1 - \mu_2$ using the t distribution.

Constructing a confidence interval for $\mu_1 - \mu_2$: small and independent samples and $\sigma_1 = \sigma_2$.

EXAMPLE 10–7 A consumer agency wanted to estimate the difference in the mean amounts of caffeine in two brands of coffee. The agency took a sample of 15 one-pound jars of Brand I coffee that showed the mean amount of caffeine in these jars to be 80 milligrams per jar with a standard deviation of 5 milligrams. Another sample of 12 one-pound jars of Brand II coffee gave a mean amount of caffeine equal to 77 milligrams per jar with a standard deviation of 6 milligrams. Construct a 95% confidence interval for the difference between the mean amounts of caffeine in one-pound jars of these two brands of coffee. Assume that the two populations are normally distributed and that the standard deviations of the two populations are equal.

Solution Let μ_1 and μ_2 be the mean amounts of caffeine per jar in all one-pound jars of Brands I and II, respectively, and let $\bar{x}_1$ and $\bar{x}_2$ be the means of the two respective samples. From the given information,

$$\begin{array}{lll} n_1 = 15 & \bar{x}_1 = 80 \text{ milligrams} & s_1 = 5 \text{ milligrams} \\ n_2 = 12 & \bar{x}_2 = 77 \text{ milligrams} & s_2 = 6 \text{ milligrams} \end{array}$$

The confidence level is $1 - \alpha = .95$.

First we calculate the standard deviation of $\bar{x}_1 - \bar{x}_2$ as follows:

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(15-1)(5)^2 + (12-1)(6)^2}{15+12-2}} = 5.46260011$$

$$s_{\bar{x}_1 - \bar{x}_2} = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = (5.46260011)\sqrt{\frac{1}{15} + \frac{1}{12}} = 2.11565593$$

Next, to find the t value from the t distribution table, we need to know the area in each tail of the t distribution curve and the degrees of freedom.

$$\text{Area in each tail} = \alpha/2 = .5 - (.95/2) = .025$$

$$\text{Degrees of freedom} = n_1 + n_2 - 2 = 15 + 12 - 2 = 25$$

The t value for $df = 25$ and .025 area in the right tail of the t distribution curve is 2.060. The 95% confidence interval for $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t s_{\bar{x}_1 - \bar{x}_2} = (80 - 77) \pm 2.060(2.11565593)$$

$$= 3 \pm 4.36 = \mathbf{-1.36 \text{ to } 7.36 \text{ milligrams}}$$

Thus, with 95% confidence we can state that based on these two sample results, the difference in the mean amounts of caffeine in one-pound jars of these two brands of coffee lies between -1.36 and 7.36 milligrams. Because the lower limit of the interval is negative, it is possible that the mean amount of caffeine is greater in the second brand than in the first brand of coffee.

Note that the value of $\bar{x}_1 - \bar{x}_2$, which is $80 - 77 = 3$, gives the point estimate of $\mu_1 - \mu_2$.

10.2.2 HYPOTHESIS TESTING ABOUT $\mu_1 - \mu_2$

When the three assumptions mentioned in Section 10.2 are satisfied, the t distribution is applied to make a hypothesis test about the difference between two population means. The test statistic in this case is t, which is calculated as follows.

TEST STATISTIC t FOR $\bar{x}_1 - \bar{x}_2$ The value of the *test statistic t for $\bar{x}_1 - \bar{x}_2$* is computed as

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

The value of $\mu_1 - \mu_2$ in this formula is substituted from the null hypothesis and $s_{\bar{x}_1 - \bar{x}_2}$ is calculated as explained on page 441.

Examples 10–8 and 10–9 illustrate how a test of hypothesis about the difference between two population means for small and independent samples that are selected from two populations with equal standard deviations is conducted using the t distribution.

Making a two-tailed test of hypothesis about $\mu_1 - \mu_2$: small and independent samples and $\sigma_1 = \sigma_2$

EXAMPLE 10–8 A sample of 14 cans of Brand I diet soda gave the mean number of calories of 23 per can with a standard deviation of 3 calories. Another sample of 16 cans of Brand II diet soda gave the mean number of calories of 25 per can with a standard deviation of 4 calories. At the 1% significance level, can you conclude that the mean numbers of calories per can are different for these two brands of diet soda? Assume that the calories per can of diet soda are normally distributed for each of the two brands and that the standard deviations for the two populations are equal.

Solution Let μ_1 and μ_2 be the mean number of calories per can for diet soda of Brand I and Brand II, respectively, and let $\bar{x}_1$ and $\bar{x}_2$ be the means of the respective samples. From the given information,

$$n_1 = 14 \quad \bar{x}_1 = 23 \quad s_1 = 3$$
$$n_2 = 16 \quad \bar{x}_2 = 25 \quad s_2 = 4$$

The significance level is $\alpha = .01$.

Step 1. *State the null and alternative hypotheses.*

We are to test for the difference in the mean numbers of calories per can for the two brands. The null and alternative hypotheses are

H_0: $\mu_1 - \mu_2 = 0$ (The mean numbers of calories are not different)

H_1: $\mu_1 - \mu_2 \neq 0$ (The mean numbers of calories are different) times spent watching television by children have a normal distribution for both populations and that the standard deviations for the two populations are equal.

Solution Let the children from New York State be referred to as population 1 and those from California as population 2. Let μ_1 and μ_2 be the mean time spent watching television by children in populations 1 and 2, respectively, and let $\bar{x}_1$ and $\bar{x}_2$ be the mean time spent watching television by children in the respective samples. From the given information,

$$n_1 = 15 \qquad \bar{x}_1 = 28.5 \text{ hours} \qquad s_1 = 4 \text{ hours}$$
$$n_2 = 16 \qquad \bar{x}_2 = 23.25 \text{ hours} \qquad s_2 = 5 \text{ hours}$$

The significance level is $\alpha = .025$.

Step 1. *State the null and alternative hypotheses.*

The two possible decisions are:

(i) The mean time spent watching television by children in New York State is not greater than that for children in California. This can be written as $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$.

(ii) The mean time spent watching television by children in New York State is greater than that for children in California. This can be written as $\mu_1 > \mu_2$ or $\mu_1 - \mu_2 > 0$.

Hence, the null and alternative hypotheses are

$$H_0\colon \mu_1 - \mu_2 = 0$$
$$H_1\colon \mu_1 - \mu_2 > 0$$

Note that the null hypothesis can also be written as $\mu_1 - \mu_2 \le 0$.

Step 2. *Select the distribution to use.*

The two populations are normally distributed, the samples are small and independent, and the standard deviations of the two populations are unknown but equal. Consequently, we use the t distribution to make the test.

Step 3. *Determine the rejection and nonrejection regions.*

The > sign in the alternative hypothesis indicates that the test is right-tailed. The significance level is .025.

$$\text{Area in the right tail of the } t \text{ distribution} = \alpha = .025$$

$$\text{Degrees of freedom} = n_1 + n_2 - 2 = 15 + 16 - 2 = 29$$

From the t distribution table, the critical value of t for $df = 29$ and .025 area in the right tail of the t distribution is 2.045. This value is shown in Figure 10.5.

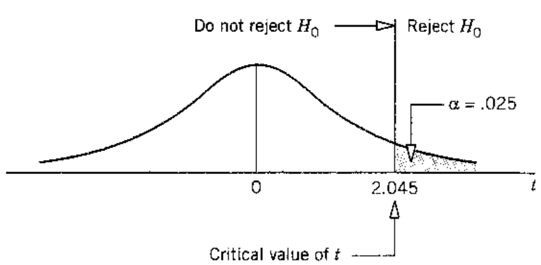

Figure 10.5

Step 4. *Calculate the value of the test statistic.*

The value of the test statistic t for $\bar{x}_1 - \bar{x}_2$ is computed as follows:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(15 - 1)(4)^2 + (16 - 1)(5)^2}{15 + 16 - 2}} = 4.54479619$$

$$s_{\bar{x}_1 - \bar{x}_2} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = (4.54479619)\sqrt{\frac{1}{15} + \frac{1}{16}} = 1.63338904$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}} = \frac{(28.5 - 23.25) - \overset{\text{From } H_0}{0}}{1.63338904} = 3.214$$

Step 5. *Make a decision.*

Because the value of the test statistic $t = 3.214$ for $\bar{x}_1 - \bar{x}_2$ falls in the rejection region, we reject the null hypothesis H_0. Hence, we conclude that children in New York State spend, on average, more time watching TV than children in California. ■

EXERCISES

■ Concepts and Procedures

10.19 Explain what conditions must hold true to use the t distribution to make a confidence interval and to test a hypothesis about $\mu_1 - \mu_2$ for two independent samples selected from two populations with unknown but equal standard deviations.

10.20 The following information was obtained from two independent samples selected from two normally distributed populations with unknown but equal standard deviations.

$$n_1 = 25 \quad \bar{x}_1 = 12.50 \quad s_1 = 3.75$$
$$n_2 = 20 \quad \bar{x}_2 = 14.60 \quad s_2 = 3.10$$

a. What is the point estimate of $\mu_1 - \mu_2$?
b. Construct a 95% confidence interval for $\mu_1 - \mu_2$.

10.21 The following information was obtained from two independent samples selected from two normally distributed populations with unknown but equal standard deviations.

$$n_1 = 18 \quad \bar{x}_1 = 33.75 \quad s_1 = 5.25$$
$$n_2 = 20 \quad \bar{x}_2 = 28.50 \quad s_2 = 4.55$$

a. What is the point estimate of $\mu_1 - \mu_2$?
b. Construct a 99% confidence interval for $\mu_1 - \mu_2$.

10.22 Refer to the information given in Exercise 10.20. Test at the 5% significance level if the two population means are different.

10.23 Refer to the information given in Exercise 10.21. Test at the 1% significance level if the two population means are different.

10.24 Refer to the information given in Exercise 10.20. Test at the 1% significance level if μ_1 is less than μ_2.

10.25 Refer to the information given in Exercise 10.21. Test at the 5% significance level if μ_1 is greater than μ_2.

10.26 The following information was obtained from two independent samples selected from two normally distributed populations with unknown but equal standard deviations.

Sample 1:	27	39	25	33	21	35	30	26	25	31	35	30	28
Sample 2:	24	28	23	25	24	22	29	26	29	28	19	29	

a. Let μ_1 be the mean of population 1 and μ_2 be the mean of population 2. What is the point estimate of $\mu_1 - \mu_2$?

b. Construct a 98% confidence interval for $\mu_1 - \mu_2$.

c. Test at the 1% significance level if μ_1 is greater than μ_2.

10.27 The following information was obtained from two independent samples selected from two normally distributed populations with unknown but equal standard deviations.

Sample 1:	13	14	9	12	8	10	5	10	9	12	16	
Sample 2:	16	18	11	19	14	17	13	16	17	18	22	12

a. Let μ_1 be the mean of population 1 and μ_2 be the mean of population 2. What is the point estimate of $\mu_1 - \mu_2$?

b. Construct a 99% confidence interval for $\mu_1 - \mu_2$.

c. Test at the 2.5% significance level if μ_1 is lower than μ_2.

■ Applications

10.28 The management of a supermarket wanted to investigate whether the male customers spend less money, on average, than the female customers. A sample of 25 male customers who shopped at this supermarket showed that they spent an average of \$80 with a standard deviation of \$17.50. Another sample of 20 female customers who shopped at the same supermarket showed that they spent an average of \$96 with a standard deviation of \$14.40. Assume that the amounts spent at this supermarket by all male and all female customers are normally distributed with equal but unknown population standard deviations.

a. Construct a 99% confidence interval for the difference between the mean amounts spent by all male and all female customers at this supermarket.

b. Using the 5% significance level, can you conclude that the mean amount spent by all male customers at this supermarket is less than that of all female customers?

10.29 A manufacturing company is interested in buying one of two different kinds of machines. The company tested the two machines for production purposes. The first machine was run for 8 hours. It produced an average of 126 items per hour with a standard deviation of 9 items. The second machine was run for 10 hours. It produced an average of 117 items per hour with a standard deviation of 6 items. Assume that the production per hour for each machine is (approximately) normally distributed. Further assume that the standard deviations of the hourly productions of the two populations are equal.

a. Make a 95% confidence interval for the difference between the two population means.

b. Using the 2.5% significance level, can you conclude that the mean number of items produced per hour by the first machine is greater than that of the second machine?

10.30 An insurance company wants to know if the average speed at which men drive cars is greater than that of women drivers. The company took a random sample of 27 cars driven by men on a highway and found the mean speed to be 72 miles per hour with a standard deviation of 2.2 miles per hour. Another sample of 18 cars driven by women on the same highway gave a mean speed of 68 miles per hour with a standard deviation of 2.5 miles per hour. Assume that the speeds at which all men and all women drive cars on this highway are both normally distributed with the same population standard deviation.

a. Construct a 98% confidence interval for the difference between the mean speeds of cars driven by all men and all women on this highway.

b. Test at the 1% significance level whether the mean speed of cars driven by all men drivers on this highway is greater than that of cars driven by all women drivers.

10.31 According to the U.S. Census Bureau, the mean annual salary for men over 25 years old with a doctoral degree is \$86,436, whereas that for women of the same age and with the same degree is \$62,169 (*USA TODAY*, September 11, 1998). Suppose that these means are based on random samples of 27 men and 25 women and that the sample standard deviations are \$5210 for men and \$5080 for women. Further assume that the annual salaries of all such men and women have normal distributions with equal but unknown standard deviations.

a. Construct a 90% confidence interval for the difference between the mean annual salaries for all such men and all such women.

b. Test at the 5% significance level whether the mean annual salaries for all such men and all such women are different.

10.32 Quadro Corporation has two supermarket stores in a city. The company's quality control department wanted to check if the customers are equally satisfied with the service provided at these two stores. A sample of 25 customers selected from Supermarket I produced a mean satisfaction index of 7.6 (on a scale of 1 to 10, 1 being the lowest and 10 being the highest) with a standard deviation of .75. Another sample of 28 customers selected from Supermarket II produced a mean satisfaction index of 8.1 with a standard deviation of .59. Assume that the customer satisfaction index for each supermarket has a normal distribution with the same population standard deviation.

a. Construct a 98% confidence interval for the difference between the mean satisfaction indexes for all customers for the two supermarkets.

b. Test at the 1% significance level whether the mean satisfaction indexes for all customers for the two supermarkets are different.

10.33 A company claims that its medicine, Brand A, provides faster relief from pain than another company's medicine, Brand B. A researcher tested both brands of medicine on two groups of randomly selected patients. The results of the test are given in the following table. The mean and standard deviation of relief times are in minutes.

Brand	Sample Size	Mean of Relief Times	Standard Deviation of Relief Times
A	25	44	11
B	22	49	9

a. Construct a 99% confidence interval for the difference between the mean relief times for the two brands of medicine.

b. Test at the 1% significance level whether the mean relief time for Brand A is less than that for Brand B.

Assume that the two populations are normally distributed with equal standard deviations.

10.34 A study of time spent on housework found that employed married men spent an average of 8.8 hours per week on housework, whereas employed unmarried men averaged 6.9 hours per week on housework (America's Use of Time Project, University of Maryland, *American Demographics*, November 1998). Suppose that these means were based on random samples of 21 employed married men and 23 employed unmarried men with sample standard deviations of 2.2 hours and 1.9 hours, respectively. Assume that the two populations from which these samples were selected are normally distributed with equal but unknown standard deviations.

a. Construct a 90% confidence interval for the difference between the population means for these two groups of men.

b. Test at the 5% significance level whether the mean time spent on housework per week is greater for the population of employed married men than for the population of employed unmarried men.

10.35 According to data from the College Board, the average cost for room and board at public four-year residential colleges for 1997–1998 was \$4361 and the corresponding average cost for private colleges was \$5549 (*The Chronicle of Higher Education Almanac*, August 28, 1998). Suppose that these means were obtained from random samples of 15 public and 14 private four-year residential colleges. Further assume that the sample standard deviations of such costs were \$1200 for public colleges and

\$1540 for private colleges. Assume that the 1997–1998 room and board costs for both groups of colleges are normally distributed with equal but unknown standard deviations.

a. Construct a 95% confidence interval for the difference between the two population means.

b. Using the 1% significance level, can you conclude that the 1997–1998 mean cost of room and board at all public four-year residential colleges was lower than the 1997–1998 mean of such costs at all private four-year residential colleges?

10.3 INFERENCES ABOUT THE DIFFERENCE BETWEEN TWO POPULATION MEANS FOR SMALL AND INDEPENDENT SAMPLES: UNEQUAL STANDARD DEVIATIONS

Section 10.2 explained how to make inferences about the difference between two population means using the t distribution when the standard deviations of the two populations are unknown but equal and certain other assumptions hold true. Now, what if all other assumptions of Section 10.2 hold true but the population standard deviations are not only unknown but also unequal? In this case, the procedures used to make confidence intervals and to test hypotheses about $\mu_1 - \mu_2$ remain similar to the ones we learned in Sections 10.2.1 and 10.2.2 except for two differences. When the population standard deviations are unknown and not equal, the degrees of freedom are no longer given by $n_1 + n_2 - 2$ and the standard deviation of $\bar{x}_1 - \bar{x}_2$ is not calculated using the pooled standard deviation s_p.

DEGREES OF FREEDOM If

1. the two populations from which the samples are drawn are (approximately) normally distributed
2. the two samples are small (that is, $n_1 < 30$ and $n_2 < 30$) and independent
3. the two population standard deviations are unknown and unequal

then the t distribution is used to make inferences about $\mu_1 - \mu_2$ and the *degrees of freedom* for the t distribution are given by

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

The number given by this formula is always rounded down for df.

Because the standard deviations of the two populations are not known, we use $s_{\bar{x}_1 - \bar{x}_2}$ as a point estimator of $\sigma_{\bar{x}_1 - \bar{x}_2}$. The following formula is used to calculate the standard deviation $s_{\bar{x}_1 - \bar{x}_2}$ of $\bar{x}_1 - \bar{x}_2$.

ESTIMATE OF THE STANDARD DEVIATION OF $\bar{x}_1 - \bar{x}_2$ The value of $s_{\bar{x}_1 - \bar{x}_2}$ is calculated as

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

10.3.1 INTERVAL ESTIMATION OF $\mu_1 - \mu_2$

Again, the difference between the two sample means, $\bar{x}_1 - \bar{x}_2$, is the point estimator of the difference between the two population means, $\mu_1 - \mu_2$. The following formula gives the confidence interval for $\mu_1 - \mu_2$ when the t distribution is used and the population standard deviations are unknown and presumed to be unequal.

CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$ The $(1 - \alpha)100\%$ *confidence interval for* $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t s_{\bar{x}_1 - \bar{x}_2}$$

where the value of t is obtained from the t distribution table for a given confidence level and the degrees of freedom given by the formula mentioned on page 449, and $s_{\bar{x}_1 - \bar{x}_2}$ is calculated as explained on the same page.

Example 10–10 describes how to construct a confidence interval for $\mu_1 - \mu_2$ when the standard deviations of the two populations are unknown and unequal.

Constructing a confidence interval for $\mu_1 - \mu_2$: small and independent samples and $\sigma_1 \neq \sigma_2$.

EXAMPLE 10–10 According to Example 10–7 of Section 10.2.1, a sample of 15 one-pound jars of coffee of Brand I showed that the mean amount of caffeine in these jars is 80 milligrams per jar with a standard deviation of 5 milligrams. Another sample of 12 one-pound coffee jars of Brand II gave a mean amount of caffeine equal to 77 milligrams per jar with a standard deviation of 6 milligrams. Construct a 95% confidence interval for the difference between the mean amounts of caffeine in one-pound coffee jars of these two brands. Assume that the two populations are normally distributed and that the standard deviations of the two populations are not equal.

Solution Let μ_1 and μ_2 be the mean amount of caffeine per jar in all one-pound jars of Brands I and II, respectively, and let $\bar{x}_1$ and $\bar{x}_2$ be the means of the two respective samples.

From the given information,

$$n_1 = 15 \quad \bar{x}_1 = 80 \text{ milligrams} \quad s_1 = 5 \text{ milligrams}$$
$$n_2 = 12 \quad \bar{x}_2 = 77 \text{ milligrams} \quad s_2 = 6 \text{ milligrams}$$

The confidence level is $1 - \alpha = .95$.

First, we calculate the standard deviation of $\bar{x}_1 - \bar{x}_2$ as follows:

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(5)^2}{15} + \frac{(6)^2}{12}} = 2.16024690$$

Next, to find the t value from the t distribution table, we need to know the area in each tail of the t distribution curve and the degrees of freedom.

$$\text{Area in each tail} = \alpha/2 = .5 - (.95/2) = .025$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(\frac{(5)^2}{15} + \frac{(6)^2}{12}\right)^2}{\frac{\left(\frac{(5)^2}{15}\right)^2}{15 - 1} + \frac{\left(\frac{(6)^2}{12}\right)^2}{12 - 1}} = 21.42 \approx 21$$

Note that the degrees of freedom are always rounded down as in this calculation. From the

t distribution table, the t value for $df = 21$ and .025 area in the right tail of the t distribution curve is 2.080. The 95% confidence interval for $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm ts_{\bar{x}_1 - \bar{x}_2} = (80 - 77) \pm 2.080(2.16024690)$$
$$= 3 \pm 4.49 = \mathbf{-1.49 \text{ to } 7.49}$$

Thus, with 95% confidence we can state that based on these two sample results, the difference in the mean amounts of caffeine in one-pound jars of these two brands of coffee is between -1.49 and 7.49 milligrams. ■

Comparing this confidence interval with the one obtained in Example 10–7, we observe that the two confidence intervals are very close. From this we can conclude that even if the standard deviations of the two populations are not equal and we use the procedure of Section 10.2.1 to make a confidence interval for $\mu_1 - \mu_2$, the margin of error will be small as long as the difference between the two standard deviations is not too large.

10.3.2 HYPOTHESIS TESTING ABOUT $\mu_1 - \mu_2$

When the standard deviations of the two populations are unknown and unequal, with the other conditons of Section 10.2.2 holding true, we use the t distribution to make a test of hypothesis about $\mu_1 - \mu_2$. This procedure differs from the one in Section 10.2.2 only in the calculation of degrees of freedom for the t distribution and the standard deviation of $\bar{x}_1 - \bar{x}_2$. The df and the standard deviation of $\bar{x}_1 - \bar{x}_2$ in this case are given by the formulas on page 449.

TEST STATISTIC t FOR $\bar{x}_1 - \bar{x}_2$ The value of the *test statistic t for* $\bar{x}_1 - \bar{x}_2$ is computed as

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

The value of $\mu_1 - \mu_2$ in this formula is substituted from the null hypothesis and $s_{\bar{x}_1 - \bar{x}_2}$ is calculated as explained earlier.

Example 10–11 illustrates the procedure used to conduct a test of hypothesis about $\mu_1 - \mu_2$ when the standard deviations of the two populations are unknown and unequal.

Making a two-tailed test of hypothesis about $\mu_1 - \mu_2$: small and independent samples and $\sigma_1 \neq \sigma_2$.

EXAMPLE 10–11 According to Example 10–8 of Section 10.2.2, a sample of 14 cans of Brand I diet soda gave the mean number of calories per can as 23 with a standard deviation of 3 calories. Another sample of 16 cans of Brand II diet soda gave the mean number of calories of 25 per can with a standard deviation of 4 calories. Test at the 1% significance level whether the mean numbers of calories per can of diet soda are different for these two brands. Assume that the calories per can of diet soda are normally distributed for each of these two brands and that the standard deviations for the two populations are not equal.

Solution Let μ_1 and μ_2 be the mean number of calories for all cans of diet soda of Brand I and Brand II, respectively, and let $\bar{x}_1$ and $\bar{x}_2$ be the means of the respective samples. From the given information,

$$n_1 = 14 \quad \bar{x}_1 = 23 \quad s_1 = 3$$
$$n_2 = 16 \quad \bar{x}_2 = 25 \quad s_2 = 4$$

The significance level is $\alpha = .01$.

Step 1. State the null and alternative hypotheses.

We are to test for the difference in the mean numbers of calories per can for the two brands. The null and alternative hypotheses are

H_0: $\mu_1 - \mu_2 = 0$ (The mean numbers of calories are not different)

H_1: $\mu_1 - \mu_2 \neq 0$ (The mean numbers of calories are different)

Step 2. Select the distribution to use.

The two populations are normally distributed, the samples are small and independent, and the standard deviations of the two populations are unknown and unequal. Consequently, we use the t distribution to make the test.

Step 3. Determine the rejection and nonrejection regions.

The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed. The significance level is .01. Hence,

$$\text{Area in each tail} = \alpha/2 = .01/2 = .005$$

The degrees of freedom are calculated as follows:

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(\dfrac{(3)^2}{14} + \dfrac{(4)^2}{16}\right)^2}{\dfrac{\left(\dfrac{(3)^2}{14}\right)^2}{14 - 1} + \dfrac{\left(\dfrac{(4)^2}{16}\right)^2}{16 - 1}} = 27.41 \approx 27$$

From the t distribution table, the critical values of t for $df = 27$ and .005 area in each tail of the t distribution curve are -2.771 and 2.771. These values are shown in Figure 10.6.

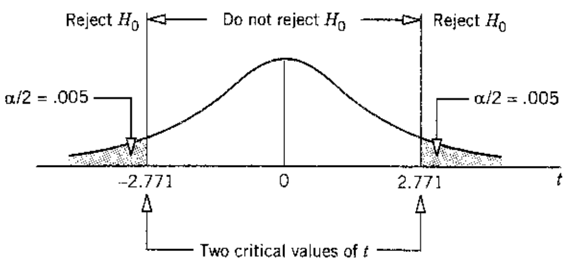

Figure 10.6

Step 4. Calculate the value of the test statistic.

The value of the test statistic t for $\bar{x}_1 - \bar{x}_2$ is computed as follows:

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(3)^2}{14} + \frac{(4)^2}{16}} = 1.28173989$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}} = \frac{(23 - 25) - \overset{\text{From } H_0}{0}}{1.28173989} = -1.560$$

Step 5. Make a decision.

Because the value of the test statistic $t = -1.560$ for $\bar{x}_1 - \bar{x}_2$ falls in the nonrejection region, we fail to reject the null hypothesis. Hence, there is no difference in the mean numbers of calories per can for the two brands of diet soda. The difference in $\bar{x}_1$ and $\bar{x}_2$ observed for the two samples may have occurred due to sampling error only. ■

☞ *Remember* The degrees of freedom for the procedures to make a confidence interval and to test a hypothesis about $\mu_1 - \mu_2$ learned in Sections 10.3.1 and 10.3.2 are always rounded down.

EXERCISES

■ Concepts and Procedures

10.36 Assuming that the two populations are normally distributed with unequal and unknown population standard deviations, construct a 95% confidence interval for $\mu_1 - \mu_2$ for the following.

$$n_1 = 24 \quad \bar{x}_1 = 20.50 \quad s_1 = 3.90$$
$$n_2 = 16 \quad \bar{x}_2 = 22.60 \quad s_2 = 5.15$$

10.37 Assuming that the two populations are normally distributed with unequal and unknown population standard deviations, construct a 99% confidence interval for $\mu_1 - \mu_2$ for the following.

$$n_1 = 15 \quad \bar{x}_1 = 52.61 \quad s_1 = 3.55$$
$$n_2 = 19 \quad \bar{x}_2 = 43.75 \quad s_2 = 5.40$$

10.38 Refer to Exercise 10.36. Test at the 5% significance level if the two population means are different.

10.39 Refer to Exercise 10.37. Test at the 1% significance level if the two population means are different.

10.40 Refer to Exercise 10.36. Test at the 1% significance level if μ_1 is less than μ_2.

10.41 Refer to Exercise 10.37. Test at the 5% significance level if μ_1 is greater than μ_2.

■ Applications

10.42 According to the information given in Exercise 10.28, a sample of 25 male customers who shopped at a supermarket showed that they spent an average of \$80 with a standard deviation of \$17.50. Another sample of 20 female customers who shopped at the same supermarket showed that they spent an average of \$96 with a standard deviation of \$14.40. Assume that the amounts spent at this supermarket by all male and all female customers are normally distributed with unequal and unknown population standard deviations.

a. Construct a 99% confidence interval for the difference between the mean amounts spent by all male and all female customers at this supermarket.
b. Using the 5% significance level, can you conclude that the mean amount spent by all male customers at this supermarket is less than that of all female customers?

10.43 According to Exercise 10.29, a manufacturing company is interested in buying one of two different kinds of machines. The company tested the two machines for production purposes. The first machine was run for 8 hours. It produced an average of 126 items per hour with a standard deviation of 9 items. The second machine was run for 10 hours. It produced an average of 117 items per hour with a standard deviation of 6 items. Assume that the production per hour for each machine is (approximately) normally distributed. Further assume that the standard deviations of the hourly productions of the two populations are unequal.

a. Make a 95% confidence interval for the difference between the two population means.
b. Using the 2.5% significance level, can you conclude that the mean number of items produced per hour by the first machine is greater than that of the second machine?

10.44 According to Exercise 10.30, an insurance company wants to know if the average speed at which men drive cars is higher than that of women drivers. The company took a random sample of 27 cars driven by men on a highway and found the mean speed to be 72 miles per hour with a standard deviation of 2.2 miles. Another sample of 18 cars driven by women on the same highway gave a mean speed of 68 miles per hour with a standard deviation of 2.5 miles. Assume that the speeds at which all men and all women drive cars on this highway are both normally distributed with unequal population standard deviations.

a. Construct a 98% confidence interval for the difference between the mean speeds of cars driven by all men and all women on this highway.
b. Test at the 1% significance level whether the mean speed of cars driven by all men drivers on this highway is higher than that of cars driven by all women drivers.

10.45 Refer to Exercise 10.31. According to the U.S. Census Bureau, the mean annual salary for men over 25 years old with a doctoral degree is $86,436, whereas that for women of the same age and with the same degree is $62,169 (*USA TODAY*, September 11, 1998). Suppose that these means are based on random samples of 27 men and 25 women and that the sample standard deviations are $5210 for men and $5080 for women. Further assume that the annual salaries of all such men and women have normal distributions with unequal and unknown standard deviations.

a. Construct a 90% confidence interval for the difference between the mean annual salaries for all such men and all such women.
b. Test at the 5% significance level whether the mean annual salaries for all such men and all such women are different.

10.46 As mentioned in Exercise 10.32, Quadro Corporation has two supermarket stores in a city. The company's quality control department wanted to check if the customers are equally satisfied with the service provided at these two stores. A sample of 25 customers selected from Supermarket I produced a mean satisfaction index of 7.6 (on a scale of 1 to 10, 1 being the lowest and 10 being the highest) with a standard deviation of .75. Another sample of 28 customers selected from Supermarket II produced a mean satisfaction index of 8.1 with a standard deviation of .59. Assume that the customer satisfaction index for each supermarket has a normal distribution with a different population standard deviation.

a. Construct a 98% confidence interval for the difference between the mean satisfaction indexes for all customers for the two supermarkets.
b. Test at the 1% significance level whether the mean satisfaction indexes for all customers for the two supermarkets are different.

10.47 As mentioned in Exercise 10.33, a company claims that its medicine, Brand A, provides faster relief from pain than another company's medicine, Brand B. A researcher tested both brands of medicine on two groups of randomly selected patients. The results of the test are given in the following table. The mean and standard deviation of relief times are in minutes.

Brand	Sample Size	Mean of Relief Times	Standard Deviation of Relief Times
A	25	44	11
B	22	49	9

a. Construct a 99% confidence interval for the difference between the mean relief times for the two brands of medicine.
b. Test at the 1% significance level whether the mean relief time for Brand A is less than that for Brand B.

Assume that the two populations are normally distributed with unequal standard deviations.

10.48 Refer to Exercise 10.34. A study of time spent on housework found that employed married men spent an average of 8.8 hours per week on housework, whereas employed unmarried men averaged 6.9 hours per week on housework (America's Use of Time Project, University of Maryland, *American De-*

mographics, November 1998). Suppose that these means were based on random samples of 21 employed married men and 23 employed unmarried men with sample standard deviations of 2.2 hours and 1.9 hours, respectively. Assume that the two populations from which these samples were selected are normally distributed with unequal and unknown standard deviations.

a. Construct a 90% confidence interval for the difference between the population means for these two groups of men.

b. Test at the 5% significance level whether the mean time spent on housework per week is greater for the population of employed married men than that for the population of employed unmarried men.

10.49 Refer to Exercise 10.35. According to data from the College Board, the average cost for room and board at public four-year residential colleges for 1997–1998 was $4361 and the corresponding average cost for private colleges was $5549 (*The Chronicle of Higher Education Almanac,* August 28, 1998). Suppose that these means were obtained from random samples of 15 public and 14 private four-year residential colleges. Further assume that the sample standard deviations of such costs were $1200 for public colleges and $1540 for private colleges. Assume that the 1997–1998 room and board costs for both groups of colleges are normally distributed with unequal and unknown standard deviations.

a. Construct a 95% confidence interval for the difference between the two population means.

b. Using the 1% significance level, can you conclude that the 1997–1998 mean cost of room and board at all public four-year residential colleges was lower than the 1997–1998 mean of such costs at all private four-year residential colleges?

10.4 INFERENCES ABOUT THE DIFFERENCE BETWEEN TWO POPULATION MEANS FOR PAIRED SAMPLES

Sections 10.1, 10.2, and 10.3 were concerned with estimation and hypothesis testing about the difference between two population means when the two samples were drawn independently from two different populations. This section describes estimation and hypothesis-testing procedures for the difference between two population means when the samples are dependent.

In a case of two dependent samples, two data values—one for each sample—are collected from the same source (or element) and, hence, these are also called **paired** or **matched samples**. For example, we may want to make inferences about the mean weight loss for members of a health club after they have gone through an exercise program for a certain period of time. To do so, suppose we select a sample of 15 members of this health club and record their weights before and after the program. In this example, both sets of data are collected from the same 15 persons, once before and once after the program. Thus, although there are two samples, they contain the same 15 persons. This is an example of paired (or dependent or matched) samples. The procedures to make confidence intervals and test hypotheses in the case of paired samples are different from the ones for independent samples discussed in earlier sections of this chapter.

PAIRED OR MATCHED SAMPLES Two samples are said to be *paired* or *matched samples* when for each data value collected from one sample there is a corresponding data value collected from the second sample, and both these data values are collected from the same source.

As another example of paired samples, suppose an agronomist wants to measure the effect of a new brand of fertilizer on the yield of potatoes. To do so, he selects 10 pieces of land and divides each piece into two portions. Then he randomly assigns one of the two portions from each piece of land to grow potatoes without using fertilizer (or using some other brand of fertilizer). The second portion from each piece of land is used to grow potatoes with the new brand of fertilizer. Thus, he will have 10 pairs of data values. Then, using the procedure to be discussed in this section, he will make inferences about the difference in the mean yields of potatoes with and without the new fertilizer.

The question arises, why does the agronomist not choose 10 pieces of land on which to grow potatoes without using the new brand of fertilizer and another 10 pieces of land to grow potatoes by using the new brand of fertilizer? If he does so, the effect of the fertilizer might be confused with the effects due to soil differences at different locations. Thus, he will not be able to isolate the effect of the new brand of fertilizer on the yield of potatoes. Consequently, the results will not be reliable. By choosing 10 pieces of land and then dividing each of them into two portions, the researcher decreases the possibility that the difference in the productivities of different pieces of land affects the results.

In paired samples, the difference between the two data values for each element of the two samples is denoted by $\boldsymbol{d}$. This value of d is called the **paired difference**. We then treat all the values of d as one sample and make inferences applying procedures similar to the ones used for one-sample cases in Chapters 8 and 9. Note that because each source (or element) gives a pair of values (one for each of the two data sets), each sample contains the same number of values. That is, both samples are the same size. Therefore, we denote the (common) **sample size** by $\boldsymbol{n}$, which gives the number of paired difference values denoted by d. The **degrees of freedom** for the paired samples are $\boldsymbol{n-1}$. Let

μ_d = the mean of the paired differences for the population

σ_d = the standard deviation of the paired differences for the population

$\bar{d}$ = the mean of the paired differences for the sample

s_d = the standard deviation of the paired differences for the sample

n = the number of paired difference values

MEAN AND STANDARD DEVIATION OF THE PAIRED DIFFERENCES FOR SAMPLES The values of $\bar{d}$ and s_d are calculated as[2]

$$\bar{d} = \frac{\Sigma d}{n}$$

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}}$$

In paired samples, instead of using $\bar{x}_1 - \bar{x}_2$ as the sample statistic to make inferences about $\mu_1 - \mu_2$, we use the sample statistic $\bar{d}$ to make inferences about μ_d. Actually the value of $\bar{d}$ is always equal to $\bar{x}_1 - \bar{x}_2$, and the value of μ_d is always equal to $\mu_1 - \mu_2$.

[2]The basic formula to calculate s_d is

$$s_d = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}}$$

However, we will not use this formula to make calculations in this chapter.

SAMPLING DISTRIBUTION, MEAN, AND STANDARD DEVIATION OF $\bar{d}$ If the number of paired values is large ($n \geq 30$), then because of the central limit theorem, the *sampling distribution* of $\bar{d}$ is approximately normal with its *mean* and *standard deviation* given as

$$\mu_{\bar{d}} = \mu_d \quad \text{and} \quad \sigma_{\bar{d}} = \frac{\sigma_d}{\sqrt{n}}$$

In cases when $n \geq 30$, the normal distribution can be used to make inferences about μ_d. However, in cases of paired samples, the sample sizes are usually small and σ_d is unknown. Then, assuming that the paired differences for the population are (approximately) normally distributed, the normal distribution is replaced by the t distribution to make inferences about μ_d. When σ_d is not known, the standard deviation of $\bar{d}$ is estimated by $s_{\bar{d}} = s_d/\sqrt{n}$.

ESTIMATE OF THE STANDARD DEVIATION OF PAIRED DIFFERENCES If

1. n is less than 30
2. σ_d is not known
3. the population of paired differences is (approximately) normally distributed

then the t distribution is used to make inferences about μ_d. The standard deviation $\sigma_{\bar{d}}$ of $\bar{d}$ is estimated by $s_{\bar{d}}$, which is calculated as

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}}$$

Sections 10.4.1 and 10.4.2 describe the procedures used to make a confidence interval and test a hypothesis about μ_d when σ_d is unknown and n is small. The inferences are made using the t distribution. However, if n is large, even if σ_d is unknown, the normal distribution can be used to make inferences about μ_d.

10.4.1 INTERVAL ESTIMATION OF μ_d

The mean $\bar{d}$ of paired differences for paired samples is the point estimator of μ_d. The following formula is used to construct a confidence interval for μ_d in the case of (approximately) normally distributed populations.

CONFIDENCE INTERVAL FOR μ_d The $(1 - \alpha)100\%$ *confidence interval for* μ_d is

$$\bar{d} \pm t s_{\bar{d}}$$

where the value of t is obtained from the t distribution table for the given confidence level and $n - 1$ degrees of freedom, and $s_{\bar{d}}$ is calculated as explained earlier.

Example 10–12 illustrates the procedure to construct a confidence interval for μ_d.

Constructing a confidence interval for μ_d: paired samples.

EXAMPLE 10-12 A researcher wanted to find the effect of a special diet on systolic blood pressure. She selected a sample of seven adults and put them on this dietary plan for three months. The following table gives the systolic blood pressures of these seven adults before and after the completion of this plan.

Before	210	180	195	220	231	199	224
After	193	186	186	223	220	183	233

Let μ_d be the mean reduction in the systolic blood pressures due to this special dietary plan for the population of all adults. Construct a 95% confidence interval for μ_d. Assume that the population of paired differences is (approximately) normally distributed.

Solution Because the information obtained is from paired samples, we will make the confidence interval for the paired difference mean μ_d of the population using the paired difference mean $\bar{d}$ of the sample. Let d be the difference in the systolic blood pressure of an adult before and after this special dietary plan. Then, d is obtained by subtracting the systolic blood pressure after the plan from the systolic blood pressure before the plan. The third column of Table 10.1 lists the values of d for the seven adults. The fourth column of the table records the values of d^2, which are obtained by squaring each of the d values.

Table 10.1

Before	**After**	**Difference d**	**d^2**
210	193	17	289
180	186	−6	36
195	186	9	81
220	223	−3	9
231	220	11	121
199	183	16	256
224	233	−9	81
		$\Sigma d = 35$	$\Sigma d^2 = 873$

The values of $\bar{d}$ and s_d are calculated as follows:

$$\bar{d} = \frac{\Sigma d}{n} = \frac{35}{7} = 5.00$$

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}} = \sqrt{\frac{873 - \frac{(35)^2}{7}}{7-1}} = 10.78579312$$

Hence, the standard deviation of $\bar{d}$ is

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}} = \frac{10.78579312}{\sqrt{7}} = 4.07664661$$

For the 95% confidence interval, the area in each tail of the t distribution curve is

$$\text{Area in each tail} = \alpha/2 = .5 - (.95/2) = .025$$

The degrees of freedom are

$$df = n - 1 = 7 - 1 = 6$$

From the t distribution table, the t value for $df = 6$ and .025 area in the right tail of the t distribution curve is 2.447. Therefore, the 95% confidence interval for μ_d is

$$\bar{d} \pm ts_{\bar{d}} = 5.00 \pm 2.447(4.07664661) = 5.00 \pm 9.98 = \mathbf{-4.98 \text{ to } 14.98}$$

Thus, we can state with 95% confidence that the mean difference between systolic blood pressures before and after the given dietary plan for all adult participants is between −4.98 and 14.98.

10.4.2 HYPOTHESIS TESTING ABOUT μ_d

A hypothesis about μ_d is tested by using the sample statistic $\bar{d}$. If n is 30 or larger, we can use the normal distribution to test a hypothesis about μ_d. However, if n is less than 30, we replace the normal distribution by the t distribution. To use the t distribution, we assume that the population of all paired differences is (approximately) normally distributed and that the population standard deviation, σ_d, of paired differences is not known. This section illustrates the case of the t distribution only. The following formula is used to calculate the value of the test statistic t when testing a hypothesis about μ_d.

TEST STATISTIC t FOR $\bar{d}$ The value of the *test statistic t for $\bar{d}$* is computed as follows:

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}}$$

The critical value of t is found from the t distribution table for the given significance level and $n - 1$ degrees of freedom.

Examples 10–13 and 10–14 illustrate the hypothesis-testing procedure for μ_d.

Conducting a left-tailed test of hypothesis about μ_d: paired samples

EXAMPLE 10–13 A company wanted to know if attending a course on "how to be a successful salesperson" can increase the average sales of its employees. The company sent six of its salespersons to attend this course. The following table gives the one-week sales of these salespersons before and after they attended this course.

Before	12	18	25	9	14	16
After	18	24	24	14	19	20

Using the 1% significance level, can you conclude that the mean weekly sales for all salespersons increase as a result of attending this course? Assume that the population of paired differences has a normal distribution.

Solution Because the data are for paired samples, we test a hypothesis about the paired differences mean μ_d of the population using the paired differences mean $\bar{d}$ of the sample.

Let

$$d = (\text{Weekly sales before the course}) - (\text{Weekly sales after the course})$$

In Table 10.2, we calculate d for each of the six salespersons by subtracting the sales after the course from the sales before the course. The fourth column of the table lists the values of d^2.

Table 10.2

Before	After	Difference d	d^2
12	18	−6	36
18	24	−6	36
25	24	1	1
9	14	−5	25
14	19	−5	25
16	20	−4	16
		$\Sigma d = -25$	$\Sigma d^2 = 139$

The values of $\bar{d}$ and s_d are calculated as follows:

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-25}{6} = -4.17$$

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}} = \sqrt{\frac{139 - \frac{(-25)^2}{6}}{6-1}} = 2.63944439$$

The standard deviation of $\bar{d}$ is

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}} = \frac{2.63944439}{\sqrt{6}} = 1.07754866$$

Step 1. *State the null and alternative hypotheses.*

We are to test if the mean weekly sales for all salespersons increase as a result of taking the course. Let μ_1 be the mean weekly sales for all salespersons before the course and μ_2 the mean weekly sales for all salespersons after the course. Then $\mu_d = \mu_1 - \mu_2$. The mean weekly sales for all salespersons will increase due to attending the course if μ_1 is less than μ_2, which can be written as $\mu_1 - \mu_2 < 0$ or $\mu_d < 0$. Consequently, the null and alternative hypotheses are

H_0: $\mu_d = 0$ ($\mu_1 - \mu_2 = 0$ or the mean weekly sales do not increase)

H_1: $\mu_d < 0$ ($\mu_1 - \mu_2 < 0$ or the mean weekly sales do increase)

Note that we can also write the null hypothesis as $\mu_d \geq 0$.

Step 2. *Select the distribution to use.*

The sample size is small ($n < 30$), the population of paired differences is normal, and σ_d is unknown. Therefore, we use the t distribution to conduct the test.

Step 3. *Determine the rejection and nonrejection regions.*

The $<$ sign in the alternative hypothesis indicates that the test is left-tailed. The significance level is .01. Hence,

$$\text{Area in left tail} = \alpha = .01$$

$$\text{Degrees of freedom} = n - 1 = 6 - 1 = 5$$

The critical value of t for $df = 5$ and .01 area in the left tail of the t distribution curve is -3.365. This value is shown in Figure 10.7.

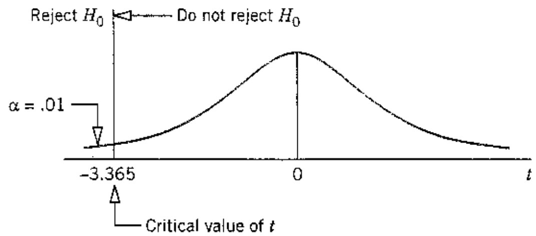

Figure 10.7

Step 4. *Calculate the value of the test statistic.*

The value of the test statistic t for $\bar{d}$ is computed as follows:

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}} = \frac{-4.17 - \overset{\text{From } H_0}{0}}{1.07754866} = -3.870$$

Step 5. *Make a decision.*

Because the value of the test statistic $t = -3.870$ for $\bar{d}$ falls in the rejection region, we reject the null hypothesis. Consequently, we conclude that the mean weekly sales for all salespersons increase as a result of this course. ■

ed test ut μ_d:

EXAMPLE 10–14 Refer to Example 10–12. The table that gives the blood pressures of seven adults before and after the completion of a special dietary plan is reproduced here.

Before	210	180	195	220	231	199	224
After	193	186	186	223	220	183	233

Let μ_d be the mean of the differences between the systolic blood pressures before and after completing this special dietary plan for the population of all adults. Using the 5% significance level, can we conclude that the mean of the paired differences μ_d is different from zero? Assume that the population of paired differences is (approximately) normally distributed.

Solution Table 10.3 gives d and d^2 for each of the seven adults.

Table 10.3

Before	**After**	**Difference** d	d^2
210	193	17	289
180	186	−6	36
195	186	9	81
220	223	−3	9
231	220	11	121
199	183	16	256
224	233	−9	81
		$\Sigma d = 35$	$\Sigma d^2 = 873$

The values of $\bar{d}$ and s_d are calculated as follows:

$$\bar{d} = \frac{\Sigma d}{n} = \frac{35}{7} = 5.00$$

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}} = \sqrt{\frac{873 - \frac{(35)^2}{7}}{7-1}} = 10.78579312$$

Hence, the standard deviation of $\bar{d}$ is

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}} = \frac{10.78579312}{\sqrt{7}} = 4.07664661$$

Step 1. *State the null and alternative hypotheses.*

H_0: $\mu_d = 0$ (The mean of the paired differences is not different from zero)

H_1: $\mu_d \neq 0$ (The mean of the paired differences is different from zero)

Step 2. *Select the distribution to use.*

Because the sample size is small, the population of paired differences is (approximately) normal, and σ_d is not known, we use the t distribution to make the test.

Step 3. *Determine the rejection and nonrejection regions.*

The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed. The significance level is .05.

$$\text{Area in each tail of the curve} = \alpha/2 = .05/2 = .025$$

$$\text{Degrees of freedom} = n - 1 = 7 - 1 = 6$$

The two critical values of t for $df = 6$ and .025 area in each tail of the t distribution curve are -2.447 and 2.447. These values are shown in Figure 10.8.

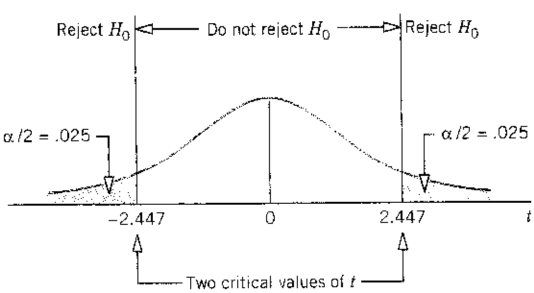

Figure 10.8

Step 4. *Calculate the value of the test statistic.*

The value of the test statistic t for $\bar{d}$ is computed as follows:

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}} = \frac{5.00 - \overset{\text{From } H_0}{0}}{4.07664661} = 1.226$$

Step 5. *Make a decision.*

Because the value of the test statistic $t = 1.226$ for $\bar{d}$ falls in the nonrejection region, we fail to reject the null hypothesis. Hence, we conclude that the mean of the population paired differences is not different from zero. In other words, we can state that the mean of the differences between the systolic blood pressures before and after completing this special dietary plan for the population of all adults is not different from zero. ■

EXERCISES

■ Concepts and Procedures

10.50 Explain when you would use the paired samples procedure to make confidence intervals and test hypotheses.

10.51 Find the following confidence intervals for μ_d assuming that the populations of paired differences are normally distributed.

a. $n = 11$, $\bar{d} = 25.4$, $s_d = 13.5$, confidence level = 99%
b. $n = 23$, $\bar{d} = 13.2$, $s_d = 4.8$, confidence level = 95%
c. $n = 18$, $\bar{d} = 34.6$, $s_d = 11.7$, confidence level = 90%

10.52 Find the following confidence intervals for μ_d assuming that the populations of paired differences are normally distributed.

a. $n = 12$, $\bar{d} = 17.5$, $s_d = 6.3$, confidence level = 99%
b. $n = 27$, $\bar{d} = 55.9$, $s_d = 14.7$, confidence level = 95%
c. $n = 16$, $\bar{d} = 29.3$, $s_d = 8.3$, confidence level = 90%

10.53 Perform the following tests of hypotheses assuming that the populations of paired differences are normally distributed.

a. H_0: $\mu_d = 0$, H_1: $\mu_d \neq 0$, $n = 9$, $\bar{d} = 6.7$, $s_d = 2.5$, $\alpha = .10$
b. H_0: $\mu_d = 0$, H_1: $\mu_d > 0$, $n = 22$, $\bar{d} = 14.8$, $s_d = 6.4$, $\alpha = .05$
c. H_0: $\mu_d = 0$, H_1: $\mu_d < 0$, $n = 17$, $\bar{d} = -9.3$, $s_d = 4.8$, $\alpha = .01$

10.54 Conduct the following tests of hypotheses assuming that the populations of paired differences are normally distributed.

a. H_0: $\mu_d = 0$, H_1: $\mu_d \neq 0$, $n = 26$, $\bar{d} = 9.6$, $s_d = 3.9$, $\alpha = .05$
b. H_0: $\mu_d = 0$, H_1: $\mu_d > 0$, $n = 15$, $\bar{d} = 8.8$, $s_d = 4.7$, $\alpha = .01$
c. H_0: $\mu_d = 0$, H_1: $\mu_d < 0$, $n = 20$, $\bar{d} = -7.4$, $s_d = 2.3$, $\alpha = .10$

■ Applications

10.55 A company sent seven of its employees to attend a course in building self-confidence. These employees were evaluated for their self-confidence before and after attending this course. The following table gives the scores (on a scale of 1 to 15, 1 being the lowest and 15 being the highest score) of these employees before and after they attended the course.

Before	8	5	4	9	6	9	5
After	10	8	5	11	6	7	9

a. Construct a 95% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the score of an employee before attending the course minus the score of the same employee after attending the course.
b. Test at the 1% significance level whether attending this course increases the mean score of employees.

Assume that the population of paired differences has a normal distribution.

10.56 Many students suffer from math anxiety. A professor who teaches statistics offered her students a 2-hour lecture on math anxiety and ways to overcome it. The following table gives the test scores in statistics of seven students before and after they attended this lecture.

differences is not different from zero. In other words, we can state that the mean of the differences between the systolic blood pressures before and after completing this special dietary plan for the population of all adults is not different from zero. ■

EXERCISES

■ Concepts and Procedures

10.50 Explain when you would use the paired samples procedure to make confidence intervals and test hypotheses.

10.51 Find the following confidence intervals for μ_d assuming that the populations of paired differences are normally distributed.

a. $n = 11$, $\bar{d} = 25.4$, $s_d = 13.5$, confidence level = 99%
b. $n = 23$, $\bar{d} = 13.2$, $s_d = 4.8$, confidence level = 95%
c. $n = 18$, $\bar{d} = 34.6$, $s_d = 11.7$, confidence level = 90%

10.52 Find the following confidence intervals for μ_d assuming that the populations of paired differences are normally distributed.

a. $n = 12$, $\bar{d} = 17.5$, $s_d = 6.3$, confidence level = 99%
b. $n = 27$, $\bar{d} = 55.9$, $s_d = 14.7$, confidence level = 95%
c. $n = 16$, $\bar{d} = 29.3$, $s_d = 8.3$, confidence level = 90%

10.53 Perform the following tests of hypotheses assuming that the populations of paired differences are normally distributed.

a. H_0: $\mu_d = 0$, H_1: $\mu_d \neq 0$, $n = 9$, $\bar{d} = 6.7$, $s_d = 2.5$, $\alpha = .10$
b. H_0: $\mu_d = 0$, H_1: $\mu_d > 0$, $n = 22$, $\bar{d} = 14.8$, $s_d = 6.4$, $\alpha = .05$
c. H_0: $\mu_d = 0$, H_1: $\mu_d < 0$, $n = 17$, $\bar{d} = -9.3$, $s_d = 4.8$, $\alpha = .01$

10.54 Conduct the following tests of hypotheses assuming that the populations of paired differences are normally distributed.

a. H_0: $\mu_d = 0$, H_1: $\mu_d \neq 0$, $n = 26$, $\bar{d} = 9.6$, $s_d = 3.9$, $\alpha = .05$
b. H_0: $\mu_d = 0$, H_1: $\mu_d > 0$, $n = 15$, $\bar{d} = 8.8$, $s_d = 4.7$, $\alpha = .01$
c. H_0: $\mu_d = 0$, H_1: $\mu_d < 0$, $n = 20$, $\bar{d} = -7.4$, $s_d = 2.3$, $\alpha = .10$

■ Applications

10.55 A company sent seven of its employees to attend a course in building self-confidence. These employees were evaluated for their self-confidence before and after attending this course. The following table gives the scores (on a scale of 1 to 15, 1 being the lowest and 15 being the highest score) of these employees before and after they attended the course.

Before	8	5	4	9	6	9	5
After	10	8	5	11	6	7	9

a. Construct a 95% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the score of an employee before attending the course minus the score of the same employee after attending the course.
b. Test at the 1% significance level whether attending this course increases the mean score of employees.

Assume that the population of paired differences has a normal distribution.

10.56 Many students suffer from math anxiety. A professor who teaches statistics offered her students a 2-hour lecture on math anxiety and ways to overcome it. The following table gives the test scores in statistics of seven students before and after they attended this lecture.

Before	56	69	48	74	65	71	60
After	62	73	44	85	71	70	73

a. Construct a 99% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the score before attending this lecture minus the score after attending this lecture.

b. Test at the 2.5% significance level whether attending this lecture increases the average score in statistics.

Assume that the population of paired differences is (approximately) normally distributed.

10.57 A private agency claims that the crash course it offers significantly increases the writing speed of secretaries. The following table gives the scores of eight secretaries before and after they attended this course.

Before	81	75	89	91	65	70	90	64
After	97	72	93	110	78	69	115	72

a. Make a 90% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the score before attending the course minus the score after attending the course.

b. Using the 5% significance level, can you conclude that attending this course increases the writing speed of secretaries?

Assume that the population of paired differences is (approximately) normally distributed.

10.58 A company claims that its 12-week special exercise program significantly reduces weight. A random sample of six persons was selected, and these persons were put on this exercise program for 12 weeks. The following table gives the weights (in pounds) of those six persons before and after the program.

Before	180	195	177	221	208	199
After	183	187	161	204	197	189

a. Make a 95% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the weight before joining this exercise program minus the weight at the end of the 12-week program.

b. Using the 1% significance level, can you conclude that the mean weight loss for all persons due to this special exercise program is greater than zero?

Assume that the population of all paired differences is (approximately) normally distributed.

10.59 The manufacturer of a gasoline additive claims that the use of this additive increases gasoline mileage. A random sample of six cars was selected and these cars were driven for one week without the gasoline additive and then for one week with the gasoline additive. The following table gives the miles per gallon for these cars without and with the gasoline additive.

Without	24.6	28.3	18.9	23.7	15.4	29.5
With	26.3	31.7	18.2	25.3	18.3	30.9

a. Construct a 99% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the miles per gallon without the gasoline additive minus the miles per gallon with the gasoline additive.

b. Using the 2.5% significance level, can you conclude that the use of the gasoline additive increases the gasoline mileage?

Assume that the population of paired differences is (approximately) normally distributed.

10.60 A company is considering installing new machines to assemble its products. The company is considering two types of machines, but it will buy only one type. The company selected eight assembly workers and asked them to use these two types of machines to assemble products. The following table gives the time taken (in minutes) to assemble one unit of the product on each type of machine for each of these eight workers.

Machine I	23	26	19	24	27	22	20	18
Machine II	21	24	23	25	24	28	24	23

a. Construct a 98% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the time taken to assemble a unit of the product on Machine I minus the time taken to assemble a unit of the product on Machine II.
b. Test at the 5% significance level whether the mean times taken to assemble a unit of the product are different for the two types of machines.

Assume that the population of paired differences is (approximately) normally distributed.

10.5 INFERENCES ABOUT THE DIFFERENCE BETWEEN TWO POPULATION PROPORTIONS FOR LARGE AND INDEPENDENT SAMPLES

Quite often we need to construct a confidence interval and test a hypothesis about the difference between two population proportions. For instance, we may want to estimate the difference between the proportions of defective items produced on two different machines. If p_1 and p_2 are the proportions of defective items produced on the first and second machine, respectively, then we are to make a confidence interval for $p_1 - p_2$. Or we may want to test the hypothesis that the proportion of defective items produced on Machine I is different from the proportion of defective items produced on Machine II. In this case, we are to test the null hypothesis $p_1 - p_2 = 0$ against the alternative hypothesis $p_1 - p_2 \neq 0$.

This section discusses how to make a confidence interval and test a hypothesis about $p_1 - p_2$ for two large and independent samples. The sample statistic that is used to make inferences about $p_1 - p_2$ is $\hat{p}_1 - \hat{p}_2$, where $\hat{p}_1$ and $\hat{p}_2$ are the proportions for two large and independent samples. As discussed in Section 7.6 of Chapter 7, we determine a sample proportion by dividing the number of elements in the sample that possess a given attribute by the sample size. Thus,

$$\hat{p}_1 = x_1/n_1 \quad \text{and} \quad \hat{p}_2 = x_2/n_2$$

where x_1 and x_2 are the number of elements that possess a given characteristic in the two samples and n_1 and n_2 are the sizes of the two samples, respectively.

10.5.1 MEAN, STANDARD DEVIATION, AND SAMPLING DISTRIBUTION OF $\hat{p}_1 - \hat{p}_2$

As discussed in Chapter 7, for a large sample the sample proportion $\hat{p}$ is (approximately) normally distributed with mean p and standard deviation $\sqrt{pq/n}$. Hence, for two large and independent samples of sizes n_1 and n_2, respectively, their sample proportions $\hat{p}_1$ and $\hat{p}_2$ are (approximately) normally distributed with means p_1 and p_2 and standard deviations $\sqrt{p_1 q_1/n_1}$

and $\sqrt{p_2q_2/n_2}$, respectively. Using these results, we can make the following statements about the shape of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ and its mean and standard deviation.

MEAN, STANDARD DEVIATION, AND SAMPLING DISTRIBUTION OF $\hat{p}_1 - \hat{p}_2$ For two large and independent samples, the *sampling distribution* of $\hat{p}_1 - \hat{p}_2$ is (approximately) normal with its *mean* and *standard deviation* given as

$$\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2$$

and

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}}$$

respectively, where $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$.

Thus, to construct a confidence interval and test a hypothesis about $p_1 - p_2$ for large and independent samples, we use the normal distribution. As was indicated in Chapter 7, in the case of proportions, the sample is large if np and nq are both greater than 5. In the case of two samples, both sample sizes are large if n_1p_1, n_1q_1, n_2p_2, and n_2q_2 are all greater than 5.

10.5.2 INTERVAL ESTIMATION OF $p_1 - p_2$

The difference between two sample proportions $\hat{p}_1 - \hat{p}_2$ is the point estimator for the difference between two population proportions $p_1 - p_2$. Because we do not know p_1 and p_2 when we are making a confidence interval for $p_1 - p_2$, we cannot calculate the value of $\sigma_{\hat{p}_1-\hat{p}_2}$. Therefore, we use $s_{\hat{p}_1-\hat{p}_2}$ as the point estimator of $\sigma_{\hat{p}_1-\hat{p}_2}$ in the interval estimation. We construct the confidence interval for $p_1 - p_2$ using the following formula.

CONFIDENCE INTERVAL FOR $p_1 - p_2$
The $(1 - \alpha)100\%$ *confidence interval for* $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z s_{\hat{p}_1-\hat{p}_2}$$

where the value of z is read from the normal distribution table for the given confidence level, and $s_{\hat{p}_1-\hat{p}_2}$ is calculated as

$$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Example 10–15 describes the procedure used to make a confidence interval for the difference between two population proportions for large samples.

Constructing a confidence interval for $p_1 - p_2$: large and independent samples.

EXAMPLE 10–15 A researcher wanted to estimate the difference between the percentages of users of two toothpastes who will never switch to another toothpaste. In a sample of 500 users of Toothpaste A taken by this researcher, 100 said that they will never switch to another toothpaste. In another sample of 400 users of Toothpaste B taken by the same researcher, 68 said that they will never switch to another toothpaste.

(a) Let p_1 and p_2 be the proportions of all users of Toothpastes A and B, respectively, who will never switch to another toothpaste. What is the point estimate of $p_1 - p_2$?
(b) Construct a 97% confidence interval for the difference between the proportions of all users of the two toothpastes who will never switch.

Solution Let p_1 and p_2 be the proportions of all users of Toothpastes A and B, respectively, who will never switch to another toothpaste and let $\hat{p}_1$ and $\hat{p}_2$ be the respective sample proportions. Let x_1 and x_2 be the number of users of Toothpastes A and B, respectively, in the two samples who said that they will never switch to another toothpaste. From the given information,

Toothpaste A:	$n_1 = 500$	and	$x_1 = 100$
Toothpaste B:	$n_2 = 400$	and	$x_2 = 68$

The two sample proportions are calculated as follows:

$$\hat{p}_1 = x_1/n_1 = 100/500 = .20$$

$$\hat{p}_2 = x_2/n_2 = 68/400 = .17$$

Then,

$$\hat{q}_1 = 1 - .20 = .80 \quad \text{and} \quad \hat{q}_2 = 1 - .17 - .83$$

(a) The point estimate of $p_1 - p_2$ is as follows:

$$\text{Point estimate of } p_1 - p_2 = \hat{p}_1 - \hat{p}_2 = .20 - .17 = \mathbf{.03}$$

(b) The values of $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, and $n_2\hat{q}_2$ are

$$n_1\hat{p}_1 = 500(.20) = 100 \qquad n_1\hat{q}_1 = 500(.80) = 400$$

$$n_2\hat{p}_2 = 400(.17) = 68 \qquad n_2\hat{q}_2 = 400(.83) = 332$$

Since each of these values is greater than 5, both sample sizes are large. Consequently we use the normal distribution to make a confidence interval for $p_1 - p_2$. The standard deviation of $\hat{p}_1 - \hat{p}_2$ is

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \sqrt{\frac{(.20)(.80)}{500} + \frac{(.17)(.83)}{400}} = .02593742$$

The z value for a 97% confidence level, obtained from the normal distribution table for $.97/2 = .4850$, is 2.17. The 97% confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z s_{\hat{p}_1 - \hat{p}_2} = (.20 - .17) \pm 2.17(.02593742)$$

$$= .03 \pm .056 = \mathbf{-.026 \text{ to } .086}$$

Thus, with 97% confidence we can state that the difference between the two population proportions is between $-.026$ and $.086$.

10.5.3 HYPOTHESIS TESTING ABOUT $p_1 - p_2$

In this section we learn how to test a hypothesis about $p_1 - p_2$ for two large and independent samples. The procedure involves the same five steps that we have used previously. Once again, we calculate the standard deviation of $\hat{p}_1 - \hat{p}_2$ as

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

When a test of hypothesis about $p_1 - p_2$ is performed, usually the null hypothesis is $p_1 = p_2$ and the values of p_1 and p_2 are not known. Assuming that the null hypothesis is true and $p_1 = p_2$, a common value of p_1 and p_2, denoted by $\bar{p}$, is calculated by using one of the following formulas:

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{or} \quad \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

Which of these formulas is used depends on whether the values of x_1 and x_2 or the values of $\hat{p}_1$ and $\hat{p}_2$ are known. Note that x_1 and x_2 are the number of elements in each of the two samples that possess a certain characteristic. This value of $\bar{p}$ is called the **pooled sample proportion**. Using the value of the pooled sample proportion, we compute an estimate of the standard deviation of $\hat{p}_1 - \hat{p}_2$ as follows:

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

where $\bar{q} = 1 - \bar{p}$.

TEST STATISTIC z FOR $\hat{p}_1 - \hat{p}_2$ The value of the *test statistic z for $\hat{p}_1 - \hat{p}_2$* is calculated as

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{s_{\hat{p}_1 - \hat{p}_2}}$$

The value of $p_1 - p_2$ is substituted from H_0, which usually is zero.

Examples 10–16 and 10–17 illustrate the procedure to test hypotheses about the difference between two population proportions for large samples.

Making a right-tailed test of hypothesis about $p_1 - p_2$: large and independent samples.

EXAMPLE 10–16 Reconsider Example 10–15 about the percentages of users of two toothpastes who will never switch to another toothpaste. At the 1% significance level, can we conclude that the proportion of users of Toothpaste A who will never switch to another toothpaste is higher than the proportion of users of Toothpaste B who will never switch to another toothpaste?

Solution Let p_1 and p_2 be the proportions of all users of Toothpastes A and B, respectively, who will never switch to another toothpaste and let $\hat{p}_1$ and $\hat{p}_2$ be the corresponding sample proportions. Let x_1 and x_2 be the number of users of Toothpastes A and B, respectively, in the two samples who said that they will never switch to another toothpaste. From the given information,

Toothpaste A: $n_1 = 500$ and $x_1 = 100$

Toothpaste B: $n_2 = 400$ and $x_2 = 68$

The significance level is $\alpha = .01$. The two sample proportions are calculated as follows:

$$\hat{p}_1 = x_1/n_1 = 100/500 = .20$$

$$\hat{p}_2 = x_2/n_2 = 68/400 = .17$$

Step 1. *State the null and alternative hypotheses.*

We are to test if the proportion of users of Toothpaste A who will never switch to another toothpaste is higher than the proportion of users of Toothpaste B who will never switch to another toothpaste. In other words, we are to test whether p_1 is greater than p_2. This can be written as $p_1 - p_2 > 0$. Thus, the two hypotheses are

$$H_0: p_1 - p_2 = 0 \qquad (p_1 \text{ is not greater than } p_2)$$

$$H_1: p_1 - p_2 > 0 \qquad (p_1 \text{ is greater than } p_2)$$

Step 2. *Select the distribution to use.*

As shown in Example 10–15, $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, and $n_2\hat{q}_2$ are all greater than 5. Consequently both samples are large, and we apply the normal distribution to make the test.

Step 3. *Determine the rejection and nonrejection regions.*

The > sign in the alternative hypothesis indicates that the test is right-tailed. From the normal distribution table, for a .01 significance level, the critical value of z is 2.33. This is shown in Figure 10.9.

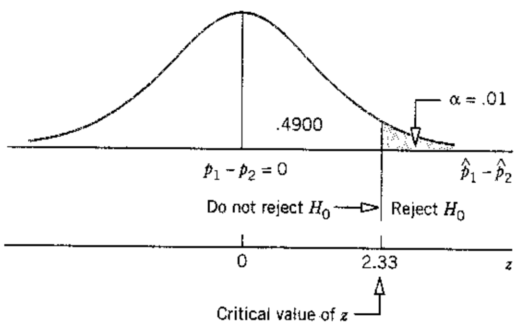

Figure 10.9

Step 4. *Calculate the value of the test statistic.*

The pooled sample proportion is

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{100 + 68}{500 + 400} = .187$$

$$\bar{q} = 1 - \bar{p} = 1 - .187 = .813$$

The estimate of the standard deviation of $\hat{p}_1 - \hat{p}_2$ is

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{(.187)\,(.813)\left(\frac{1}{500} + \frac{1}{400}\right)} = .02615606$$

The value of the test statistic z for $\hat{p}_1 - \hat{p}_2$ is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{s_{\hat{p}_1 - \hat{p}_2}} = \frac{(.20 - .17) - \overset{\text{From } H_0}{0}}{.02615606} = 1.15$$

Step 5. *Make a decision.*

Since the value of the test statistic $z = 1.15$ for $\hat{p}_1 - \hat{p}_2$ falls in the nonrejection region, we fail to reject the null hypothesis. Therefore, we conclude that the proportion of users of Toothpaste A who will never switch to another toothpaste is not greater than the proportion of users of Toothpaste B who will never switch to another toothpaste.

Conducting a two-tailed test of hypothesis about $p_1 - p_2$: large and independent samples.

EXAMPLE 10-17 According to a poll conducted for *Men's Health* magazine/CNN by Opinion Research, 67% of men and 77% of women said that a good diet is very important to good health (*USA TODAY*, January 25, 1999). Suppose that this study is based on samples of 900 men and 1200 women. Test whether the percentages of all men and all women who hold this view are different. Use the 1% significance level.

Solution Let p_1 and p_2 be the proportions of all men and all women, respectively, who hold the view that a good diet is very important to good health. Let $\hat{p}_1$ and $\hat{p}_2$ be the corresponding sample proportions. From the given information,

$$\text{For men:} \quad n_1 = 900 \quad \text{and} \quad \hat{p}_1 = .67$$

$$\text{For women:} \quad n_2 = 1200 \quad \text{and} \quad \hat{p}_2 = .77$$

The significance level is $\alpha = .01$.

Step 1. *State the null and alternative hypotheses.*

The null and alternative hypotheses are

H_0: $p_1 - p_2 = 0$ (The two population proportions are not different)

H_1: $p_1 - p_2 \neq 0$ (The two population proportions are different)

Step 2. *Select the distribution to use.*

Because the samples are large and independent, we apply the normal distribution to make the test. (The reader should check that $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, and $n_2\hat{q}_2$ are all greater than 5.)

Step 3. *Determine the rejection and nonrejection regions.*

The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed. For a 1% significance level, the critical values of z are -2.58 and 2.58. These values are shown in Figure 10.10.

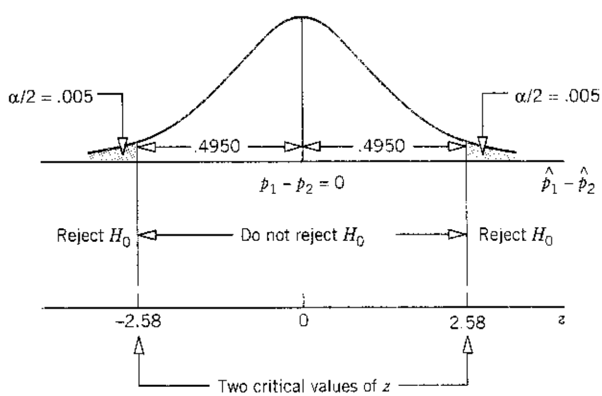

Figure 10.10

Step 4. *Calculate the value of the test statistic.*

The pooled sample proportion is

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{900(.67) + 1200(.77)}{900 + 1200} = .727$$

$$\bar{q} = 1 - \bar{p} = 1 - .727 = .273$$

The estimate of the standard deviation of $\hat{p}_1 - \hat{p}_2$ is

$$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{(.727)(.273)\left(\frac{1}{900}+\frac{1}{1200}\right)} = .01964474$$

The value of the test statistic z for $\hat{p}_1 - \hat{p}_2$ is

$$z = \frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{s_{\hat{p}_1-\hat{p}_2}} = \frac{(.67-.77)-\overset{\text{From } H_0}{0}}{.01964474} = -5.09$$

Step 5. *Make a decision.*

The value of the test statistic $z = -5.09$ for $\hat{p}_1 - \hat{p}_2$ falls in the rejection region. Consequently, we reject the null hypothesis. As a result, we conclude that the percentages of all men and all women who hold the view that good diet is very important to good health are different. ∎

whereas 52% of the households without children feel this way. If we know the sample sizes for the two types of households, we can construct a confidence interval and conduct a test of hypothesis for the difference in the two percentages for each category.

Suppose that this survey is based on a random sample of 1000 households, of which 495 have children and 505 do not. For a particular category of concern, let p_1 be the proportion of households with children who share that level of concern and p_2 be the proportion of households without children who feel that way. Let $\hat{p}_1$ and $\hat{p}_2$ be the corresponding sample proportions. Then for the first category (that meeting their financial obligations is a major concern):

$$n_1 = 495 \qquad \hat{p}_1 = .60$$

$$n_2 = 505 \qquad \hat{p}_2 = .52$$

Below we find a confidence interval and test a hypothesis for the difference $p_1 - p_2$ for this category.

1. Confidence interval for $p_1 - p_2$

Suppose we want to construct a 97% confidence interval for $p_1 - p_2$. The z value for the 97% confidence level is 2.17. The standard deviation of $\hat{p}_1 - \hat{p}_2$ is

$$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \sqrt{\frac{(.60)(.40)}{495} + \frac{(.52)(.48)}{505}} = .03129067$$

Hence, a 97% confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z s_{\hat{p}_1-\hat{p}_2} = (.60 - .52) \pm 2.17(.03129067) = .08 \pm .068$$

$$= .012 \text{ to } .148 \quad \text{or} \quad 1.2\% \text{ to } 14.8\%$$

Thus, we can say with 97% confidence that the difference in the proportions of all households with and without children who say that meeting their financial obligations is a major concern is in the interval .012 to .148 or 1.2% to 14.8%.

2. Test of hypothesis about $p_1 - p_2$

Suppose we want to test, at the 1% significance level, whether p_1 is greater than p_2. Then,

$$H_0: p_1 = p_2 \text{ or } p_1 - p_2 = 0$$

$$H_1: p_1 > p_2 \text{ or } p_1 - p_2 > 0$$

Note that the test is right-tailed. For $\alpha = .01$, we will reject the null hypothesis if the observed value of z is 2.33 or larger. The pooled sample proportion is

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{495(.60) + 505(.52)}{495 + 505} = .5596$$

and

$$\bar{q} = 1 - \bar{p} = 1 - .5596 = .4404$$

The estimate of the standard deviation of $\hat{p}_1 - \hat{p}_2$ is

$$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{(.5596)(.4404)\left(\frac{1}{495} + \frac{1}{505}\right)} = .03139888$$

whereas 52% of the households without children feel this way. If we know the sample sizes for the two types of households, we can construct a confidence interval and conduct a test of hypothesis for the difference in the two percentages for each category.

Suppose that this survey is based on a random sample of 1000 households, of which 495 have children and 505 do not. For a particular category of concern, let p_1 be the proportion of households with children who share that level of concern and p_2 be the proportion of households without children who feel that way. Let $\hat{p}_1$ and $\hat{p}_2$ be the corresponding sample proportions. Then for the first category (that meeting their financial obligations is a major concern):

$$n_1 = 495 \qquad \hat{p}_1 = .60$$

$$n_2 = 505 \qquad \hat{p}_2 = .52$$

Below we find a confidence interval and test a hypothesis for the difference $p_1 - p_2$ for this category.

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Suppose we want to construct a 97% confidence interval for $p_1 - p_2$. The z value for the 97% confidence level is 2.17. The standard deviation of $\hat{p}_1 - \hat{p}_2$ is

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Hence, a 97% confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z s_{\hat{p}_1-\hat{p}_2} = (.60 - .52) \pm 2.17(.03129067) = .08 \pm .068$$

$$= .012 \text{ to } .148 \quad \text{or} \quad 1.2\% \text{ to } 14.8\%$$

Thus, we can say with 97% confidence that the difference in the proportions of all households with and without children who say that meeting their financial obligations is a major concern is in the interval .012 to .148 or 1.2% to 14.8%.

2. Test of hypothesis about $p_1 - p_2$

Suppose we want to test, at the 1% significance level, whether p_1 is greater than p_2. Then,

$$H_0\colon p_1 = p_2 \text{ or } p_1 - p_2 = 0$$

$$H_1\colon p_1 > p_2 \text{ or } p_1 - p_2 > 0$$

Note that the test is right-tailed. For $\alpha = .01$, we will reject the null hypothesis if the observed value of z is 2.33 or larger. The pooled sample proportion is

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{495(.60) + 505(.52)}{495 + 505} = .5596$$

and

$$\bar{q} = 1 - \bar{p} = 1 - .5596 = .4404$$

The estimate of the standard deviation of $\hat{p}_1 - \hat{p}_2$ is

$$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{(.5596)(.4404)\left(\frac{1}{495} + \frac{1}{505}\right)} = .03139888$$

The value of the test statistic z for $\hat{p}_1 - \hat{p}_2$ is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{s_{\hat{p}_1 - \hat{p}_2}} = \frac{(.60 - .52) - 0}{.03139888} = 2.55$$

Since the observed value of $z = 2.55$ is greater than the critical value of 2.33, we reject the null hypothesis. As a result, we conclude that p_1 is greater than p_2.

Source: The chart reproduced with permission from *USA TODAY*, May 3, 1999. Copyright © 1999, *USA TODAY*.

EXERCISES

Concepts and Procedures

10.61 What is the shape of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ for two large samples? What are the mean and standard deviation of this sampling distribution?

10.62 When are the samples considered large enough for the sampling distribution of the difference between two sample proportions to be (approximately) normal?

10.63 Construct a 99% confidence interval for $p_1 - p_2$ for the following.

$$n_1 = 300 \qquad \hat{p}_1 = .55 \qquad n_2 = 200 \qquad \hat{p}_2 = .62$$

10.64 Construct a 95% confidence interval for $p_1 - p_2$ for the following.

$$n_1 = 100 \qquad \hat{p}_1 = .81 \qquad n_2 = 150 \qquad \hat{p}_2 = .77$$

10.65 Refer to the information given in Exercise 10.63. Test at the 1% significance level if the two population proportions are different.

10.66 Refer to the information given in Exercise 10.64. Test at the 5% significance level if $p_1 - p_2$ is different from zero.

10.67 Refer to the information given in Exercise 10.63. Test at the 1% significance level if p_1 is less than p_2.

10.68 Refer to the information given in Exercise 10.64. Test at the 2.5% significance level if p_1 is greater than p_2.

10.69 A sample of 500 observations taken from the first population gave $x_1 = 305$. Another sample of 600 observations taken from the second population gave $x_2 = 348$.

a. Find the point estimate of $p_1 - p_2$.
b. Make a 97% confidence interval for $p_1 - p_2$.
c. Show the rejection and nonrejection regions on the sampling distribution of $\hat{p}_1 - \hat{p}_2$ for H_0: $p_1 = p_2$ versus H_1: $p_1 > p_2$. Use a significance level of 2.5%.
d. Find the value of the test statistic z for the test of part c.
e. Will you reject the null hypothesis mentioned in part c at a significance level of 2.5%?

10.70 A sample of 1000 observations taken from the first population gave $x_1 = 290$. Another sample of 1200 observations taken from the second population gave $x_2 = 396$.

a. Find the point estimate of $p_1 - p_2$.
b. Make a 98% confidence interval for $p_1 - p_2$.
c. Show the rejection and nonrejection regions on the sampling distribution of $\hat{p}_1 - \hat{p}_2$ for H_0: $p_1 = p_2$ versus H_1: $p_1 < p_2$. Use a significance level of 1%.
d. Find the value of the test statistic z for the test of part c.
e. Will you reject the null hypothesis mentioned in part c at a significance level of 1%?

Applications

10.71 In a 1998 survey, teachers and members of the public were asked, How much do you think computers have helped improve student learning? Fifty-eight percent of the members of the public and 31% of the teachers stated that computers have helped a "great amount" (MCI National Poll on the Internet in Education, 1998, *Education Week*, October 1, 1998). Assume that these estimates were based on random samples of 200 members of the public and 192 teachers.

a. Determine a 99% confidence interval for the difference between the two population proportions.
b. At the 1% significance level, can you conclude that the proportion of all members of the public who hold this view is greater than the proportion of all teachers who hold this view?

10.72 A company has two restaurants in two different areas of New York City. The company wants to estimate the percentages of patrons who think that the food and service at each of these restaurants are excellent. A sample of 200 patrons taken from the restaurant in Area A showed that 118 of them think that the food and service are excellent at this restaurant. Another sample of 250 patrons taken from the restaurant in Area B showed that 160 of them think that the food and service are excellent at this restaurant.

a. Construct a 97% confidence interval for the difference between the two population proportions.
b. Testing at the 2.5% significance level, can you conclude that the proportion of patrons at the restaurant in Area A who think that the food and service are excellent is lower than the corresponding proportion at the restaurant in Area B?

10.73 According to two polls conducted for AARP (American Association of Retired Persons), 14% of grown children say that they provide some housekeeping help for their elderly parents, whereas only 9% of parents agree with this statement (*USA TODAY*, January 28, 1999). These estimates are based on samples of 1689 grown children and 896 parents for the two polls.

a. Construct a 98% confidence interval for the difference between the two population proportions.
b. Test at the 1% level of significance whether the two population proportions are different.

10.74 In a survey conducted for *Men's Health* magazine/CNN by Opinion Research, 61% of men and 68% of women rated exercise as "very important" to good health (*USA TODAY*, January 25, 1999). Assume that the percentages were based on random samples of 250 men and 250 women.

a. Construct a 95% confidence interval for the difference between the proportions of all men and all women who rate exercise as "very important" to good health.
b. Testing at the 2.5% significance level, can you conclude that the proportion of all men who consider exercise "very important" to good health is lower than the proportion of all women who hold this view?

10.75 The management of a supermarket wanted to investigate if the percentages of men and women who prefer to buy national brand products over the store brand products are different. A sample of 600 men shoppers at the company's supermarkets showed that 246 of them prefer to buy national brand products over the store brand products. Another sample of 700 women shoppers at the company's supermarkets showed that 266 of them prefer to buy national brand products over the store brand products.

a. What is the point estimate of the difference between the two population proportions?
b. Construct a 95% confidence interval for the difference between the proportions of all men and all women shoppers at these supermarkets who prefer to buy national brand products over the store brand products.
c. Testing at the 5% significance level, can you conclude that the proportions of all men and all women shoppers at these supermarkets who prefer to buy national brand products over the store brand products are different?

10.76 The lottery commissioner's office in a state wanted to find if the percentages of men and women who play the lottery often are different. A sample of 500 men taken by the commissioner's office showed that 160 of them play the lottery often. Another sample of 300 women showed that 66 of them play the lottery often.

a. What is the point estimate of the difference between the two population proportions?
b. Construct a 99% confidence interval for the difference between the proportions of all men and all women who play the lottery often.

c. Testing at the 1% significance level, can you conclude that the proportions of all men and all women who play the lottery often are different?

10.77 A mail-order company has two warehouses, one on the West Coast and the second on the East Coast. The company's policy is to mail all orders placed with it within 72 hours. The company's quality control department checks quite often whether or not this policy is maintained at the two warehouses. A recently taken sample of 400 orders placed with the warehouse on the West Coast showed that 364 of them were mailed within 72 hours. Another sample of 300 orders placed with the warehouse on the East Coast showed that 279 of them were mailed within 72 hours.

a. Construct a 97% confidence interval for the difference between the proportions of all orders placed at the two warehouses that are mailed within 72 hours.
b. Using the 2.5% significance level, can you conclude that the proportion of all orders placed at the warehouse on the West Coast that are mailed within 72 hours is lower than the corresponding proportion for the warehouse on the East Coast?
*c. Find the p-value for the test mentioned in part b.

10.78 A company that has many department stores in the southern states wanted to find the percentage of sales at two such stores for which at least one of the items was returned. A sample of 800 sales randomly selected from Store A showed that for 240 of them at least one item was returned. Another sample of 900 sales randomly selected from Store B showed that for 279 of them at least one item was returned.

a. Construct a 98% confidence interval for the difference between the proportions of all sales at the two stores for which at least one item is returned.
b. Using the 1% significance level, can you conclude that the proportions of all sales at the two stores for which at least one item is returned are different?
*c. Find the p-value for the test mentioned in part b.

GLOSSARY

d The difference between two matched values in two samples collected from the same source. It is called the paired difference.

$\bar{d}$ The mean of the paired differences for a sample.

Independent samples Two samples drawn from two populations such that the selection of one does not affect the selection of the other.

Paired or **matched samples** Two samples drawn in such a way that they include the same elements and two data values are obtained from each element, one for each sample. Also called **dependent samples**.

μ_d The mean of the paired differences for the population.

s_d The standard deviation of the paired differences for a sample.

σ_d The standard deviation of the paired differences for the population.

KEY FORMULAS

1. **Mean of the sampling distribution of $\bar{x}_1 - \bar{x}_2$**

$$\mu_{\bar{x}_1-\bar{x}_2} = \mu_1 - \mu_2$$

2. **Standard deviation of $\bar{x}_1 - \bar{x}_2$ for two large and independent samples**

$$\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

3. **The $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ for two large and independent samples**

$$(\bar{x}_1 - \bar{x}_2) \pm z\sigma_{\bar{x}_1 - \bar{x}_2}$$

If σ_1 and σ_2 are not known, then $\sigma_{\bar{x}_1 - \bar{x}_2}$ is replaced by its point estimator $s_{\bar{x}_1 - \bar{x}_2}$, which is calculated as

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

4. **Value of the test statistic z for $\bar{x}_1 - \bar{x}_2$ for two large and independent samples**

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

If σ_1 and σ_2 are not known, then $\sigma_{\bar{x}_1 - \bar{x}_2}$ is replaced by $s_{\bar{x}_1 - \bar{x}_2}$.

5. **Pooled standard deviation for two small and independent samples taken from two populations with equal standard deviations**

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

6. **Estimator of the standard deviation of $\bar{x}_1 - \bar{x}_2$ for two small and independent samples taken from two populations with equal standard deviations**

$$s_{\bar{x}_1 - \bar{x}_2} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

7. **The $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ for two small and independent samples taken from two normally distributed populations with equal standard deviations**

$$(\bar{x}_1 - \bar{x}_2) \pm t s_{\bar{x}_1 - \bar{x}_2}$$

8. **Value of the test statistic t for $\bar{x}_1 - \bar{x}_2$ for two small and independent samples taken from two normally distributed populations with equal standard deviations**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

9. **Degrees of freedom to make inferences about $\mu_1 - \mu_2$ for two small and independent samples taken from two normally distributed populations with unequal standard deviations**

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

10. **Estimate of the standard deviation of $\bar{x}_1 - \bar{x}_2$ for two small and independent samples taken from two normally distributed populations with unequal standard deviations**

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

11. **The $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ for two small and independent samples taken from two normally distributed populations with unequal standard deviations**

$$(\bar{x}_1 - \bar{x}_2) \pm t s_{\bar{x}_1 - \bar{x}_2}$$

12. **Value of the test statistic t for $\bar{x}_1 - \bar{x}_2$ for two small and independent samples taken from two normally distributed populations with unequal standard deviations**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

13. **Sample mean for paired differences**

$$\bar{d} = \frac{\Sigma d}{n}$$

14. **Sample standard deviation for paired differences**

$$s_d = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n - 1}}$$

15. **Mean and standard deviation of the sampling distribution of $\bar{d}$**

$$\mu_{\bar{d}} = \mu_d \quad \text{and} \quad s_{\bar{d}} = \frac{s_d}{\sqrt{n}}$$

16. **The $(1 - \alpha)100\%$ confidence interval for μ_d**

$$\bar{d} \pm t s_{\bar{d}}$$

17. **Value of the test statistic t for $\bar{d}$**

$$t = \frac{\bar{d} - \mu_d}{s_{\bar{d}}}$$

18. **Mean of the sampling distribution of $\hat{p}_1 - \hat{p}_2$**

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$$

19. **Estimate of the standard deviation of $\hat{p}_1 - \hat{p}_2$ for two large and independent samples**

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

20. **The $(1 - \alpha)100\%$ confidence interval for $p_1 - p_2$ for two large and independent samples**

$$(\hat{p}_1 - \hat{p}_2) \pm z s_{\hat{p}_1 - \hat{p}_2}$$

21. **Pooled sample proportion for two samples**

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \quad \text{or} \quad \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$

22. **Estimator of the standard deviation of $\hat{p}_1 - \hat{p}_2$ using the pooled sample proportion**

$$s_{\hat{p}_1 - \hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

23. **Value of the test statistic z for $\hat{p}_1 - \hat{p}_2$ for large and independent samples**

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{s_{\hat{p}_1 - \hat{p}_2}}$$

SUPPLEMENTARY EXERCISES

10.79 According to the College and University Personnel Association, in 1997–1998 the average salary of economics professors was \$60,021 at four-year public institutions of higher education and \$61,515 at similar private institutions (*The Chronicle of Higher Education Almanac*, August 28, 1998). Suppose that these results are based on random samples of 400 economics professors from public institutions and 360 economics professors from private institutions. Further assume that the two sample standard deviations are \$9400 and \$11,000, respectively.

a. Construct a 99% confidence interval for the difference between the 1997–1998 mean salaries of economics professors at four-year public and private institutions of higher education.
b. Using the 1% significance level, can you conclude that the 1997–1998 mean salary of economics professors at four-year public institutions of higher education was lower than this mean at four-year private institutions?
c. What would your decision in part b be if the probability of making a Type I error were zero? Explain.

10.80 According to the Centers for Disease Control and Prevention/National Center for Health Statistics, the average length of stay for cases of appendicitis in short-stay hospitals in the United States was 3.5 days for patients under 15 years of age and 3.0 days for patients aged 15–44 (*Advance Data*, August 31, 1998). Suppose that these averages are based on random samples of 160 patients under 15 years of age and 200 patients aged 15–44. Further assume that the standard deviations for the two samples are 1.1 days and 1.0 day, respectively.

a. Construct a 95% confidence interval for the difference between the two population means.
b. Test at the 5% significance level whether the mean length of stay for cases of appendicitis for all patients under 15 years of age is greater than that for all patients in the 15–44 age group.
c. What would your decision in part b be if the probability of making a Type I error were zero? Explain.

10.81 A consulting agency was asked by a large insurance company to investigate if business majors were better salespersons than those with other majors. A sample of 40 salespersons with a business degree showed that they sold an average of 11 insurance policies per week with a standard deviation of 1.80 policies. Another sample of 45 salespersons with a degree other than business showed that they sold an average of 9 insurance policies per week with a standard deviation of 1.35 policies.

a. Construct a 99% confidence interval for the difference between the two population means.
b. Using the 1% significance level, can you conclude that persons with a business degree are better salespersons than those who have a degree in another area?

10.82 According to an estimate, the average earnings of female workers who are not union members are $388 per week and those of female workers who are union members are $505 per week. Suppose that these average earnings are calculated based on random samples of 1500 female workers who are not union members and 2000 female workers who are union members. Further assume that the standard deviations for these two samples are $30 and $35, respectively.

a. Construct a 95% confidence interval for the difference between the two population means.
b. Test at the 2.5% significance level whether the mean weekly earnings of female workers who are not union members are less than those of female workers who are union members.

10.83 A researcher wants to test if the mean GPAs (grade point averages) of all male and all female college students who actively participate in sports are different. She took a random sample of 28 male students and 24 female students who are actively involved in sports. She found the mean GPAs of the two groups to be 2.62 and 2.74, respectively, with the corresponding standard deviations equal to .43 and .38.

a. Test at the 5% significance level whether the mean GPAs of the two populations are different.
b. Construct a 90% confidence interval for the difference between the two population means.

Assume that the GPAs of all male and all female student athletes both have a normal distribution with the same standard deviation.

10.84 According to data from Runzheimer International, the average monthly cost of day care for a three-year-old in a for-profit center, 5 days per week, 8 hours a day, was $606 in New York and $622 in Boston (*USA TODAY*, August 18, 1998). Suppose that these means were obtained from random samples of 23 day-care centers in New York and 27 day-care centers in Boston. Further assume that the two sample standard deviations are $110 and $105, respectively, and that the two populations are normally distributed with equal but unknown standard deviations.

a. Construct a 90% confidence interval for the difference in the mean monthly day-care costs for the two cities.

b. Test at the 5% significance level whether the mean monthly day-care costs are different for the two cities.

10.85 An agency wanted to estimate the difference between the auto insurance premiums paid by drivers insured with two different insurance companies. A random sample of 25 drivers insured with insurance company A showed that they paid an average monthly insurance premium of \$97 with a standard deviation of \$14. Another random sample of 20 drivers insured with insurance company B showed that these drivers paid an average monthly insurance premium of \$89 with a standard deviation of \$12. Assume that the insurance premiums paid by all drivers insured with companies A and B are both normally distributed with equal standard deviations.

a. Construct a 99% confidence interval for the difference between the two population means.
b. Test at the 1% significance level whether the mean monthly insurance premium paid by drivers insured with company A is higher than that of drivers insured with company B.

10.86 A random sample of 28 children selected from families with only one child gave a mean tolerance level of 2.8 (on a scale of 1 to 8) with a standard deviation of .62. Another random sample of 25 children selected from families with more than one child gave a mean tolerance level of 4.0 with a standard deviation of .47.

a. Construct a 99% confidence interval for the difference between the two population means.
b. Test at the 5% significance level whether the mean tolerance level for children from families with only one child is lower than that for children from families with more than one child.

Assume that the tolerance levels for all children in both groups have normal distributions with the same standard deviation.

10.87 Repeat Exercise 10.83, but now assume that the GPAs of all male and all female student athletes are both normally distributed with unequal standard deviations.

10.88 Repeat Exercise 10.84, but now assume that the two populations have normal distributions with unequal standard deviations.

10.89 Repeat Exercise 10.85, but now assume that the insurance premiums paid by all drivers insured with companies A and B both are normally distributed with unequal standard deviations.

10.90 Repeat Exercise 10.86, but now assume that the tolerance levels for all children in both groups are normally distributed with unequal standard deviations.

10.91 Which day of the week is most productive in the workplace? In a 1998 survey of company executives by Accountemps, over half of those executives said that more work was accomplished on Tuesday than on any other day of the week (*The Willimantic Chronicle,* October 13, 1998). A health insurance company executive wanted to compare the productivity of claim processors on Mondays to that on Tuesdays. He took a random sample of eight such workers and recorded the numbers of claims each of them processed on four Mondays and on four Tuesdays. The table below shows the total number of claims processed on these four Mondays and four Tuesdays by each worker.

Worker	A	B	C	D	E	F	G	H
Mondays	47	41	51	48	44	50	47	48
Tuesdays	53	49	44	53	49	53	55	49

a. Construct a 95% confidence interval for the mean μ_d of population paired differences, where a paired difference is defined as the number of claims processed on Mondays minus the number of claims processed on Tuesdays.
b. Test at the 2.5% significance level whether the average productivity of all such workers is lower on Mondays than on Tuesdays.

10.92 A random sample of nine students was selected to test for the effectiveness of a special course designed to improve memory. The following table gives the results of a memory test given to these students before and after this course.

Before	43	57	48	65	81	49	38	69	58
After	49	56	55	77	89	57	36	64	69

a. Construct a 95% confidence interval for the mean μ_d of the population paired differences, where a paired difference is defined as the difference between the memory test scores of a student before and after attending this course.
b. Test at the 1% significance level whether this course makes any statistically significant improvement in the memory of all students.

Assume that the population of the paired differences has a normal distribution.

10.93 According to a survey of 1000 men and 900 women, 21% of men and 28% of women read for fun almost every day.

a. Construct a 95% confidence interval for the difference between the proportions of all men and all women who read for fun every day.
b. Test at the 2% significance level whether the proportions of all men and all women who read for fun every day are different.

10.94 The National Television Violence Study, released in 1998, analyzed 2750 different programs (*Education Week,* April 22, 1998). According to this study, 53% of music videos and 67% of children's programs contained at least one violent act. Suppose that these percentages were based on random samples of 380 music videos and 440 children's programs.

a. Construct a 99% confidence interval for the difference between the two population proportions.
b. At the 2.5% significance level, can you conclude that the percentage of music videos with violent acts is lower than the percentage of children's programs with violent acts?

10.95 In a 1998 poll, 1151 adults were asked whether teacher unionization had helped, hurt, or made no difference in the quality of public education in the United States (1998 Phi Delta Kappa/Gallup Poll of the Public's Attitudes Toward the Public Schools, *Education Week,* October 7, 1998). Twenty-eight percent of public school parents and 23% of nonpublic school parents said that unionization had helped. Suppose that the sample of 1151 adults included 410 public school parents and 245 nonpublic school parents (the remaining adults had no children in school).

a. Find a 95% confidence interval for the difference between the two population proportions.
b. At the 2.5% significance level, can you conclude that the percentage of all public school parents who feel that unionization has helped is different from the percentage of all nonpublic school parents who hold this view?

10.96 According to data from Teenage Research Unlimited, 21% of U.S. teenage boys described Levi's as a "cool" brand in 1994; however, only 7% held this opinion in 1998 (*Fortune,* April 12, 1999). Suppose that these percentages are based on random samples of 300 U.S. teenage boys in 1994 and 290 in 1998.

a. Construct a 98% confidence interval for the difference between the two population proportions.
b. Using the 1% significance level, can you conclude that the proportion of all U.S. teenage boys who considered Levi's a "cool" brand in 1998 was less than the corresponding proportion in 1994?

Advanced Exercises

10.97 Manufacturers of two competing automobile models, Gofer and Diplomat, each claim to have the lowest mean fuel consumption. Let μ_1 be the mean fuel consumption in miles per gallon (mpg) for the Gofer and μ_2 the mean fuel consumption in mpg for the Diplomat. The two manufacturers have agreed to a test in which several cars of each model will be driven on a 100-mile test run. Then the fuel consumption, in mpg, will be calculated for each test run. The average of the mpg for all 100-mile test runs for each model gives the corresponding mean. Assume that for each model the gas mileages for the test runs are normally distributed with $\sigma = 2$ mpg. Note that each car is driven for one and only one 100-mile test run.

a. How many cars (i.e., sample size) for each model are required to estimate $\mu_1 - \mu_2$ with a 90% confidence level and with a maximum error of estimate of 1.5 mpg? Use the same number of cars (i.e., sample size) for each model.

b. If μ_1 is actually 33 mpg and μ_2 is actually 30 mpg, what is the probability that five cars for each model would yield $\bar{x}_1 \geq \bar{x}_2$?

10.98 Maria and Ellen both specialize in throwing the javelin. Maria throws the javelin a mean distance of 200 feet with a standard deviation of 10 feet, whereas Ellen throws the javelin a mean distance of 210 feet with a standard deviation of 12 feet. Assume that the distances each of these athletes throws the javelin are normally distributed with these means and standard deviations. If Maria and Ellen each throw the javelin once, what is the probability that Maria's throw is longer than Ellen's?

10.99 A new type of sleeping pill is tested against an older standard pill. Two thousand insomniacs are randomly divided into two equal groups. The first group is given the old standard pill, while the second group receives the new pill. The time required to fall asleep after the pill is administered is recorded for each person. The results of the experiment are given in the following table, where $\bar{x}$ and s represent the mean and standard deviation, respectively, for the times required to fall asleep for people in each group after the pill is taken.

	Group 1 (Old Pill)	**Group 2 (New Pill)**
n	1000	1000
$\bar{x}$	15.4 minutes	15.0 minutes
s	3.5 minutes	3.0 minutes

Consider the test of hypothesis H_0: $\mu_1 - \mu_2 = 0$ versus H_1: $\mu_1 - \mu_2 > 0$, where μ_1 and μ_2 are the mean times required for all potential users to fall asleep using the old pill and the new pill, respectively.

a. Find the p-value for this test.
b. Does your answer to part a indicate that the result is statistically significant? Use $\alpha = .025$.
c. Find the 95% confidence interval for $\mu_1 - \mu_2$.
d. Does your answer to part c imply that this result is of great *practical* significance?

10.100 Gamma Corporation is considering the installation of governors on cars driven by its sales staff. These devices would limit the car speeds to a preset level, which is expected to improve fuel economy. The company is planning to test several cars for fuel consumption without governors for one week. Then governors would be installed in the same cars, and fuel consumption will be monitored for another week. Gamma Corporation wants to estimate the mean difference in fuel consumption with a maximum error of estimate of 2 mpg with a 90% confidence level. Assume that the differences in fuel consumption are normally distributed and that previous studies suggest that an estimate of $s_d = 3$ mpg is reasonable. How many cars should be tested? (Note that the critical value of t will depend on n, so it will be necessary to use trial and error.)

10.101 Refer to Exercise 10.100. Suppose Gamma Corporation decides to test governors on seven cars. However, the management is afraid that the speed limit imposed by the governors will reduce the number of contacts the salespersons can make each day. Thus, both the fuel consumption and the number of contacts made are recorded for each car/salesperson for each week of the testing period, both before and after the installation of governors.

	Number of Contacts		**Fuel Consumption (mpg)**	
Salesperson	**Before**	**After**	**Before**	**After**
A	50	49	25	26
B	63	60	21	24
C	42	47	27	26
D	55	51	23	25
E	44	50	19	24
F	65	60	18	22
G	66	58	20	23

Suppose that as a statistical analyst with the company, you are directed to prepare a brief report that includes statistical analysis and interpretation of the data. Management will use your report to help decide whether or not to install governors on all salespersons' cars. Use the 90% confidence intervals and .05 significance levels for any hypothesis tests to make suggestions. Assume that the differences in fuel consumption and the differences in the number of contacts are both normally distributed.

10.102 Two competing airlines, Alpha and Beta, fly a route between Des Moines, Iowa, and Wichita, Kansas. Each airline claims to have a lower percentage of flights that arrive late. Let p_1 be the proportion of Alpha's flights that arrive late and p_2 the proportion of Beta's flights that arrive late.

a. You are asked to observe a random sample of arrivals for each airline to estimate $p_1 - p_2$ with a 90% confidence level and a maximum error of estimate of .05. How many arrivals for each airline would you have to observe? (Assume that you will observe the same number of arrivals, n, for each airline. To be sure of taking a large enough sample, use $p_1 = p_2 = .50$ in your calculations for n.)
b. Suppose that p_1 is actually .30 and p_2 is actually .23. What is the probability that a sample of 100 flights for each airline (200 in all) would yield $\hat{p}_1 \geq \hat{p}_2$?

10.103 Refer to Exercise 10.59, in which a random sample of six cars was selected to test a gasoline additive. The six cars were driven for one week without the gasoline additive and then for one week with the additive. The data reproduced here from that exercise show miles per gallon without and with the additive.

Without	24.6	28.3	18.9	23.7	15.4	29.5
With	26.3	31.7	18.2	25.3	18.3	30.9

Suppose that instead of the study with six cars, a random sample of 12 cars is selected and these cars are divided randomly into two groups of six cars each. The cars in the first group are driven for one week without the additive, and the cars in the second group are driven for one week with the additive. Suppose that the top row of the table lists the gas mileages for the six cars without the additive, and the bottom row gives the gas mileages for the cars with the additive. Assume that the distributions of the gas mileages with or without the additive are (approximately) normal with equal standard deviations.

a. Would a paired sample test as described in Section 10.4 be appropriate in this case? Why or why not? Explain.
b. If the paired sample test is inappropriate here, carry out a suitable test whether the mean gas mileage is lower without the additive. Use $\alpha = .025$.
c. Compare your conclusion in part b with the result of the hypothesis test in Exercise 10.59.

10.104 Does the use of cellular telephones increase the risk of brain tumors? Suppose that a manufacturer of cellular telephones hires you to answer this question because of concern about public liability suits. How would you conduct an experiment to address this question? Be specific. Explain how you would observe, how many observations you would take, and how you would analyze the data once you collect them. What are your null and alternative hypotheses? Would you want to use a higher or a lower significance level for the test? Explain.

10.105 We wish to estimate the difference between the mean scores on a standardized test of students taught by Instructors A and B. The scores of all students taught by Instructor A have a normal distribution with a standard deviation of 15, and the scores of all students taught by Instructor B have a normal distribution with a standard deviation of 10. To estimate the difference between the two means, you decide that the same number of students from each instructor's class should be observed.

a. Assuming that the sample size is the same for each instructor's class, how large a sample should be taken from each class to estimate the difference between the mean scores of the two populations to within 5 points with 90% confidence?
b. Suppose that samples of the size computed in part a will be selected in order to test for the difference between the two population mean scores using a .05 level of significance. How large does the difference between the two sample means have to be for us to conclude that the two population means are different?

10.106 The weekly weight losses of all dieters on Diet I have a normal distribution with a mean of 1.3 pounds and a standard deviation of .4 pound. The weekly weight losses of all dieters on Diet II have a normal distribution with a mean of 1.5 pounds and a standard deviation of .7 pound. A random sample of 25 dieters on Diet I and another sample of 36 dieters on Diet II are observed.

a. What is the probability that the difference between the two sample means, $\bar{x}_1 - \bar{x}_2$, will be less than .15 pound?

b. What is the probability that the average weight loss $\bar{x}_1$ for dieters on Diet I will be greater than the average weight loss $\bar{x}_2$ for dieters on Diet II?

c. If the average weight loss of the 25 dieters using Diet I is computed to be 2.0 pounds, what is the probability that the difference between the two sample means, $\bar{x}_1 - \bar{x}_2$, will be less than .15 pound?

10.107 Sixty-five percent of all male voters and 40% of all female voters favor a particular candidate. A sample of 100 male voters and another sample of 100 female voters will be polled. What is the probability that at least 10 more male voters than female voters will favor this candidate?

SELF-REVIEW TEST

1. To test the hypothesis that the mean blood pressure of university professors is lower than that of company executives, which of the following would you use?

   a. A left-tailed test **b.** A two-tailed test **c.** A right-tailed test

2. Briefly explain the meaning of independent and dependent samples. Give one example of each of these cases.

3. A company psychologist wanted to test if company executives have job-related stress scores higher than those of university professors. He took a sample of 40 executives and 50 professors and tested them for job-related stress. The sample of 40 executives gave a mean stress score of 7.6 with a standard deviation of .8. The sample of 50 professors produced a mean stress score of 5.4 with a standard deviation of 1.3.

   a. Construct a 99% confidence interval for the difference between the mean stress scores of all executives and all professors.

   b. Test at the 2.5% significance level whether the mean stress score of all executives is higher than that of all professors.

4. A sample of 20 alcoholic fathers showed that they spend an average of 2.3 hours per week playing with their children with a standard deviation of .54 hour. A sample of 25 nonalcoholic fathers gave a mean of 4.6 hours per week with a standard deviation of .8 hour.

   a. Construct a 95% confidence interval for the difference between the mean times spent per week playing with their children by all alcoholic and all nonalcoholic fathers.

   b. Test at the 1% significance level whether the mean time spent per week playing with their children by all alcoholic fathers is less than that of nonalcoholic fathers.

Assume that the times spent per week playing with their children by all alcoholic and all nonalcoholic fathers both are normally distributed with equal but unknown standard deviations.

5. Repeat Problem 4 assuming that the times spent per week playing with their children by all alcoholic and all nonalcoholic fathers both are normally distributed with unequal and unknown standard deviations.

6. The table on page 484 gives the number of items made in one hour by seven randomly selected workers on two different machines.

Worker	1	2	3	4	5	6	7
Machine I	15	18	14	20	16	18	21
Machine II	16	20	13	23	19	18	20

a. Construct a 99% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the number of items made by an employee in one hour on Machine I minus the number of items made by the same employee in one hour on Machine II.

b. Test at the 5% significance level whether the mean μ_d of the population paired differences is different from zero.

Assume that the population of paired differences is (approximately) normally distributed.

7. A sample of 500 male registered voters showed that 57% of them voted in the last presidential election. Another sample of 400 female registered voters showed that 55% of them voted in the same election.

a. Construct a 97% confidence interval for the difference between the proportion of all male and all female registered voters who voted in the last presidential election.

b. Test at the 1% significance level whether the proportion of all male voters who voted in the last presidential election is different from that of all female voters.

MINI-PROJECTS

Mini-Project 10–1

Medical science is continually seeking safe and effective drugs for the prevention of influenza. Zanamivir, a promising antiviral drug, was tested in a double-blind, randomized, placebo-controlled clinical trial for 28 days during the 1997–1998 flu season (Arnold S. Monto et al., "Zanamivir in the Prevention of Influenza Among Healthy Adults: A Randomized Controlled Trial," *JAMA* 282 [1], July 7, 1999). The researchers recruited 1107 healthy adults for the experiment and divided them randomly into two groups: a treatment group of 553 participants who received zanamivir and a control group of 554 who were given a placebo. During the experiment, 11 participants in the treatment group and 34 participants in the control group contracted influenza. Explain how to construct a 95% confidence interval for the difference between the relevant population proportions. Also describe an appropriate hypothesis test, using the given data, to evaluate the effectiveness of zanamivir in preventing flu.

Find a similar article in a journal of medicine, psychology, or another field that lends itself to confidence intervals and hypothesis tests for differences in two means or proportions. First explain how to make the confidence intervals and hypothesis tests; then do so using the data given in that article.

Mini-Project 10–2

A researcher conjectures that cities in the more populous states of the United States tend to have higher costs for hospital rooms. Using "CITY DATA" that accompany this text, select a random sample of 10 cities from the six most populous states (California, Texas, New York, Florida, Pennsylvania, and Illinois). Then take a random sample of 10 cities from the remaining states in the data set. For each of the 20 cities, record the average daily cost of a semiprivate hospital room. Assume that such costs are approximately normally distributed for all cities in each of the two groups of states. Further assume that the cities you selected make random samples of all cities for the two groups of states.

a. Construct a 95% confidence interval for the difference in the means of such hospital costs for all cities in the two groups of states.

b. At the 5% level of significance, can you conclude that the average daily cost of a semiprivate hospital room for all cities in the six most populous states is higher than that of such a room for all cities in the remaining states?

For instructions on using MINITAB for this chapter, please see Section B.9 of Appendix B.

See Appendix C for *Excel Adventure 10* on inferences on two population means.

COMPUTER ASSIGNMENTS

CA10.1 A random sample of 13 male college students who hold jobs gave the following data on their GPAs.

3.12	2.84	2.43	2.15	3.92	2.45	2.73
3.06	2.36	1.93	2.81	3.27	1.83	

Another random sample of 16 female college students who also hold jobs gave the following data on their GPAs.

2.76	3.84	2.24	2.81	1.79	3.89	2.96	3.77
2.36	2.81	3.29	2.08	3.11	1.69	2.84	3.02

a. Using MINITAB or any other statistical software, construct a 99% confidence interval for the difference between the mean GPAs of all male and all female college students who hold jobs.
b. Using MINITAB or any other statistical software, test at the 5% significance level whether the mean GPAs of all male and all female college students who hold jobs are different.

Assume that the GPAs of all such male and female college students are normally distributed with equal but unknown population standard deviations.

CA10.2 A company recently opened two supermarkets in two different areas. The management wants to know if the mean sales per day for these two supermarkets are different. A sample of 10 days for Supermarket A produced the following data on daily sales (in thousand dollars).

47.56	57.66	51.23	58.29	43.71
49.33	52.35	50.13	47.45	53.86

A sample of 12 days for Supermarket B produced the following data on daily sales (in thousand dollars).

56.34	63.55	61.64	63.75	54.78	58.19
55.40	59.44	62.33	67.82	56.65	67.90

Assume that the daily sales of the two supermarkets are both normally distributed with equal but unknown standard deviations.

a. Using MINITAB or any other statistical software, construct a 99% confidence interval for the difference between the mean daily sales for these two supermarkets.
b. Using MINITAB or any other statistical software, test at the 1% significance level whether the mean daily sales for these two supermarkets are different.

CA10.3 Refer to Computer Assignment CA10.1. Now do that assignment assuming the GPAs of all such male and female college students are normally distributed with unequal and unknown population standard deviations.

CA10.4 Refer to Computer Assignment CA10.2. Now do that assignment assuming the daily sales of the two supermarkets are both normally distributed with unequal and unknown standard deviations.

CA10.5 The manufacturer of a gasoline additive claims that the use of this additive increases gasoline mileage. A random sample of six cars was selected. These cars were driven for one week without the gasoline additive and then for one week with the gasoline additive. The table at the top of page 486 gives the miles per gallon for these cars without and with the gasoline additive.

Without	24.6	28.3	18.9	23.7	15.4	29.5
With	26.3	31.7	18.2	25.3	18.3	30.9

a. Using MINITAB or any other statistical software, construct a 99% confidence interval for the mean μ_d of the population paired differences.
b. Using MINITAB or any other statistical software, test at the 1% significance level whether the use of the gasoline additive increases the gasoline mileage.

Assume that the population of paired differences is (approximately) normally distributed.

CA10.6 A company is considering installing new machines to assemble its products. The company is considering two types of machines, but it will buy only one type. The company selected eight assembly workers and asked them to use these two types of machines to assemble products. The following table gives the time taken (in minutes) to assemble one unit of the product on each type of machine for each of these eight workers.

Machine I	23	26	19	24	27	22	20	18
Machine II	21	24	23	25	24	28	24	23

a. Using MINITAB or any other statistical software, construct a 98% confidence interval for the mean μ_d of the population paired differences, where a paired difference is equal to the time taken to assemble a unit of the product on Machine I minus the time taken to assemble a unit of the product on Machine II by the same worker.
b. Using MINITAB or any other statistical software, test at the 5% significance level whether the mean time taken to assemble a unit of the product is different for the two types of machines.

Assume that the population of paired differences is (approximately) normally distributed.

CA10.7 A company has two restaurants in two different areas of New York City. The company wants to estimate the percentages of patrons who think that the food and service at each of these restaurants are excellent. A sample of 200 patrons taken from the restaurant in Area A showed that 118 of them think that the food and service are excellent at this restaurant. Another sample of 250 patrons selected from the restaurant in Area B showed that 160 of them think that the food and service are excellent at this restaurant.

a. Construct a 97% confidence interval for the difference between the two population proportions.
b. Testing at the 2.5% significance level, can you conclude that the proportion of patrons at the restaurant in Area A who think that the food and service are excellent is lower than the corresponding proportion at the restaurant in Area B?

CA10.8 The management of a supermarket wanted to investigate whether the percentages of all men and women who prefer to buy national-brand products over the store-brand products are different. A sample of 600 men shoppers at the company's supermarkets showed that 246 of them prefer to buy national-brand products over the store-brand products. Another sample of 700 women shoppers at the company's supermarkets showed that 266 of them prefer to buy national-brand products over the store-brand products.

a. Construct a 99% confidence interval for the difference between the proportions of all men and all women shoppers at these supermarkets who prefer to buy national-brand products over the store-brand products.
b. Testing at the 2% significance level, can you conclude that the proportions of all men and all women shoppers at these supermarkets who prefer to buy national-brand products over the store-brand products are different?