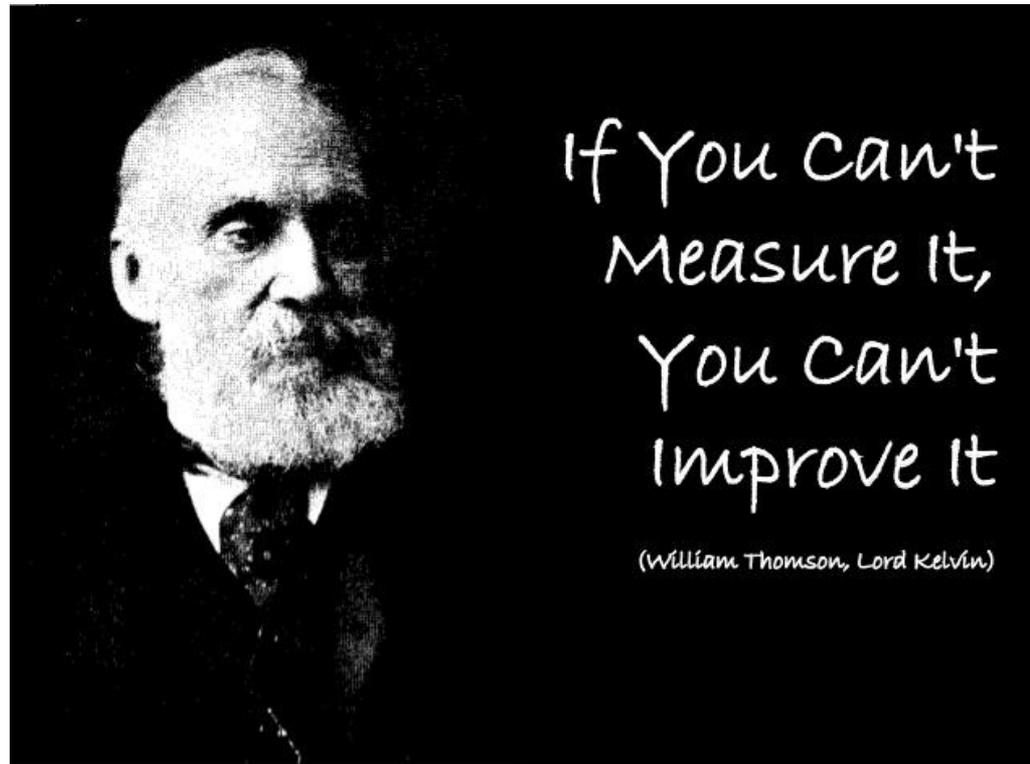


# DESIGN DEL PRODOTTO INDUSTRIALE

FISICA TECNICA PER IL DESIGN

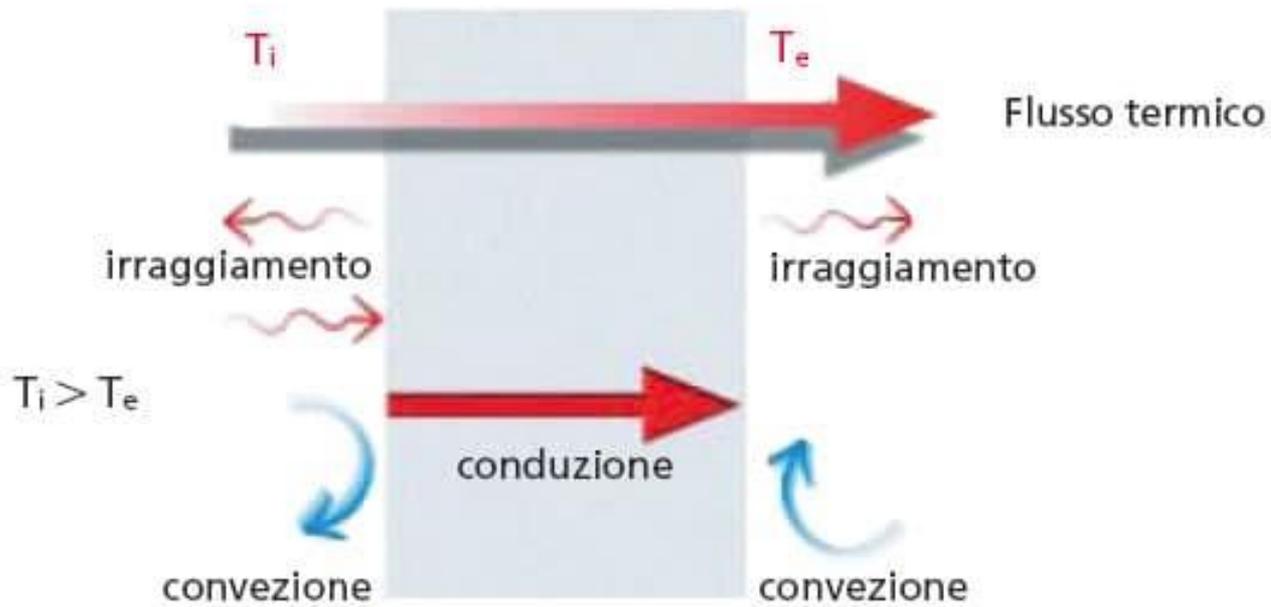
AA 2019-20

*Termocinetica*  
*principi*  
*S.03*

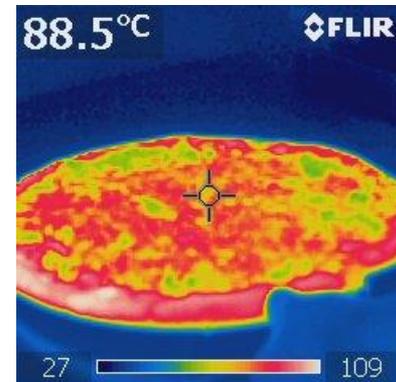
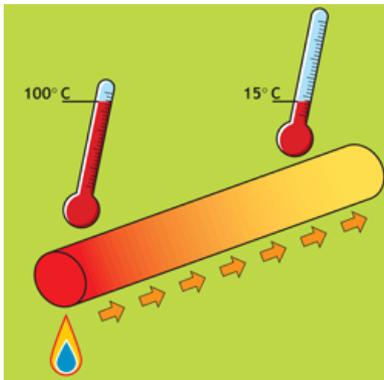


Michele Bottarelli - Dipartimento di Architettura di Ferrara  
[michele.bottarelli@unife.it](mailto:michele.bottarelli@unife.it)





<https://www.youtube.com/watch?v=KSxUX1XPiSk>



# ***HEAT TRANSFER***

## **CONDUCTION**

Heat transfer in the presence of mass but without mass exchanges  
It's the heat transfer that occurs in massive systems without a field velocity.  
(it is a singular case of convection)

## **CONVECTION**

Heat transfer in the presence of mass exchanges.  
Solid & Fluid

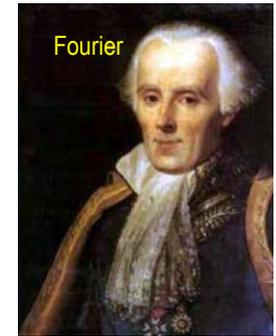
## **RADIATION**

Heat transfer in the absence of matter for mutual exposure of the radiating bodies  
Phenomenon linked to electromagnetism

# HEAT CONDUCTION

In steady state, the Fourier's law is

$$\dot{q} = -\lambda \cdot \frac{\Delta T}{s}$$

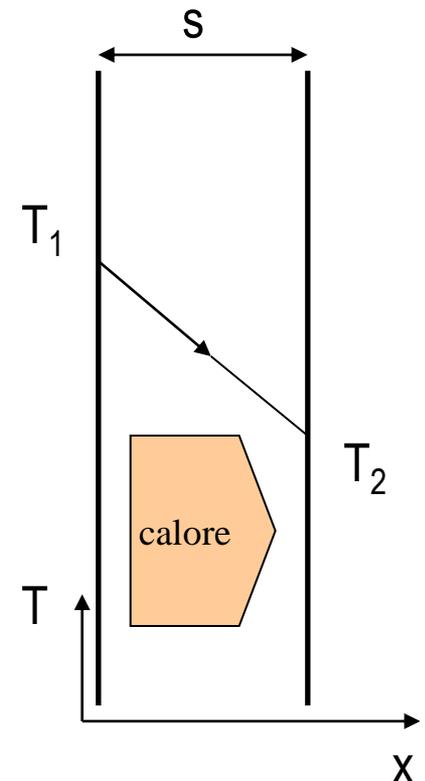


where:

- q W/m<sup>2</sup> heat flux
- λ W/(m K) thermal conductivity
- ΔT K wall difference temperature
- s m thickness

Materiale	λ [W/(m K)]
<i>Gas alla pressione atmosferica</i>	0,007 ÷ 0,2
<i>Materiali isolanti</i>	0,02 ÷ 0,2
<i>Liquidi non metallici</i>	0,05 ÷ 0,7
<i>Solidi non metallici</i>	0,3 ÷ 2,3
<i>Metalli liquidi</i>	8 ÷ 80
<i>Metalli e leghe metalliche</i>	14 ÷ 420

[https://it.wikipedia.org/wiki/Conducibilit%C3%A0\\_termica](https://it.wikipedia.org/wiki/Conducibilit%C3%A0_termica)



## HEAT CONDUCTION (single layer)

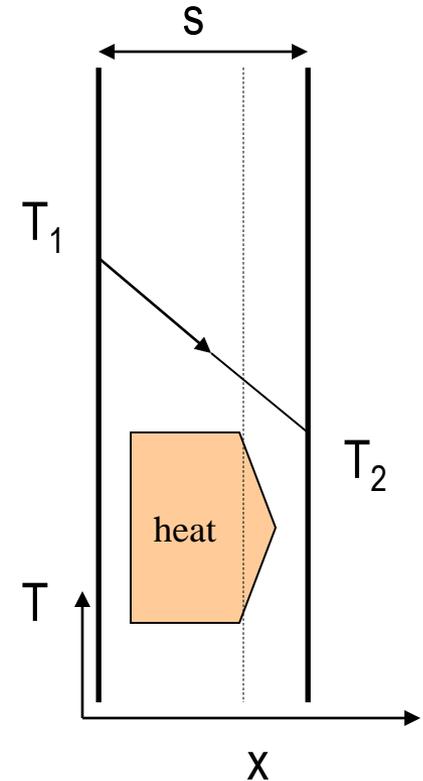
A	1	m <sup>2</sup>
$\lambda$	1	W/(m K)
$\Delta T$	20	K
s	0,30	m

$$|\dot{Q}| = A \cdot |\dot{q}| = A \cdot \lambda \cdot \left| \frac{\Delta T}{s} \right| \cong 67W$$

In steady state, known the wall temperatures:

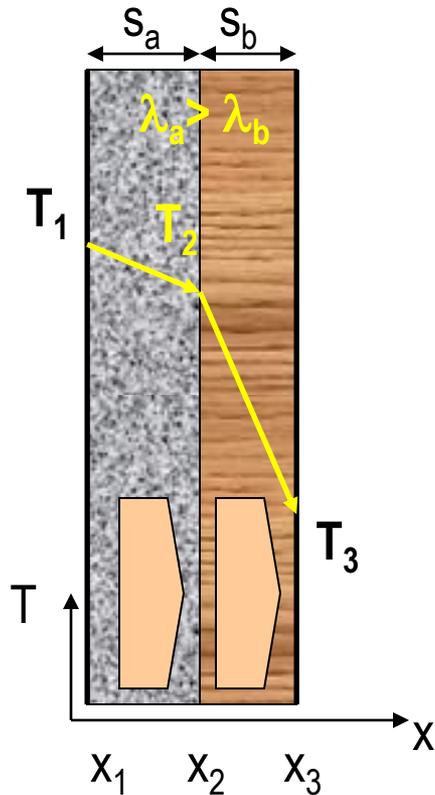
$$\begin{aligned}\dot{q}(s) &= \dot{q}(x) \\ \Rightarrow \lambda \cdot \frac{T_1 - T_2}{s} &= \lambda \cdot \frac{T_1 - T(x)}{x} \\ \Rightarrow T(x) &= T_1 - \frac{x}{s} \cdot (T_1 - T_2)\end{aligned}$$

In steady state, T is independent from  $\lambda$ , if  $\lambda = \text{const.}$



## HEAT CONDUCTION (double layer)

In steady state:  $\dot{q}_a = \dot{q}_b$



$$\lambda_a \cdot \frac{T_1 - T_2}{s_a} = \lambda_b \cdot \frac{T_2 - T_3}{s_b}$$

$$\Rightarrow T_2 = \frac{\frac{\lambda_a}{s_a} \cdot T_1 + \frac{\lambda_b}{s_b} \cdot T_3}{\frac{\lambda_a}{s_a} + \frac{\lambda_b}{s_b}}$$

CONDUCTANCE:  $c = \frac{\lambda}{s}$

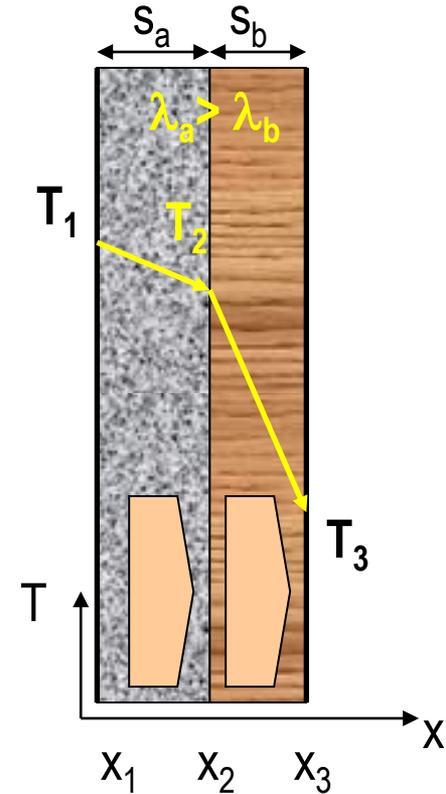
$$\Rightarrow T_2 = \frac{c_a \cdot T_1 + c_b \cdot T_3}{c_a + c_b}$$

## HEAT CONDUCTION (double layer)

$$\dot{q} = \dot{q}_a = \frac{\lambda_a}{s_a} \cdot (T_1 - T_2) = \dot{q}_b = \frac{\lambda_b}{s_b} \cdot (T_2 - T_3)$$

$$\dot{q} = c_a \cdot \left( T_1 - \frac{c_a \cdot T_1 + c_b \cdot T_3}{c_a + c_b} \right) = \frac{c_a \cdot c_b}{c_a + c_b} \cdot (T_1 - T_3)$$

$$\dot{q} = \frac{T_1 - T_3}{\frac{1}{c_a} + \frac{1}{c_b}}$$



Thermal resistance of heat conduction:  $R_{cond} = \frac{1}{A \cdot c} = \frac{s}{A \cdot \lambda}$

$$\Rightarrow \dot{Q} = \frac{T_1 - T_3}{R_{cond,a} + R_{cond,b}}$$

$$[R_{cond}] = \frac{K}{W}$$

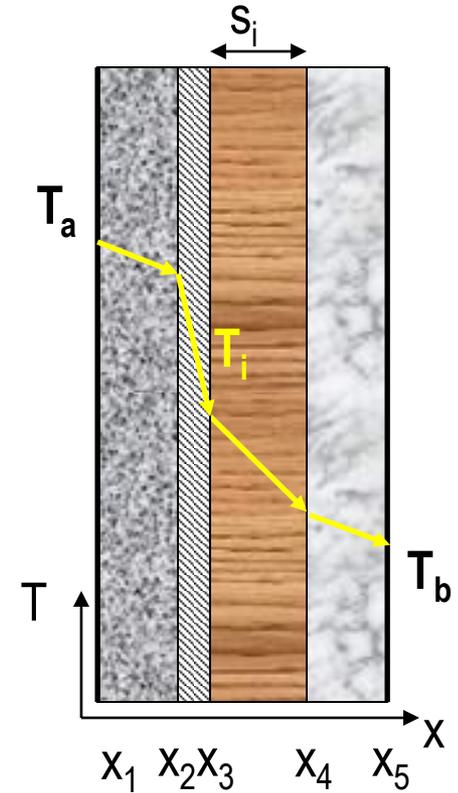
## HEAT CONDUCTION (multilayer)

For every layer:

$$R_{cond,i} = \frac{1}{A \cdot c_i} = \frac{s_i}{A \cdot \lambda_i}$$

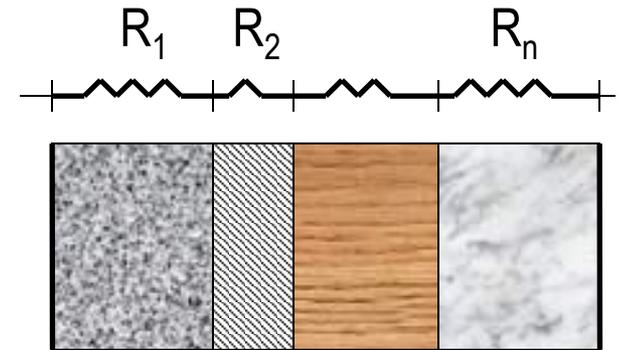
$$T_i = T_a - (T_a - T_b) \cdot \frac{\sum_{j=1}^i R_{cond,j}}{\sum_{k=1}^n R_{cond,k}}$$

$$\dot{Q} = \frac{T_a - T_b}{\sum_{i=1}^n R_{cond,i}}$$



# ELECTRICAL ANALOGY

$$\begin{array}{lcl} V & \leftrightarrow & \Delta T \\ i & \leftrightarrow & q \\ R_e & \leftrightarrow & R_{\text{cond},i} \end{array}$$



Kirchhoff's law

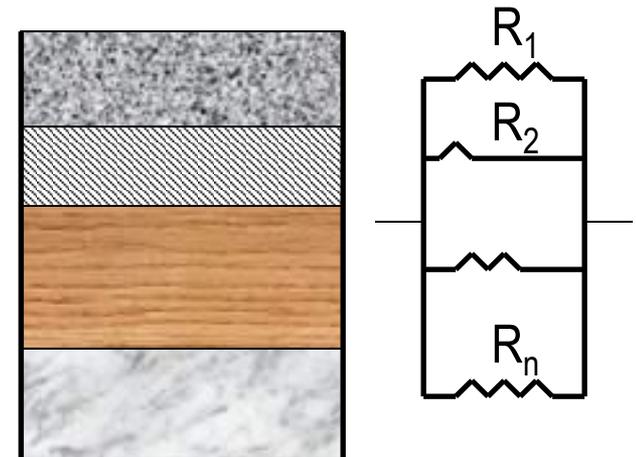
for thermal resistances in series

for thermal resistances in parallel

$$V = R_{\text{cond}}^{\text{tot}} \cdot i$$

$$R_{\text{cond}}^{\text{tot}} = \sum_{i=1}^n R_{\text{cond},i}$$

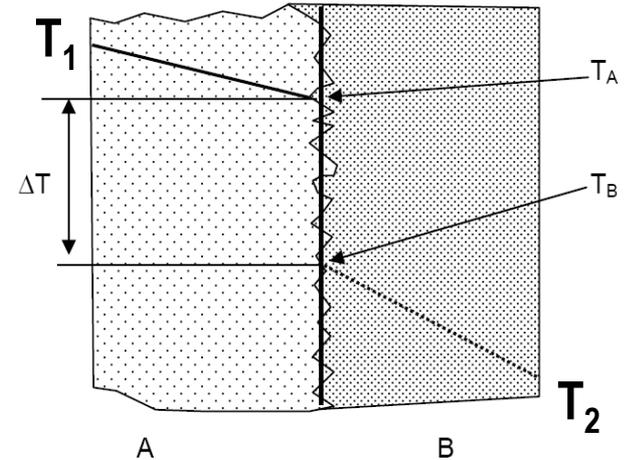
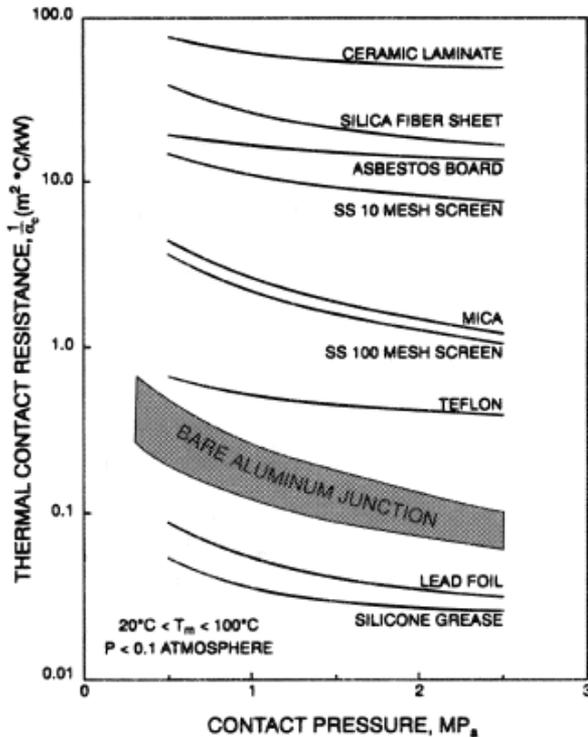
$$R_{\text{cond}}^{\text{tot}} = \frac{1}{\sum_{i=1}^n \frac{1}{R_{\text{cond},i}}}$$



# THERMAL CONTACT RESISTANCE

When a junction is formed by pressing two similar or dissimilar metallic materials together, only a small fraction of the nominal surface area is actually in contact because of the nonflatness and roughness of the contacting surfaces

$$R_{\text{contatto}} = 10^{-5} \div 10^{-3} \text{ m}^2 \text{ K} / \text{W}$$

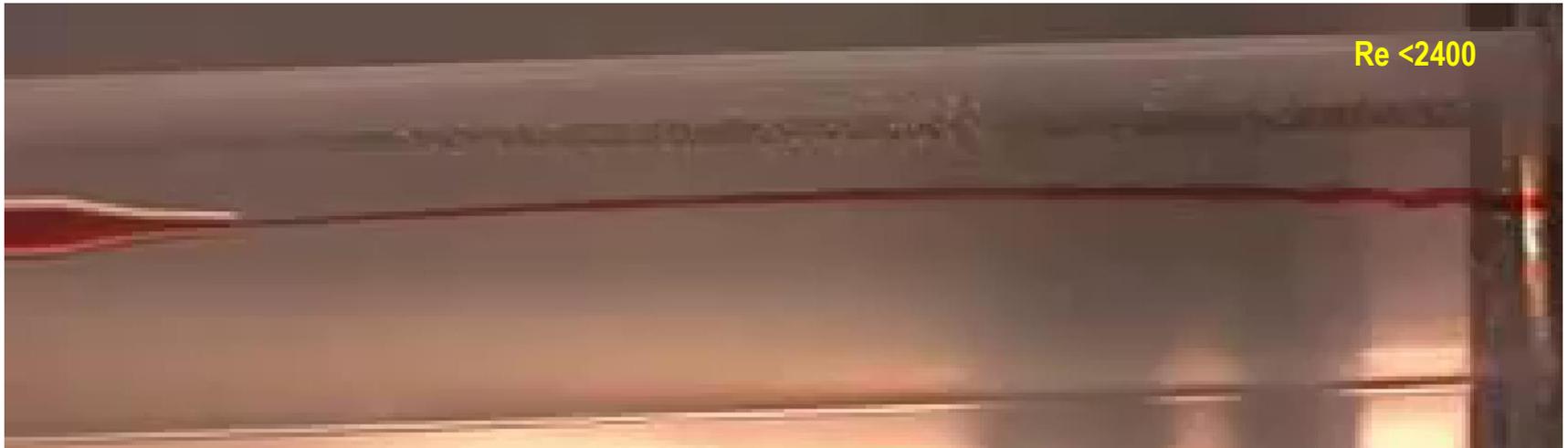


$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{cond},A} + R_{\text{contatto}} + R_{\text{cond},B}}$$

$$T_B = T_1 - (T_1 - T_2) \cdot \frac{R_{\text{cond},A} + R_{\text{contatto}}}{R_{\text{cond},A} + R_{\text{contatto}} + R_{\text{cond},B}}$$



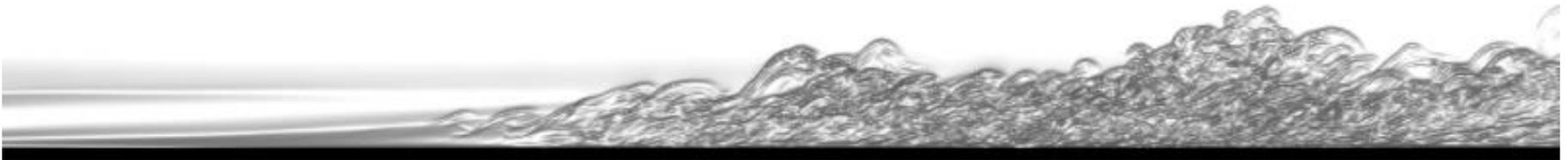
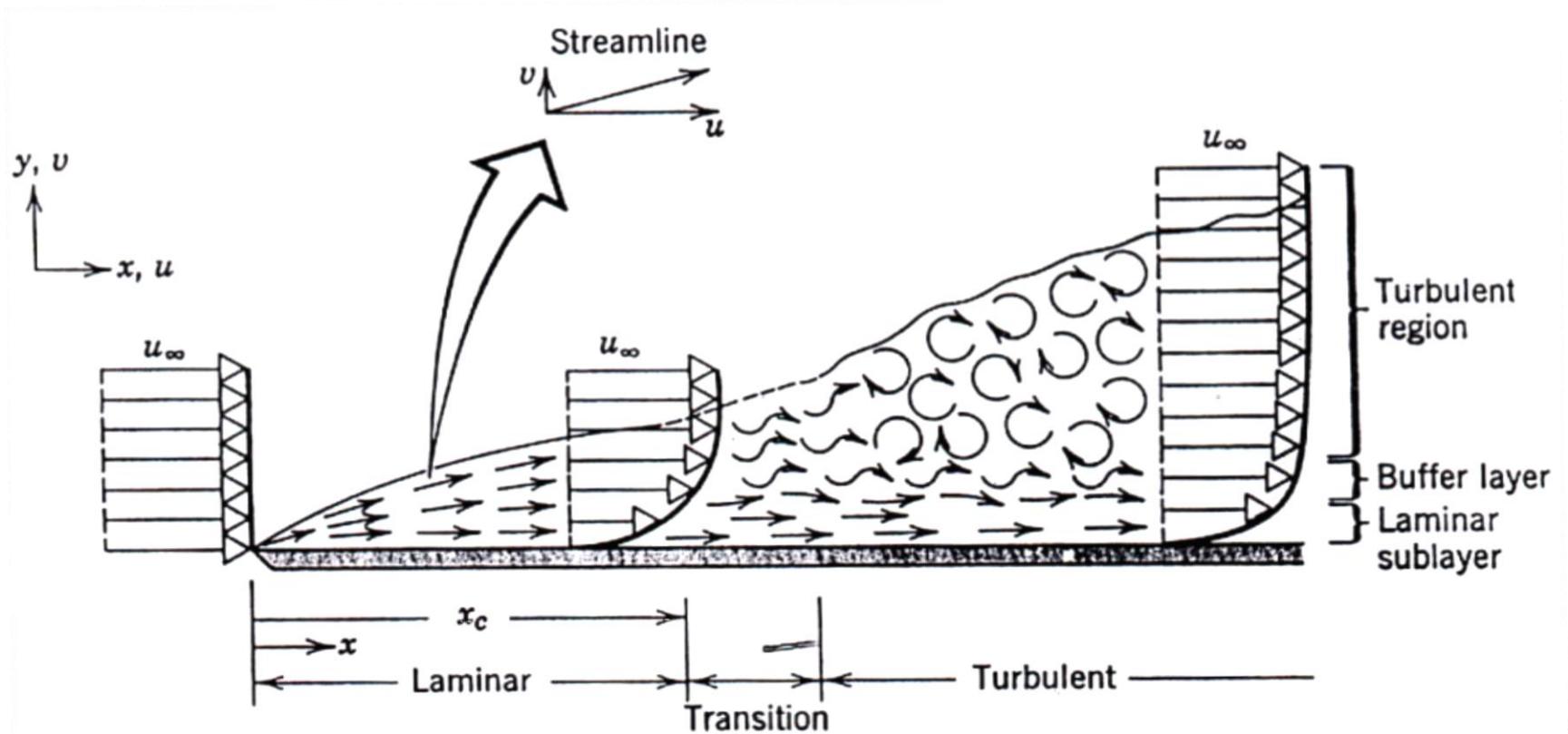
# MOTI LAMINARI E TURBOLENTI



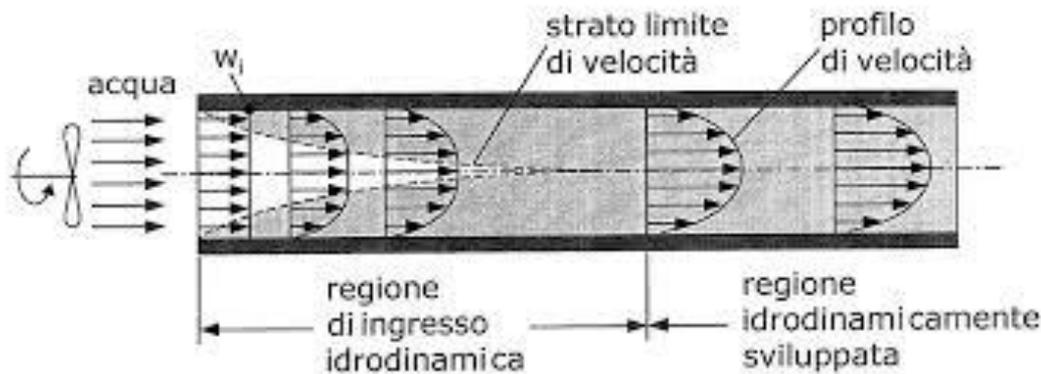
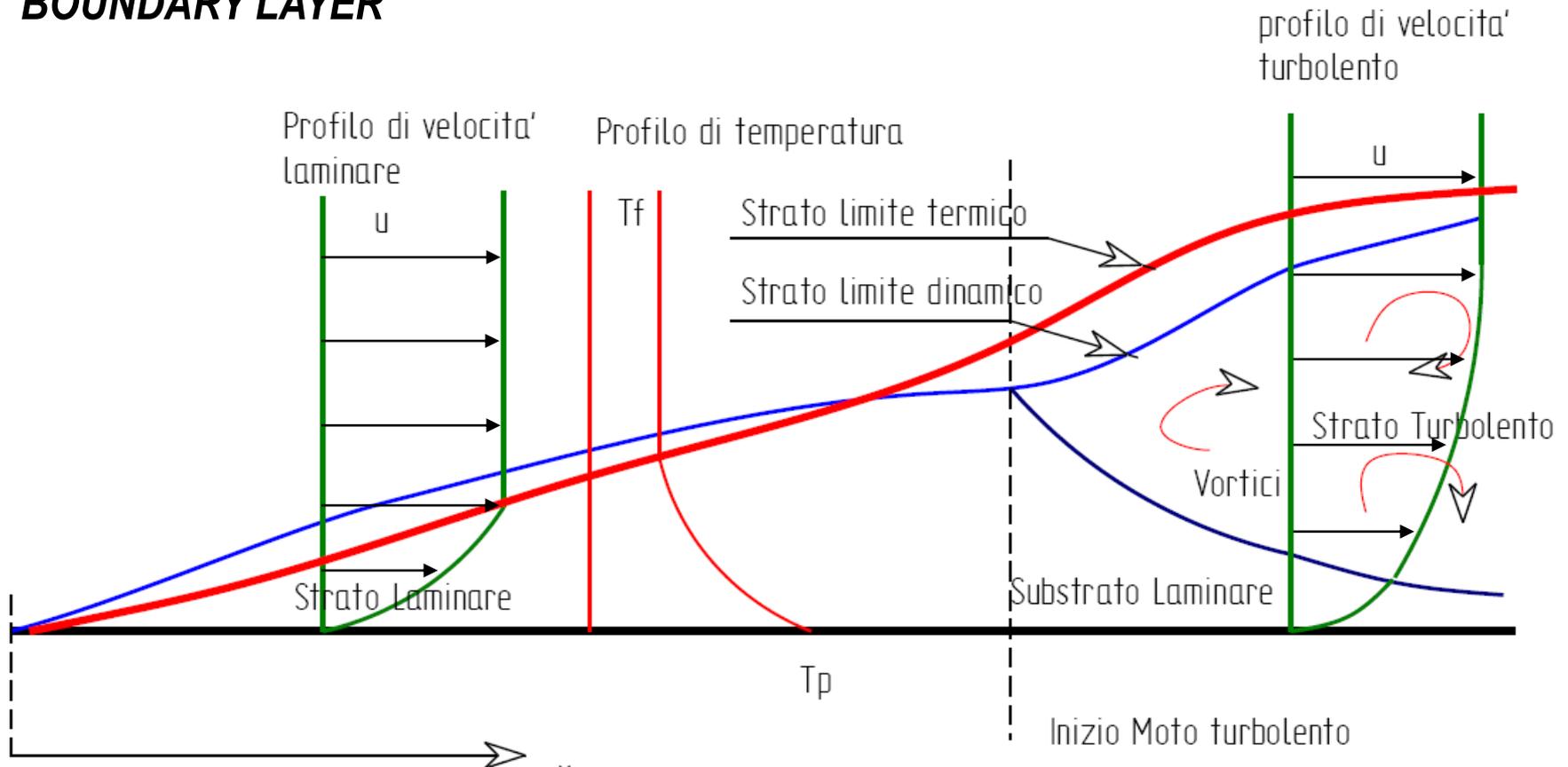
NUMERO di REYNOLDS

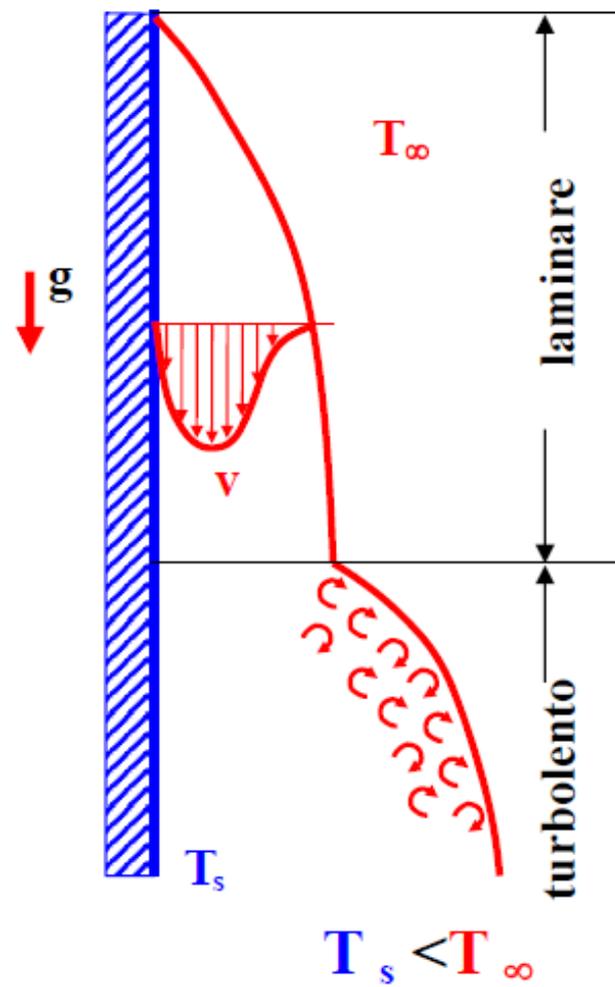
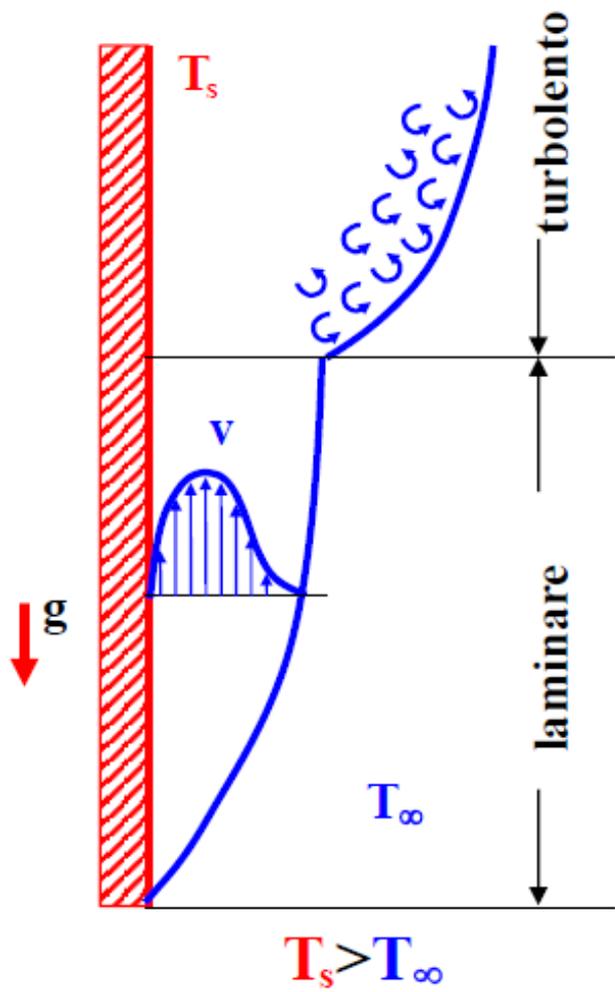
$$\text{Re} = \frac{\rho \cdot w \cdot D}{\mu} \propto \frac{\text{forze d'inerzia}}{\text{forze viscosse}} = \frac{\rho \cdot w^2 \cdot D}{\mu \cdot w}$$

# BOUNDARY LAYER



# BOUNDARY LAYER





## ADIMENSIONALIZZAZIONE E SIMILITUDINE

La formulazione rigorosa del fenomeno convettivo può essere adimensionalizzata attraverso:

$$\text{NUMERO DI REYNOLDS} \quad \text{Re} = \frac{\rho \cdot w \cdot D}{\mu} \propto \frac{\text{forze d'inerzia}}{\text{forze viscosse}}$$

$$\text{NUMERO DI GRASHOF} \quad \text{Gr} = \frac{\rho_p \cdot \beta \cdot g \cdot D \cdot \Delta T}{\mu^2} \propto \frac{\text{forze d'inerzia} \times \text{forze di galleg.}}{(\text{forze viscosse})^2}$$

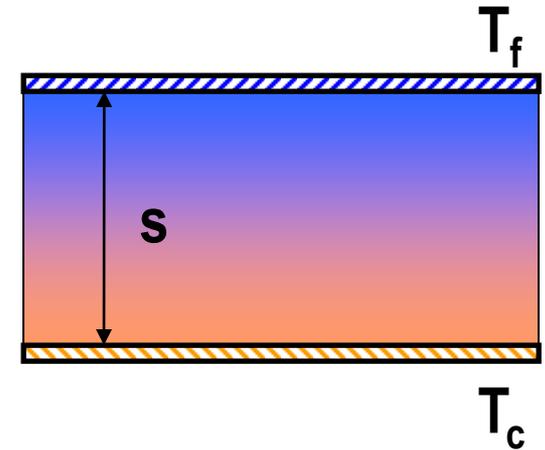
$$\text{NUMERO DI PRANDTL} \quad \text{Pr} = \frac{\nu}{\alpha} \propto \frac{\text{diffusivit\`a di moto}}{\text{diffusivit\`a termica}}$$

che consente di generalizzare le soluzioni in relazione ad una scala di similitudine:

$$(\text{Re}_1, \text{Gr}_1, \text{Pr}_1) = (\text{Re}_2, \text{Gr}_2, \text{Pr}_2)$$

## NUMERO DI NUSSELT

$$Nu = \frac{h \cdot s}{\lambda} = \frac{h \cdot (T_c - T_f)}{\frac{\lambda}{s} \cdot (T_c - T_f)} = \frac{\dot{q}_{conv}}{\dot{q}_{cond}}$$



Si può dimostrare che:

- quando  $Re^2 \gg Gr$  CONVENZIONE FORZATA

$$Nu = f(Re, Pr) = C \cdot Re^m \cdot Pr^n$$

- quando  $Re^2 \approx Gr$  CONVENZIONE MISTA

$$Nu = f(Re, Gr, Pr) = C \cdot Re^m \cdot Gr^n \cdot Pr^o$$

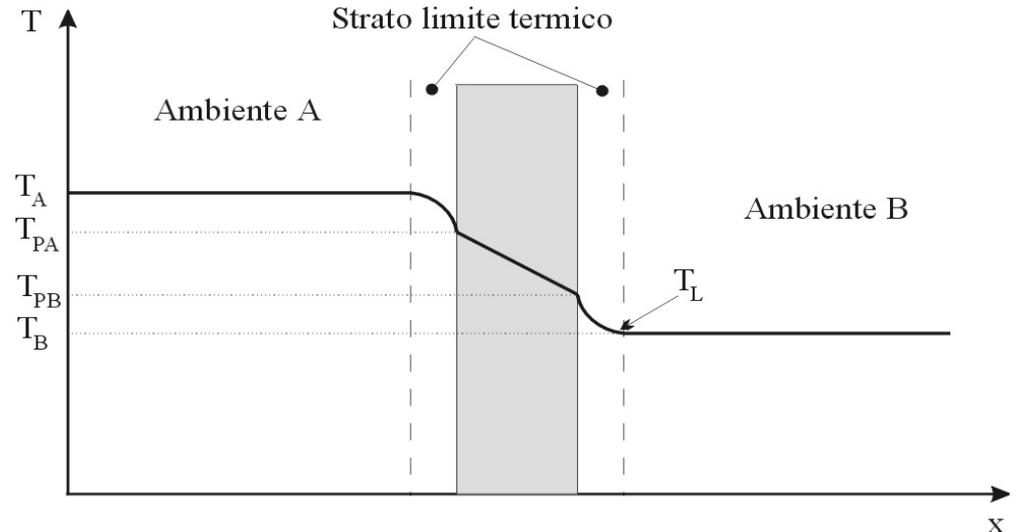
- quando  $Re^2 \ll Gr$  CONVENZIONE NATURALE

$$Nu = f(Gr, Pr) = C \cdot Gr^m \cdot Pr^n (= C' \cdot Ra^m)$$

# HEAT CONVECTION

## Newton's law

$$\dot{q}_{conv} = h \cdot (T_A - T_A^p)$$



$h$  is the coefficient of convective heat transfer ( $W / (m^2 K)$ ), which depends from fluid, thermal field (scalar) e velocity field (vectorial).

As for the electrical analogy:

$$\dot{q}_{conv} = \frac{(T_A - T_A^p)}{R_{conv}} \quad \text{where} \quad R_{conv} = \frac{1}{h}$$

<b>Tipo di convezione</b>	<b><math>h</math> <math>W / (m^2 K)</math></b>
Libera, gas	2÷25
Libera, liquidi	10÷1000
Forzata, gas	25÷250
Forzata, liquidi	50÷20.000
Ebollizione, Condensazione	2.500÷100.000

Riquadro

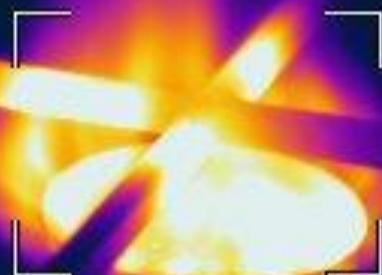
°C

150

Max. 150 >

Min. 23.5

Media 115 >



 FLIR

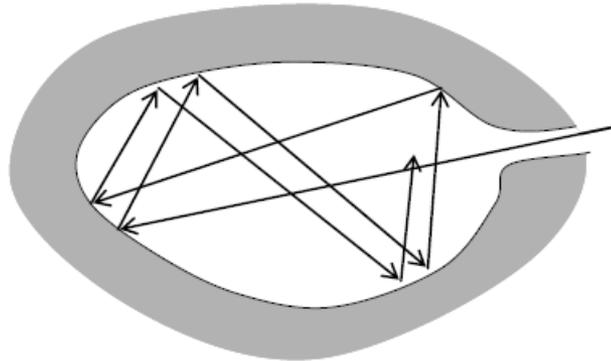
Dist = 1.0 Trefl = 20.0  $\epsilon = 0.95$

19.7

## TRASMISSIONE DEL CALORE PER IRRAGGIAMENTO

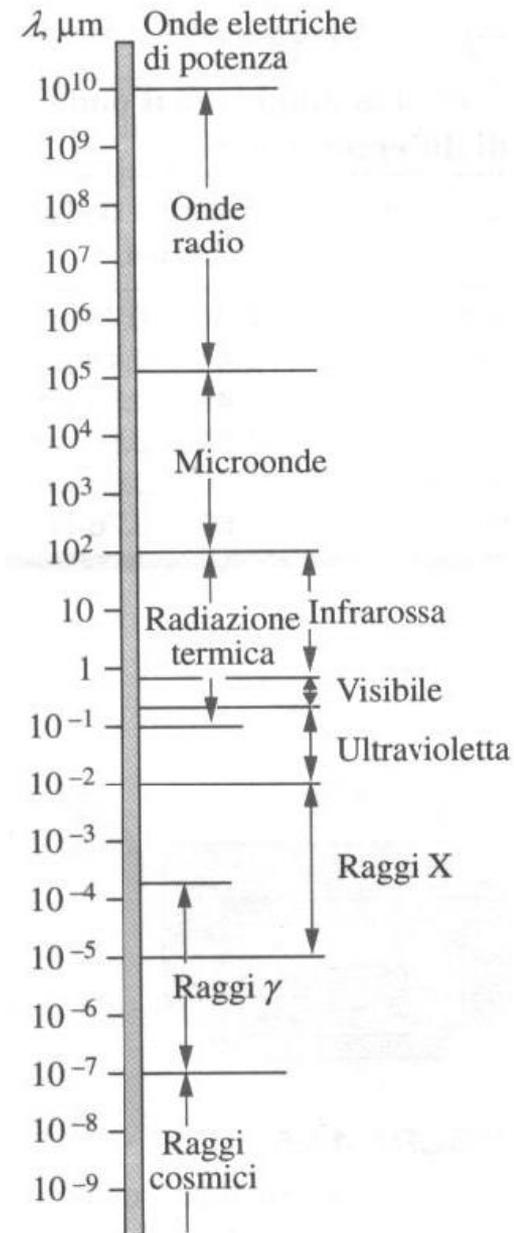
Ogni corpo con temperatura diversa dallo zero assoluto, irraggia energia.

L'irraggiamento è la radiazione termica, cioè l'energia trasportata da onde comprese tra  $0,10$  e  $100 \mu\text{m}$  ( $10^{12} \div 10^{15}$ Hz).

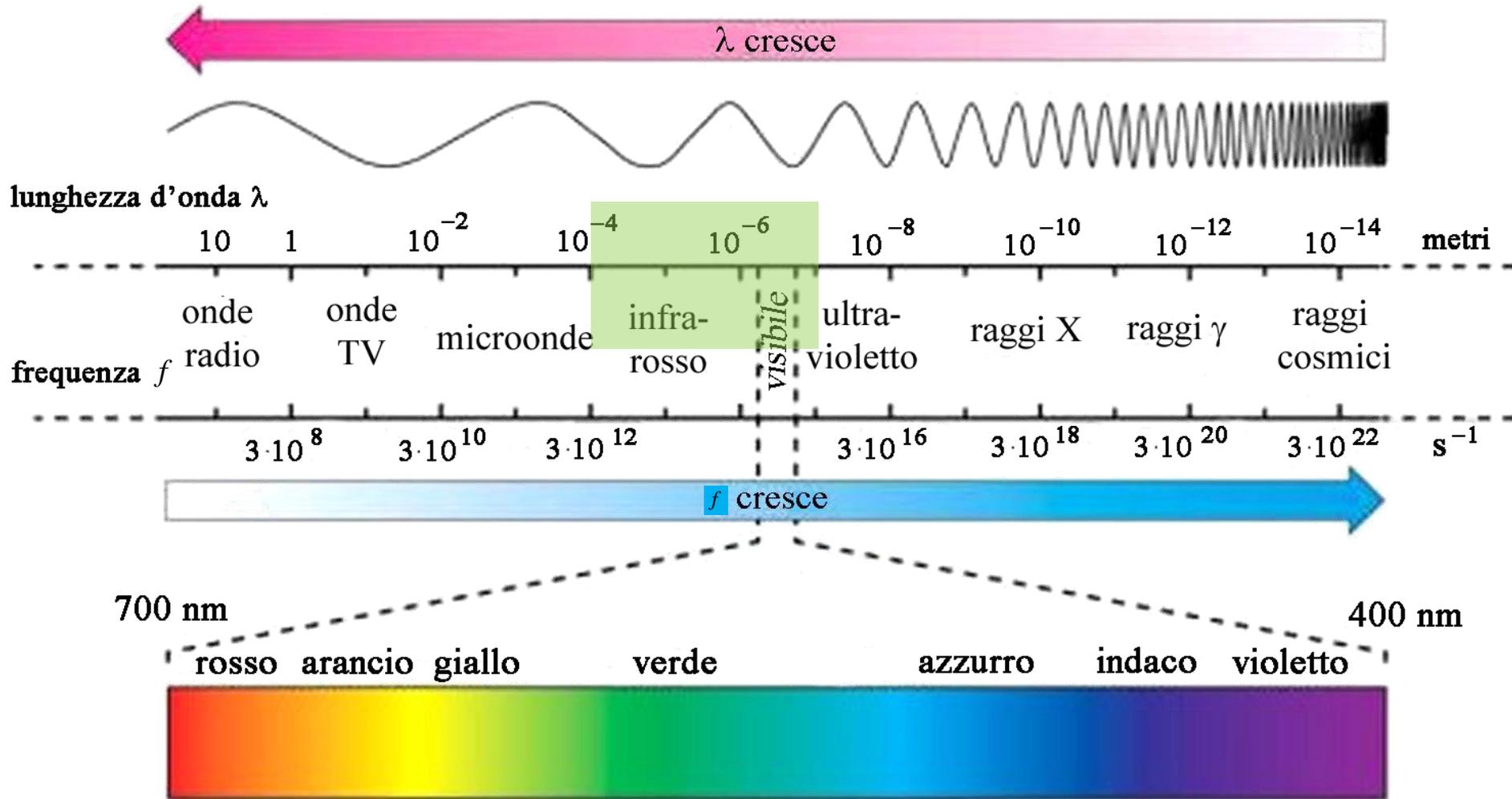


Si dice “*corpo nero*” un sistema

- capace di assorbire completamente la radiazione termica incidente;
- emettere la massima radiazione per assegnata temperatura.



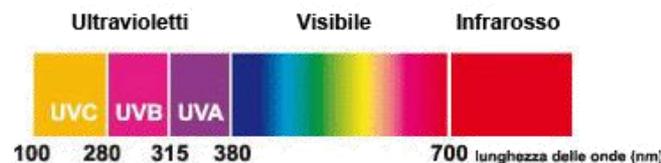
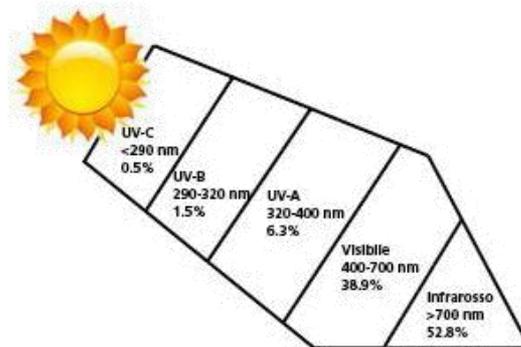
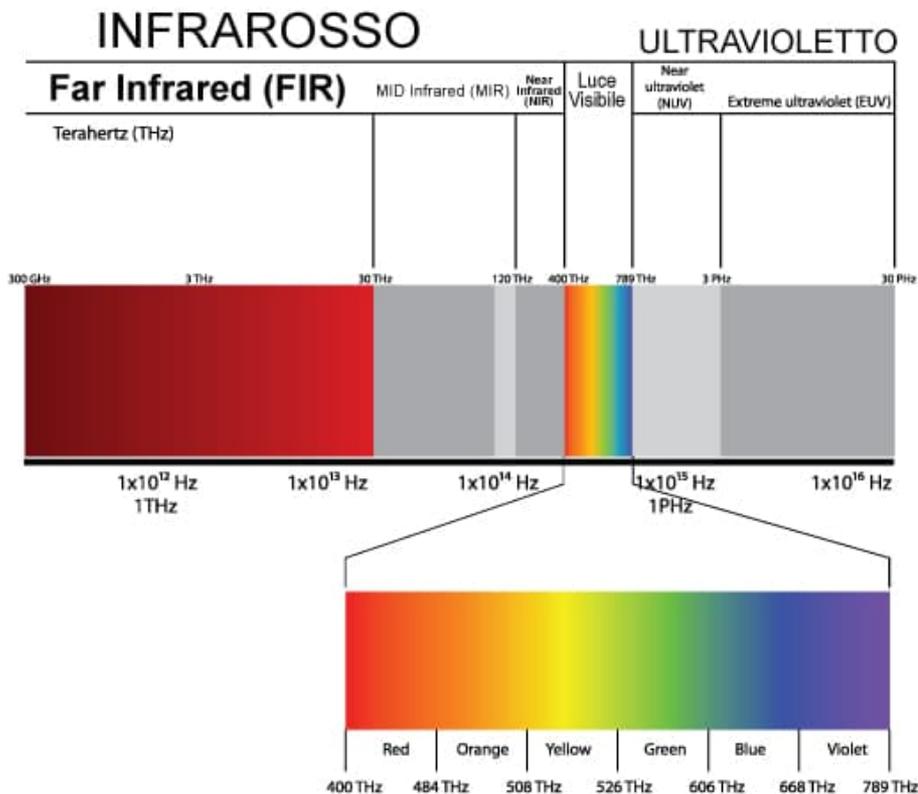
# RADIAZIONI ELETTRROMAGNETICHE



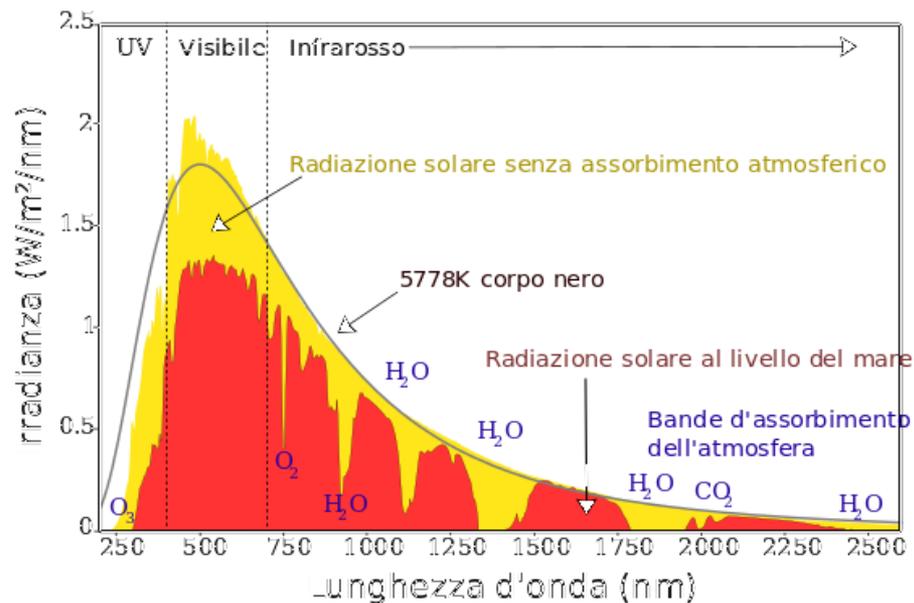
$$f = \frac{1}{T}$$

$$c = \lambda \cdot f$$

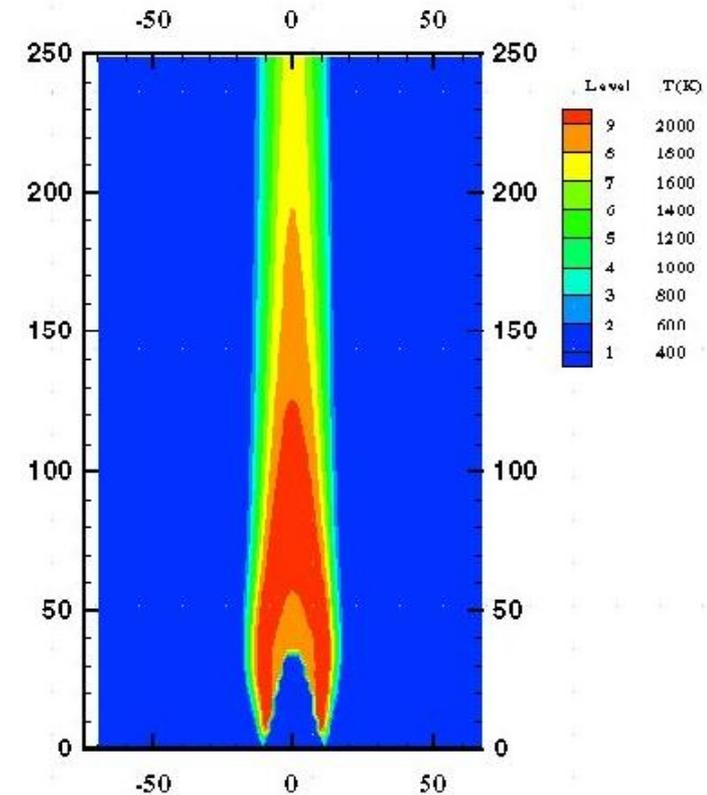
# RADIAZIONE SOLARE



## Spettro della radiazione solare (Terra)



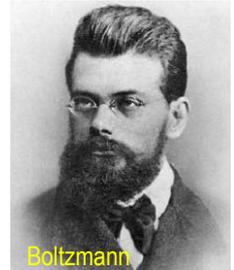
Colore fiamma	Temperatura in °C
Amaranto pallido	480
Amaranto	525
Rosso sangue	585
Rosso scuro	635
Rosso	675
Rosso chiaro	740
Rosso pallido	845
Rosa	900
Arancione	940
Giallo	995
Giallo pallido	1080
Bianco	1205
Azzurro	1400



## POTERE EMISSIVO TOTALE DI CORPO NERO

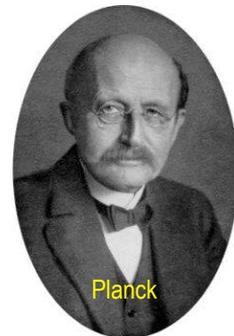
Legge di Stefan/Boltzmann  $E_n = \sigma \cdot T^4$

- $E_n$  = potere emissivo totale di corpo nero, [W/m<sup>2</sup>]  
 $\sigma$  = costante di Stefan/Boltzmann:  $5,67 \times 10^{-8}$  W/(m<sup>2</sup> K<sup>4</sup>)  
 $T$  = temperatura del corpo nero, [K]



1<sup>a</sup> Legge di Planck  $e = h \cdot f$

- $e$  = energia del fotone, [J]  
 $h$  = costante di Planck:  $6,625 \times 10^{-34}$  J\*s  
 $f$  = frequenza d'onda =  $c_{\text{luce}} / \lambda$ , [s<sup>-1</sup>]



N.B. Più aumenta la temperatura del corpo, più diminuisce la lunghezza d'onda, più aumenta il potere emissivo e l'energia del fotone

$$e \rightarrow E \propto T^4 \Leftrightarrow \frac{1}{\lambda} \propto T^4 \Leftrightarrow \lambda \propto \frac{1}{T^4}$$

# POTERE EMISSIVO MONOCROMATICO DI CORPO NERO

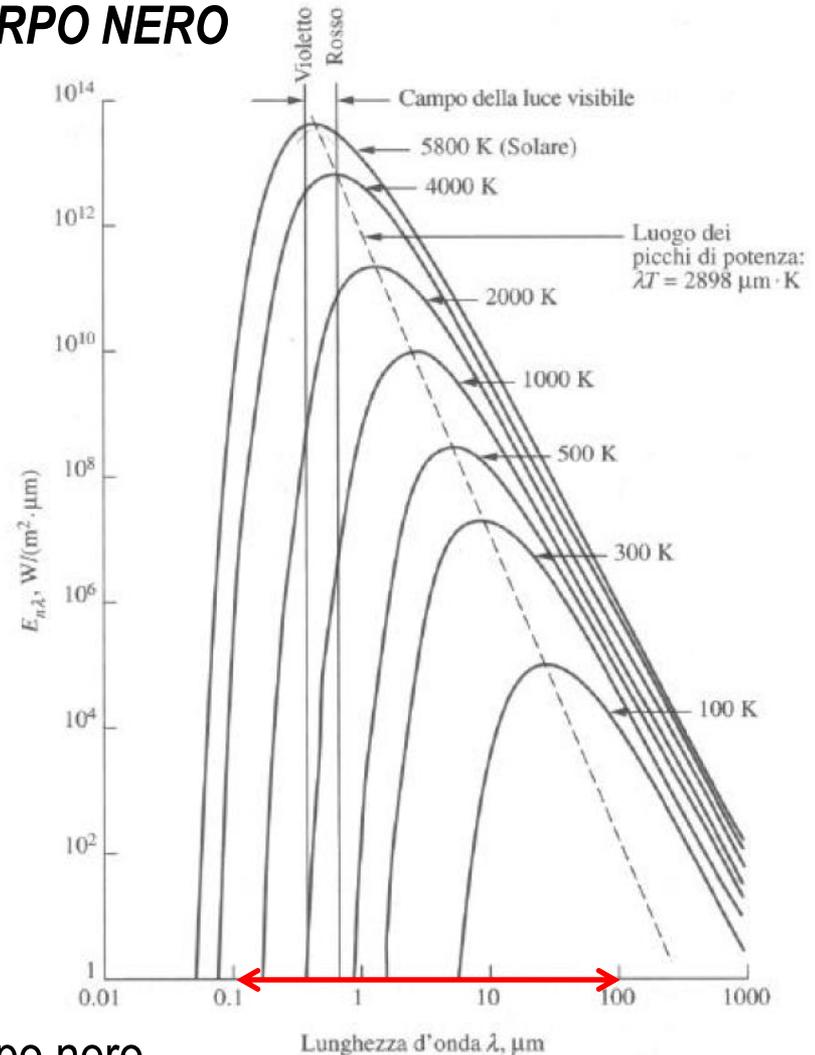
## II<sup>a</sup> Legge di Planck

$$E_{n,\lambda} = \frac{c_1}{\lambda^5 \cdot \left( e^{\frac{c_2}{\lambda \cdot T}} - 1 \right)}$$

$$c_1 = \frac{2\pi}{n^2} \cdot h \cdot c_{luce}^2$$

$$c_2 = \frac{h \cdot c_{luce}}{k}$$

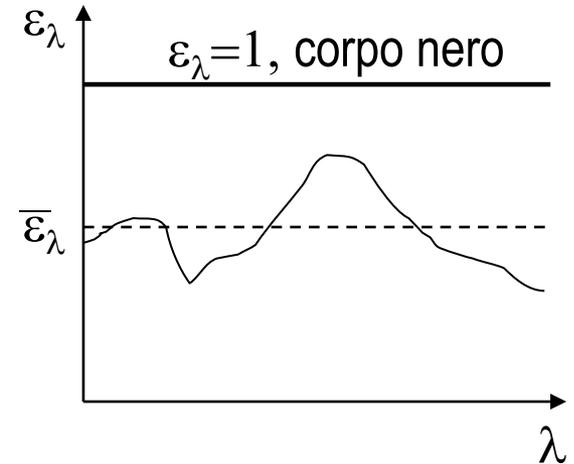
- $E_{n,\lambda}$  = potere emissivo monocromatico di corpo nero  
 $h$  = costante di Planck:  $6,625 \times 10^{-34}$  J\*s  
 $n$  = indice di rifrazione del mezzo (1 per il vuoto)  
 $k$  = costante di Boltzmann:  $1,3805 \times 10^{-23}$  J/K



## EMISSIVITA'

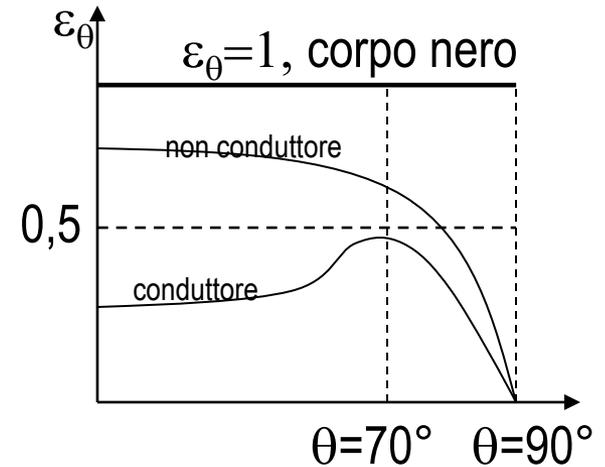
Emissività monocromatica

$$\varepsilon_{\lambda} = \frac{E_{\lambda}}{E_{n,\lambda}}$$



Emissività direzionale

$$\varepsilon_{\theta} = \frac{E_{\theta}}{E_n}$$



Per semplicità, si parla di corpi

- "grigi", la cui emissività è la media di quella monocromatica
- "diffondenti", la cui emissività è invariante per direzione

# IRRADIAZIONE

$G$  = irradiazione, cioè la radiazione incidente

$$G = (r + a + t) \cdot G$$

dove:

$r \rightarrow \rho$  = coefficiente di riflessione

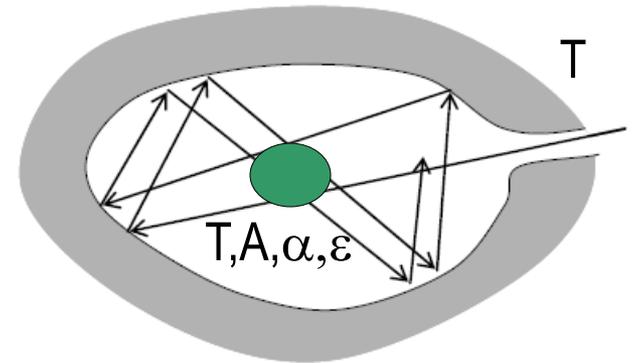
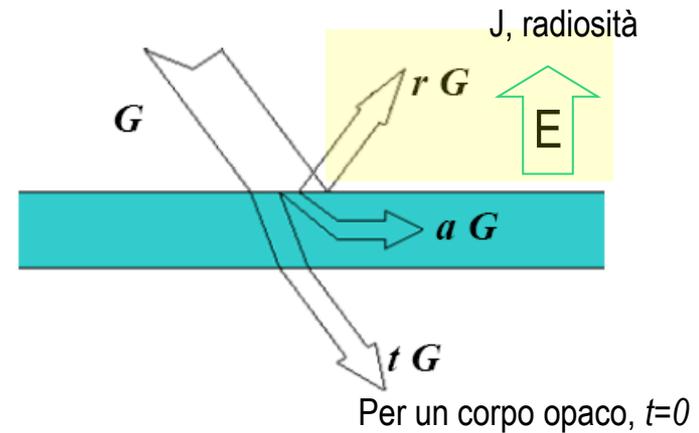
$a \rightarrow \alpha$  = coefficiente di assorbimento

$t \rightarrow \tau$  = coefficiente di trasmissione

## Legge di Kirchhoff

L'emissività emisferica totale è uguale al coefficiente di assorbimento emisferico totale per radiazione da corpo nero alla medesima temperatura

$$\varepsilon = \alpha$$



$$E \cdot A = \alpha \cdot G \cdot A$$

$$\varepsilon \cdot E_n = \alpha \cdot G$$

$$\varepsilon \cdot \sigma \cdot T^4 = \alpha \cdot \sigma \cdot T^4$$

## **ANALOGIA ELETTRICA (irraggiamento)**

$$\dot{Q} = G - E = \alpha \cdot \sigma \cdot T_{amb}^4 \cdot A - \varepsilon \cdot \sigma \cdot T_s^4 \cdot A$$

da cui per un corpo grigio

$$\dot{Q} = \varepsilon \cdot \sigma \cdot A \cdot (T_{amb}^4 - T_s^4)$$

Ponendo

$$h_{irr} = \varepsilon \cdot \sigma \cdot (T_{amb}^2 + T_s^2) \cdot (T_{amb} + T_s)$$

è possibile esprimere la potenza termica trasmessa come

$$\dot{Q} = h_{irr} \cdot A \cdot (T_{amb} - T_s)$$

Quindi

$$\dot{Q} = \frac{(T_{amb} - T_s)}{R_{irr}} \quad \text{dove} \quad R_{irr} = \frac{1}{A \cdot h_{irr}}$$

## **RESISTENZA TERMICA SUPERFICIALE**

Una superficie è soggetta simultaneamente a convezione e irraggiamento, quindi la trasmissione di calore globale si ottiene sommando (o sottraendo) le rispettive componenti, riconoscendole come resistenze in serie.

Se  $T_{\text{amb}} \approx T^{\infty}$ , l'effetto radioattivo può essere considerato assumendo:

$$h = h_{\text{conv}} + h_{\text{irr}}$$

$h_{\text{conv}}$  *coefficiente di scambio termico per convezione*

$h_{\text{irr}}$  *coefficiente di scambio termico per irraggiamento*

Secondo la UNI EN ISO 6946: 
$$h_{\text{irr}} = \varepsilon \cdot 4 \cdot \sigma \cdot T_m^3$$

dove:

$T_m$  è la temperatura assoluta media delle superfici coinvolte (265÷300 K)

$\varepsilon$  è l'emissività superficiale ( $\approx 0,9$ )

$\sigma$  è la costante di Stefan-Boltzmann ( $5,67 \times 10^{-8} \text{ W}/(\text{m}^2 \text{K}^4)$ )

$$\Rightarrow h_{\text{irr}} = 4 \div 6 \text{ W}/(\text{m}^2 \text{K})$$

## TRASMISSIONE DI CALORE PER IRRAGGIAMENTO - sintesi

$$\dot{q}_{irr} = \varepsilon \cdot \sigma \cdot (T_{amb}^4 - T_s^4) \quad \sigma = \text{costante di Stefan-Boltzmann} \\ 5,67 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4)$$

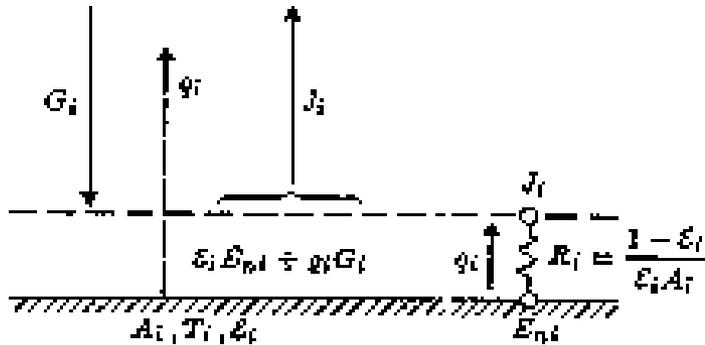
Ponendo:  $h_{irr} = \varepsilon \cdot \sigma \cdot (T_{amb}^2 + T_s^2) \cdot (T_{amb} + T_s)$

$$\dot{q}_{irr} = h_{irr} \cdot (T_{amb} - T_s) \quad h_{irr} = \varepsilon \cdot 4 \cdot \sigma \cdot T_m^3$$

$$\dot{Q}_{irr} = A \cdot h_{irr} \cdot (T_{amb} - T_s)$$

$$\dot{Q}_{irr} = \frac{(T_{amb} - T_s)}{R_{irr}} \quad R_{irr} = \frac{1}{A \cdot h_{irr}}$$

## SCAMBIO TERMICO FRA SUPERFICI GRIGIE CHE FORMANO UNA CAVITA'



La potenza che lascia una superficie è pari alla sua radiosità  $J$ , ovvero l'emissione diretta e la parte riflessa dell'irradiazione  $G$  incidente:

$$J_i = E_i + \rho_i G_i$$

Poichè per un corpo grigio  $E = \varepsilon E_n$  e per un corpo opaco  $\rho = 1 - \varepsilon$ :

$$J_i = \varepsilon_i E_{ni} + (1 - \varepsilon_i) G_i$$

Il flusso termico netto uscente è dunque:

$$\dot{Q}_i = A_i (J_i - G_i)$$

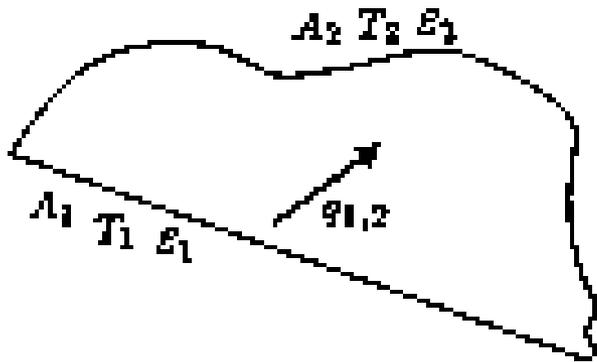
Risolvendo rispetto a  $G_i$ :

$$\dot{Q}_i = \frac{E_{ni} - J_i}{\frac{1 - \varepsilon_i}{\varepsilon_i A_i}}$$

Analogia elettrica:

$$R_i = \frac{1 - \varepsilon_i}{\varepsilon_i A_i}$$

# CAVITA' FORMATA DA DUE SUPERFICI GRIGIE



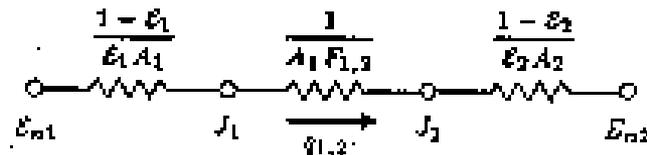
La potenza uscente dalla superficie 1 ed incidente sulla superficie 2 è:

$$\dot{Q}_{1 \rightarrow 2} = J_1 \cdot A_1 F_{1,2}$$

Analogamente, da 2 a 1:  $\dot{Q}_{2 \rightarrow 1} = J_2 \cdot A_2 F_{2,1}$

Complessivamente: 
$$\dot{Q}_{1,2} = A_1 F_{1,2} (J_1 - J_2) = \frac{J_1 - J_2}{\frac{1}{A_1 F_{1,2}}}$$

$$\dot{Q}_{1,2} = \frac{E_{n1} - J_1}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1}} = \frac{J_2 - E_{n2}}{\frac{1 - \varepsilon_2}{\varepsilon_2 A_2}} \Rightarrow \dot{Q}_{1,2} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1,2}} + \frac{1 - \varepsilon_2}{\varepsilon_2 A_2}}$$



## APPLICAZIONI PARTICOLARI

- *Piani paralleli indefiniti*:  $A_1 = A_2 = A$ ;  $F_{1,2} = 1$   $\Rightarrow$   $\dot{Q}_{1,2} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$

- *Cilindri concentrici indefiniti*:  $F_{1,2} = 1$   $\Rightarrow$   $\dot{Q}_{1,2} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \frac{r_1}{r_2}}$

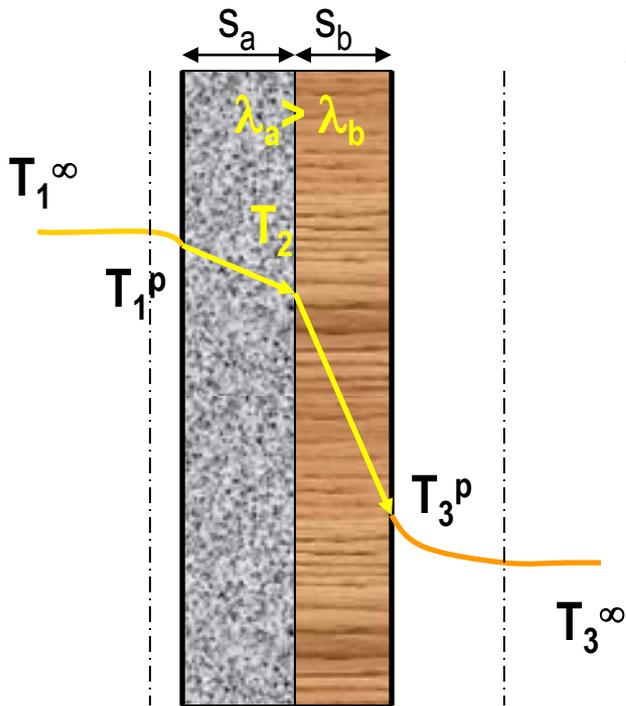
- *Sfere concentriche*:  $F_{1,2} = 1$   $\Rightarrow$   $\dot{Q}_{1,2} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2}\right)^2}$

- *Corpo convesso di piccole dimensioni (1) in una grande cavità (2)*:  $F_{1,2} = 1$ ;  $A_1/A_2 \cong 0$   $\Rightarrow$   $\dot{Q}_{1,2} = A_1\varepsilon_1\sigma(T_1^4 - T_2^4)$



*È la somma che fa il totale*  
Totò

## COEFFICIENTE DI SCAMBIO TERMICO GLOBALE



$$U = \frac{1}{\frac{1}{h_1^{conv} + h_1^{irr}} + \frac{s_1}{\lambda_1} + \frac{s_2}{\lambda_2} + \frac{1}{h_2^{conv} + h_2^{irr}}}$$

$$\dot{Q} = U \cdot A \cdot (T_1^\infty - T_3^\infty)$$

$$\dot{Q} = U \cdot A \cdot \Delta T$$

# ANALOGIA ELETTRICA (sistemi in serie e parallelo, esempio)

$$R_{t,conv\_A} = \frac{1}{S_A \cdot h_A} = \frac{1}{3 \cdot 200} = 0,00167 \frac{K}{W}$$

$$R_{t,cond\_A} = \frac{s_A}{S_A \cdot \lambda_A} = \frac{0,05}{3 \cdot 60} = 0,00028 \frac{K}{W}$$

$$R_{t,cond\_B1} = \frac{s_{B1}}{S_{B1} \cdot \lambda_{B1}} = \frac{0,10}{1 \cdot 1} = 0,1 \frac{K}{W}$$

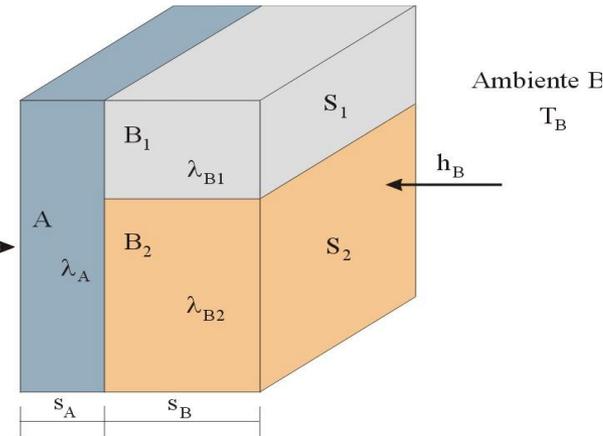
$$R_{t,cond\_B2} = \frac{s_{B2}}{S_{B2} \cdot \lambda_{B2}} = \frac{0,10}{2 \cdot 0,1} = 0,5 \frac{K}{W}$$

$$R_{t,conv\_B1} = \frac{1}{S_{B1} \cdot h_{B1}} = \frac{1}{1 \cdot 10} = 0,1 \frac{K}{W}$$

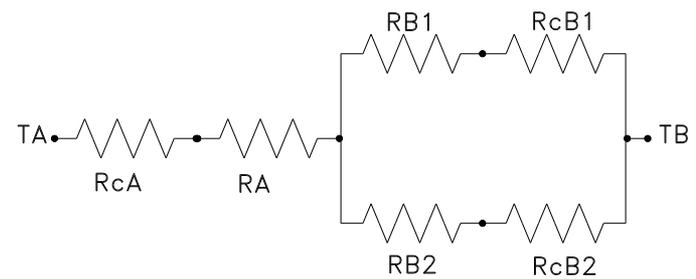
$$R_{t,conv\_B2} = \frac{1}{S_{B2} \cdot h_{B2}} = \frac{1}{2 \cdot 10} = 0,05 \frac{K}{W}$$

$$R_{parallelo} = \frac{1}{\sum_{i=1}^n \frac{1}{R_{t,i}}} = \frac{1}{\frac{1}{R_{t,cond\_B1} + R_{t,conv\_B1}} + \frac{1}{R_{t,cond\_B2} + R_{t,conv\_B2}}} = 0,1467 \frac{K}{W}$$

$$R_{serie} = R_{t,conv\_A} + R_{t,cond\_A} = 0,00195 \frac{K}{W}$$



- $s_A = 0,05 \text{ m}$
- $s_B = 0,10 \text{ m}$
- $T_A = 100^\circ \text{ C}$
- $T_B = 0^\circ \text{ C}$
- $h_A = 200 \text{ W}/(\text{m}^2 \text{ K})$
- $h_B = 10 \text{ W}/(\text{m}^2 \text{ K})$
- $S_1 = 1 \text{ m}^2$
- $S_2 = 2 \text{ m}^2$
- $\lambda_A = 60 \text{ W}/(\text{mK})$
- $\lambda_{B1} = 1 \text{ W}/(\text{mK})$
- $\lambda_{B2} = 0,1 \text{ W}/(\text{mK})$



$$\Rightarrow R_{tot} = R_{parallelo} + R_{serie} = 0,14865 \frac{K}{W}$$