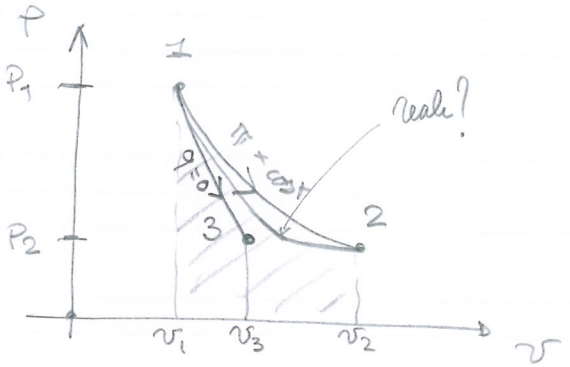
e calore

Valutare il differenziale lavoro specifico scambiato lungo una isoterma e una adiabatica, a parità di salto di pressione, per un sistema chiuso contenente aria.

$pv = cost$ isoterma

$pv^{\gamma} = cost$ adiab. $\gamma = \frac{c_p}{c_v}$ (aria = 1,4)



$pv = RT \rightarrow p = \frac{RT}{v}$

1) Isoterma $\Rightarrow pv = cost$

$p_1 v_1 = p_2 v_2$

$\frac{p_1}{p_2} = \frac{v_2}{v_1} \Leftrightarrow v_2 = v_1 \cdot \beta$ ($\beta = \frac{p_1}{p_2}$)

Dati

$p_1 = 10\ p_2$

$p_2 = 1$ bar

$R_{aria} = 0{,}288\ \frac{kJ}{kg\,k}$

$T = 300$ k

Sistema chiuso $du = \delta q - \delta l$

$c_v dT = \delta q - p dv$

$\delta q = \frac{RT}{v} dv$

$q_{12} = RT \cdot \ln v \Big|_{v_1}^{v_2}$

$= RT \ln \frac{v_2}{v_1}$

$= RT \ln \frac{\frac{RT}{p_2}}{\frac{RT}{p_1}}$

$= RT \ln \frac{p_1}{p_2}$

$= RT \ln \beta$

$\Rightarrow q_{12}^{T=cost} = 198{,}9\ \frac{kJ}{kg}$

$v_1 = \frac{RT}{p_1} = 0{,}0864\ \frac{m^3}{kg}$

$v_2 = \frac{RT}{p_2} = 0{,}864\ \frac{m^3}{kg}$

2) Adiabatica $\rightarrow q = 0$ $(pv^{\gamma} = cost.)$

$\Rightarrow du = -\delta l$

$c_v dT = -p dv$

$c_v dT = -\frac{RT}{v} dv$

$c_v \frac{dT}{T} = -R \frac{dv}{v}$

$c_v \ln T \Big|_{T_1}^{T_3} = -R \ln v \Big|_{v_1}^{v_2}$

$l_{13} = -c_v (T_3 - T_1) = 103{,}7\ \frac{kJ}{kg}$

$p \cdot \left(\frac{RT}{p}\right)^{\gamma}$

$p^{1-\gamma} \cdot T^{\gamma}$

$T_1 \cdot p_1^{\frac{1-\gamma}{\gamma}} = T_3 \cdot p_3^{\frac{1-\gamma}{\gamma}}$

$\frac{T_3}{T_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} = \beta^{\frac{1-\gamma}{\gamma}}$

$\Rightarrow T_3 = T_1 \cdot \beta^{\frac{1-\gamma}{\gamma}} = 155$ k

Se fosse stata CO_2 invece che aria?

$CO_2 \rightarrow 44 \frac{kg}{kmol}$ $\quad 8\ bar \rightarrow 1\ bar$

$$R = \frac{8{,}314 \frac{kJ}{kmol\,k}}{44 \frac{kg}{kmol}} = 0{,}189 \frac{kJ}{kg\,k}$$

$$T_2 = T_1 \cdot \beta^{\frac{1-\gamma}{\gamma}}$$

$\downarrow$

$183\ k \rightsquigarrow -85°C$!

$c_p = 0{,}838 \frac{kJ}{kg\,k}$

$c_v = 0{,}649 \frac{kJ}{kg\,k}$

$\gamma = 1{,}291$