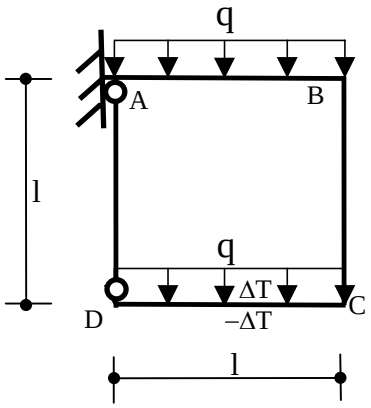
**CORSO DI LAUREA IN INGEGNERIA MECCANICA**
**UNIVERSITÀ DEGLI STUDI DI FERRARA**
**PROVA SCRITTA DI STATICA**
**FERRARA, 08/07/2008**

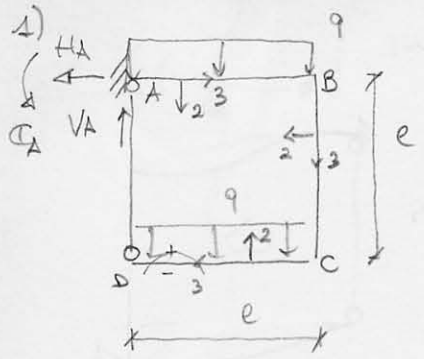



$l = 2$ m, $q = 2$ t/m, $\sigma_{AMM} = 2400$ kg/cm$^2$
$E = 2.1\ 10^6$ kg/cm$^2$, $\Delta T = 10°C$, $\alpha = 10^{-5}°C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Risolvere la travatura in presenza del solo carico q e disegnare i diagrammi delle caratteristiche di sollecitazione (N, T, M).
2. Progettare la travatura.
3. Calcolare lo spostamento verticale in C.
4. Risolvere nuovamente la travatura considerando anche il carico termico (a farfalla) nel tratto CD. Disegnare i nuovi diagrammi delle caratteristiche di sollecitazione (N, T, M).

1)



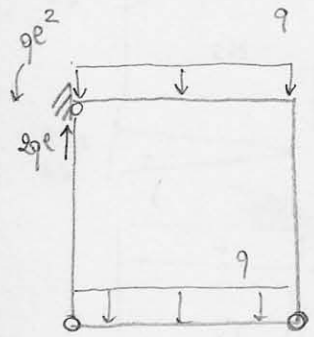

Eq.ni cardinali della Statica:

$(\leftarrow)\quad H_A = 0$

$(\uparrow)\quad V_A = 2ql$

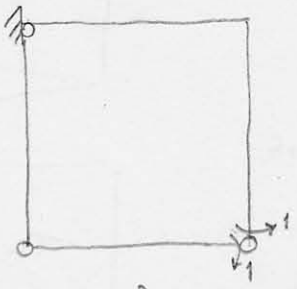
$(A\uparrow)\quad \Gamma_A = \cancel{2}\, ql \frac{l}{\cancel{2}} = ql^2$

Trattasi isostatica per vincoli esterni, 1 volta iperstatica per vincoli interni



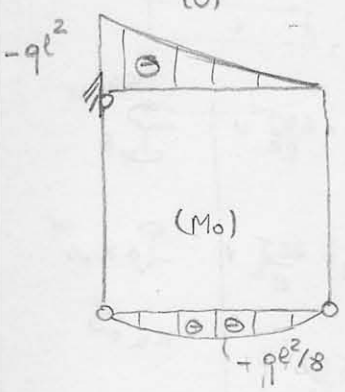

(0)

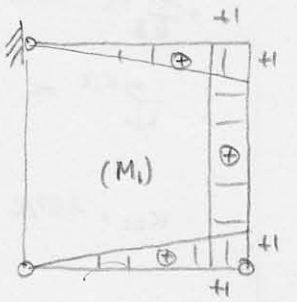
$+ X_1$

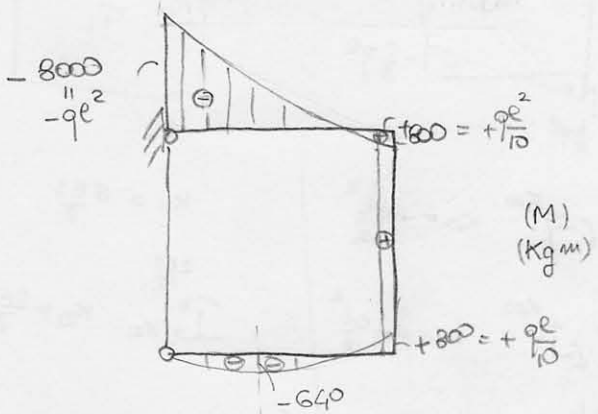


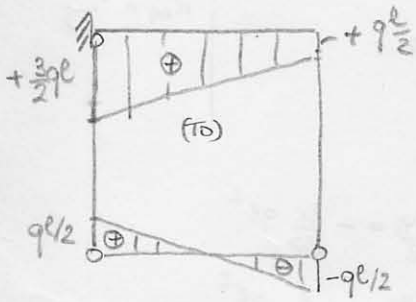

(1)

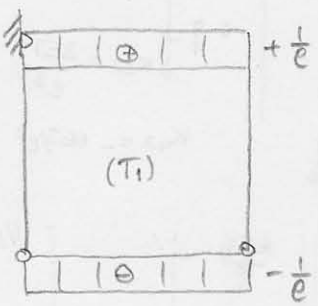
Diagrammi finali

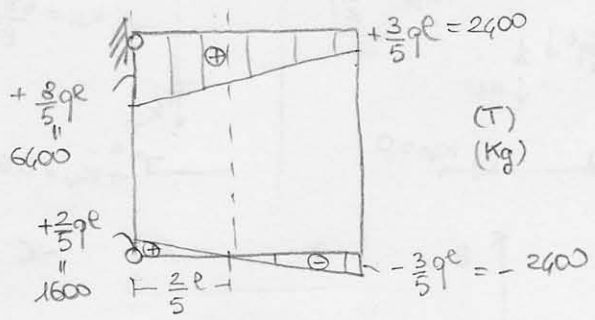


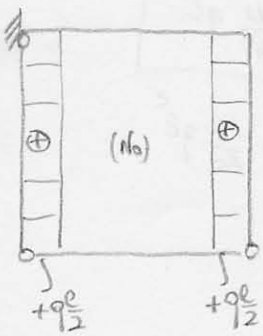



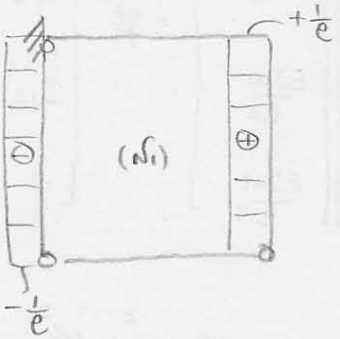



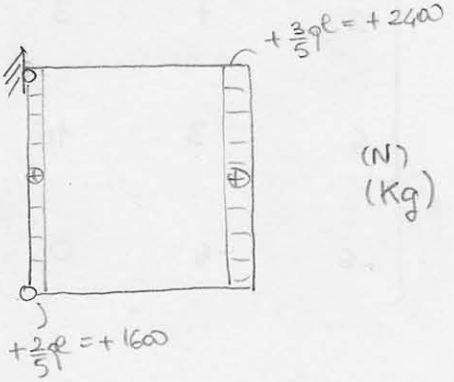



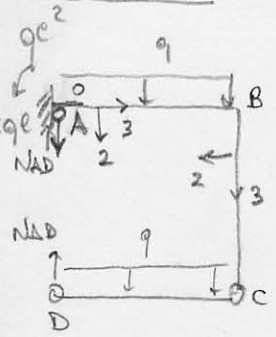



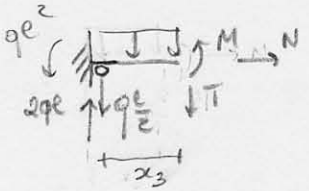



## Studio di (0)




$(C\circlearrowleft)_{CD}$ $\quad N_{AD}\ell = q\frac{\ell^2}{2}$

### Tratto AB

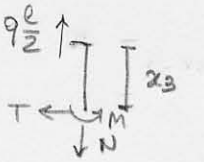



$$\begin{cases} N = 0 \\ T = 2q\ell - q\frac{\ell}{2} - qx_3 = \frac{3}{2}q\ell - qx_3 \\ M = -qe^2 + \frac{3}{2}q\ell x_3 - q\frac{x_3^2}{2} \end{cases}$$

$M(\ell) = -q\ell^2 + \frac{3}{2}q\ell^2 - q\frac{\ell^2}{2} = 0$

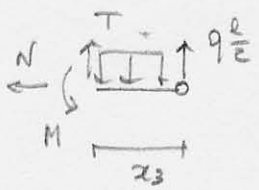
$T(\ell) = \frac{3}{2}q\ell - q\ell = q\ell/2$

### Tratto BC



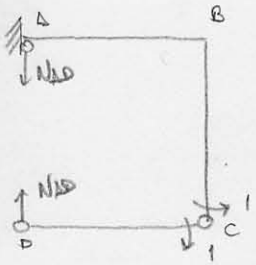

$$\begin{cases} N = q\ell/2 \\ T = 0 \\ M = 0 \end{cases}$$

### Tratto CD



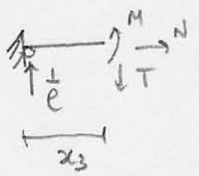

$$\begin{cases} N = 0 \\ T = qx_3 - q\ell/2 \\ M = q\frac{x_3^2}{2} - q\frac{\ell}{2}x_3 \end{cases}$$

## Studio di (1)



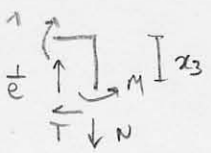

$(c\circlearrowleft)_{CD}$ $\quad N_{AD} = -\frac{1}{\ell}$

### Tratto AB



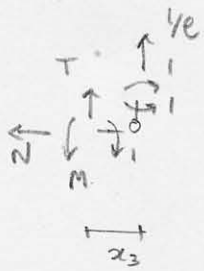

$$\begin{cases} N = 0 \\ T = 1/\ell \\ M = x_3/\ell \end{cases}$$

### Tratto BC




$$\begin{cases} N = \frac{1}{\ell} \\ T = 0 \\ M = 1 \end{cases}$$

### Tratto CD




$$\begin{cases} N = 0 \\ T = -\frac{1}{\ell} \\ M = 1 - \frac{x_3}{\ell} \end{cases}$$

$$EI_1 \gamma_{10} = -\frac{1}{24} q\ell^3 + \int_0^\ell \left(\frac{x_3}{\ell}\right)\left(-q\ell^2 + \frac{3}{2} q\ell x_3 - q\frac{x_3^2}{2}\right) dx_3$$

$$= -\frac{q\ell^3}{24} + \frac{1}{\ell}\left[-q\ell^2 \frac{x_3^2}{2} + \frac{3}{2} q\ell \frac{x_3^3}{3} - \frac{q}{2}\frac{x_3^4}{4}\right]_0^\ell$$

$$= -\frac{q\ell^3}{24} + q\ell^3\left[-\frac{1}{2} + \frac{1}{2} - \frac{1}{8}\right] = \frac{-1-3}{24} q\ell^3 = -\frac{q\ell^3}{6}$$

$$EI_1 \gamma_{11} = 2 \cdot \frac{1}{3}\ell + \ell = \frac{5}{3}\ell$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} = \frac{q\ell^3}{8_2}\frac{3}{5\ell} = \frac{q\ell^2}{10} = 800 \text{ kgm}$$

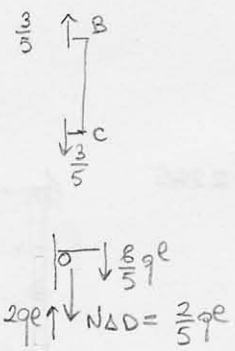
Calcolo per il disegno dei diagrammi finali:

$$T_A = \frac{3}{5} q\ell + \frac{q\ell}{10} = \frac{8}{5} q\ell$$

$$T_B = \frac{q\ell}{2} + \frac{q\ell}{10} = \frac{3}{5} q\ell$$

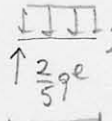
$$T_{C_D} = -\frac{q\ell}{2} - \frac{q\ell}{10} = -\frac{3}{5} q\ell$$

$$T_D = \frac{q\ell}{2} - \frac{q\ell}{10} = \frac{2}{5} q\ell$$









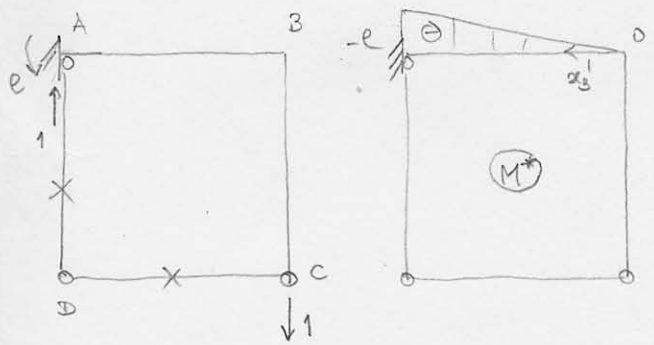

$$\bar{M} = \frac{1}{2} \frac{4^2}{25} q\ell^2 = 640 \text{ kgm}$$

2) PROGETTO

$$W_1 \geq \frac{M_1}{\sigma_{AMM}} = \frac{8000 \cdot 100}{2400} \text{ cm}^3 = 333 \text{ cm}^3 \longrightarrow \text{IPE 270}$$

$I_1 = 5790 \text{ cm}^4$
$W_1 = 429 \text{ cm}^3$
$A = 45,9 \text{ cm}^2$
$H = 270 \text{ mm} = 27 \text{ cm}$

3) Spostamento verticale in C.




$$1 \cdot v_C = \frac{1}{EI_1} \int_0^\ell (-x_3')\left(\frac{q\ell^2}{10} - \frac{3}{5} q\ell x_3' - q\frac{x_3'^2}{2}\right) dx_3'$$

$$= \frac{1}{EI_1}\left[-\frac{q\ell^2}{10}\frac{\ell^2}{2} + \frac{3}{5} q\ell \frac{\ell^3}{3} + \frac{q}{2}\frac{\ell^4}{4}\right]$$

$$= \frac{1}{EI_1} q\ell^4\left[-\frac{1}{20} + \frac{1}{5} + \frac{1}{8}\right] = \frac{11}{40}\frac{q\ell^4}{EI_1}$$

$$= \frac{11}{40} \cdot \frac{20 \cdot 200^4}{2,1 \cdot 10^6 \cdot 5790} = 0,72 \text{ cm}$$

4) Carico termico

$$\gamma_{1t} + \gamma_{10} + \gamma_{11} X_1 = 0$$

$$\gamma_{1t} = \int_{CD} M_1 k_t = k_t \frac{\ell}{2} = + \frac{\cancel{2}\Delta T \alpha}{H} \frac{\ell}{\cancel{2}} =$$

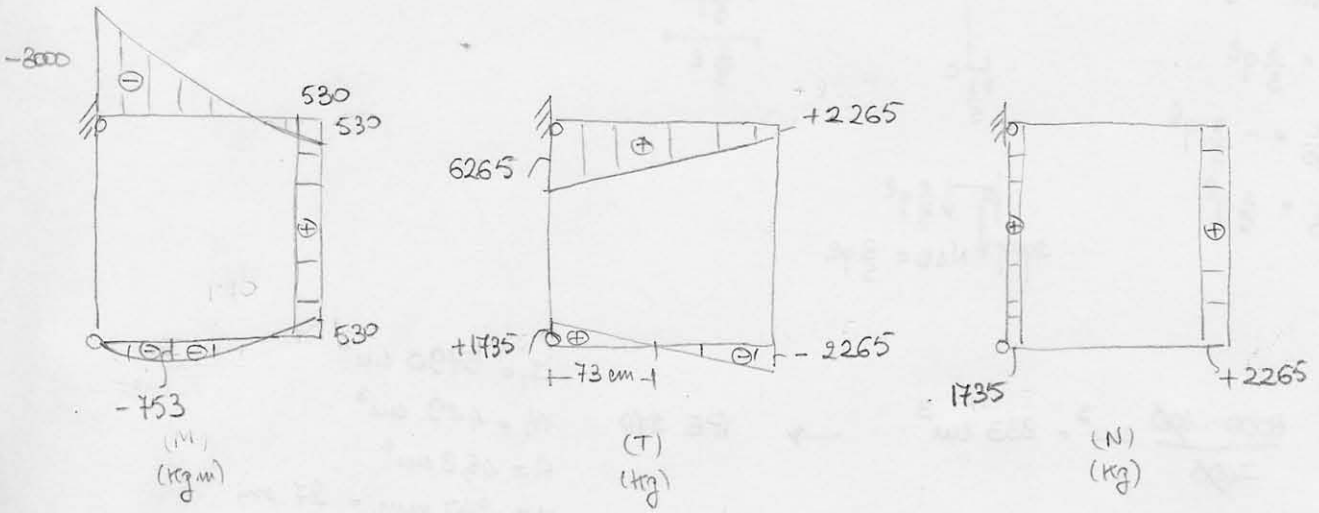
$$\rightarrow X_1 = - \frac{\gamma_{10}}{\gamma_{11}} - \frac{\gamma_{1t}}{\gamma_{11}} = + \frac{q\ell^2}{10} - \frac{\alpha \Delta T}{H} \not{\ell} \frac{3EI_1}{5\not{\ell}}$$

$$= \frac{q\ell^2}{10} - \frac{3EI_1 \alpha \Delta T}{5H}$$

$$= 800 - 270 = 530 \text{ kgm}$$

$$\frac{3 \cdot 2{,}1 \cdot 10^{\cancel{6}} \cdot 10^{\cancel{3}} \cdot 10 \cdot 5730}{5 \cdot 27} = 270{,}20 \text{ kgcm}$$

Diagrammi del punto (4):




$$T_A = 6000 + \frac{530}{2} = 6265 \text{ kg}$$

$$T_B = 6265 - 4000 = 2265 \text{ kg}$$

$$T_D = 2000 - \frac{530}{2} = 1735 \text{ kg}$$