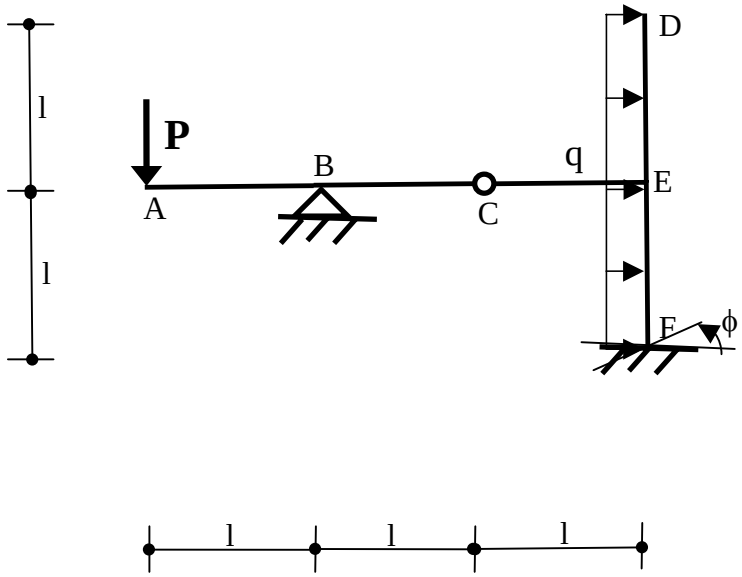
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 07/12/2007

A

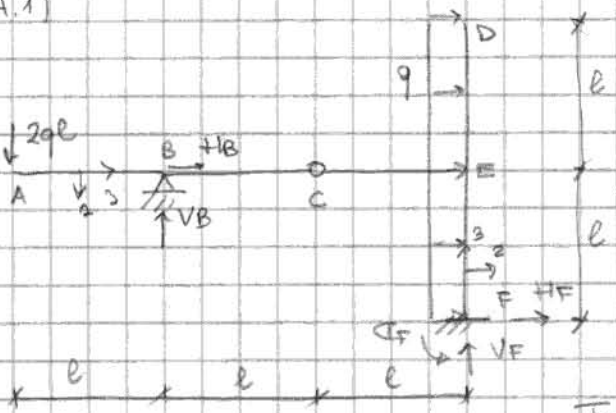


l = 1 m, q = 1 t/m, P=2ql, σ_{AMM} = 2400 kg/cm^2
E = 2.1 10^6 kg/cm^2, ϕ = 0.1°

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). Trascurare le deformazioni assiali.
2. Progettare la travatura.
3. Calcolare la rotazione del nodo B.
4. Risolvere nuovamente la travatura considerando anche il cedimento δ del vincolo in B e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).

A.1)



Eq.ne ausiliaria:

$(\circlearrowleft)_{ABC}$ $V_B \ell = 2q\ell \cdot 2\ell \rightarrow V_B = 4q\ell$

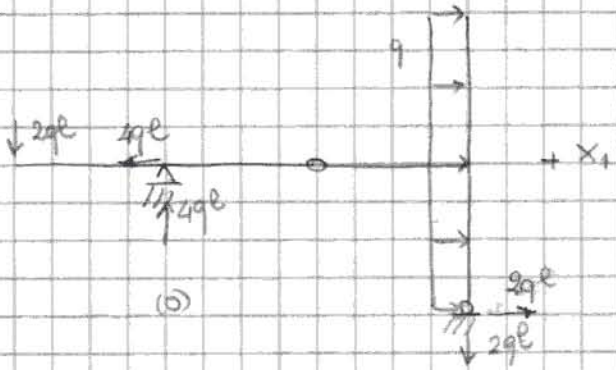
Eq.ni cardinali della Statica:

$(\uparrow)$ $V_F = 2q\ell - V_B = -2q\ell$

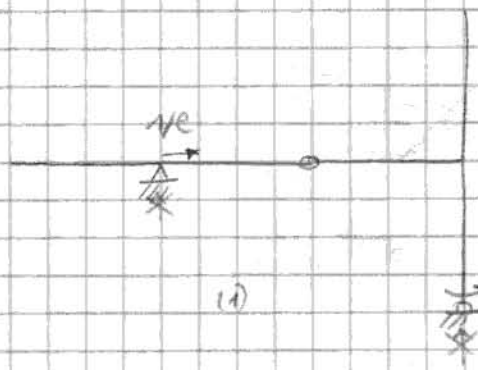
$(\rightarrow)$ $H_B + H_F = 2q\ell$

$(\circlearrowleft)$ $\Phi_F + V_F\ell + H_F\ell - V_B\ell + 2q\ell \cdot 2\ell = 0$ } 2 eq.ni, 3 incognite

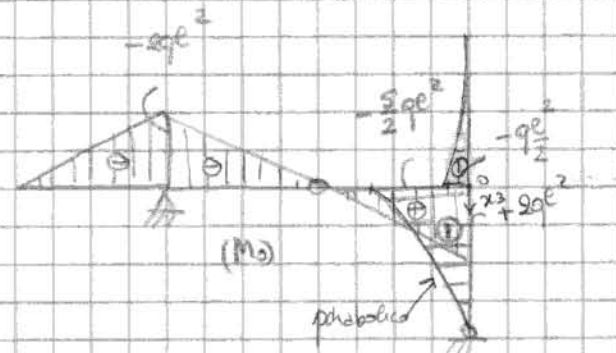
Trovata una volta iperstatica. $X_1 = \Phi_F$.



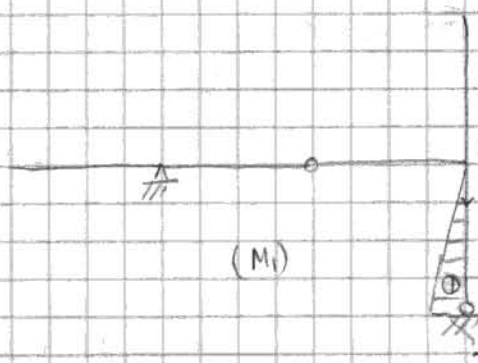
(0)



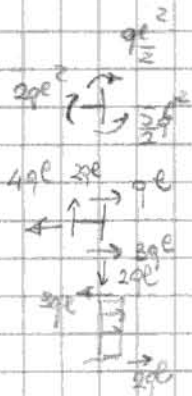
(1)

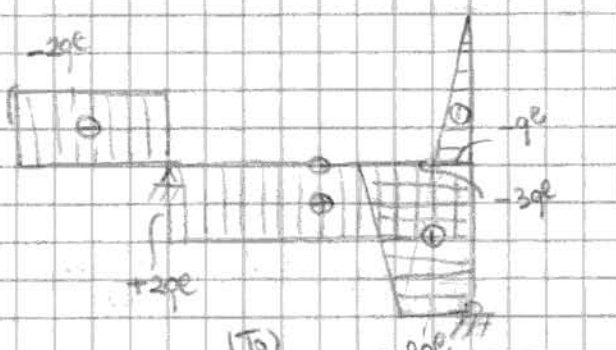


(M_0)



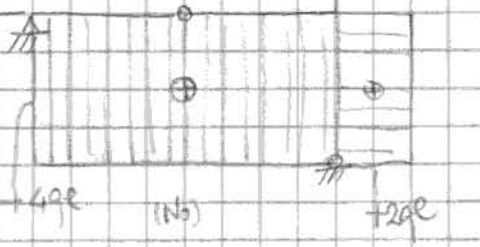
(M_1)



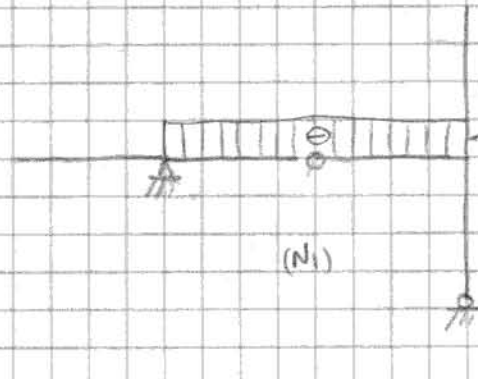


(T_0)

(T_1)



(N_0)



(N_1)

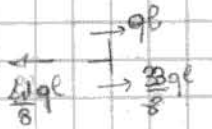
$$EI_1 M_{10} = \int_0^{\ell} \left(-\frac{x_3}{\ell}\right)\left(-\frac{5}{2}q\ell^2 + 3q\ell x_3 - q\frac{x_3^2}{2\ell}\right) dx_3 = \left[\frac{5}{2}q\ell\frac{\ell^2}{2} - \frac{3}{3}q\ell^3 + \frac{q}{2\ell}\frac{\ell^4}{4}\right] = q\ell^3\left[\frac{5}{4} - 1 + \frac{1}{8}\right] = \frac{3}{8}q\ell^3$$

$$EI_1 M_{11} = \frac{\ell}{3}$$

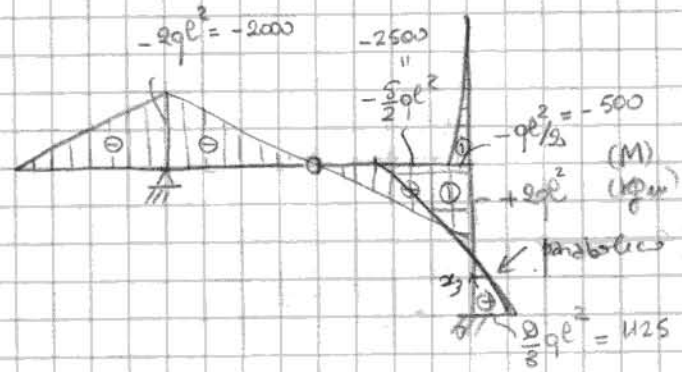
$$X_1 = -\frac{M_{10}}{M_{11}} = -\frac{3}{8}q\ell^3\frac{3}{\ell} = -\frac{9}{8}q\ell^2 = -1125 \text{ kgm}$$

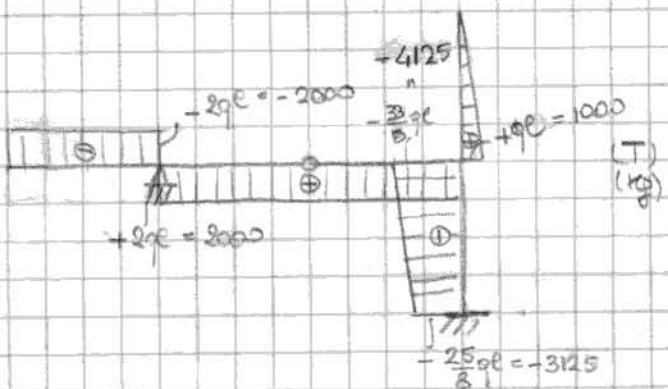
$$T_F = -2q\ell - \frac{9}{8}q\ell = -\frac{25}{8}q\ell = -3125 \text{ kg}$$

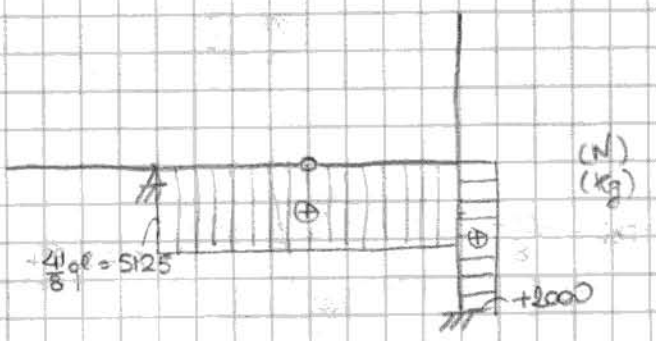
$$T_E = -3q\ell - \frac{9}{8}q\ell = -\frac{33}{8}q\ell = -4125 \text{ kg}$$



Diagrammi sui punti A.1:



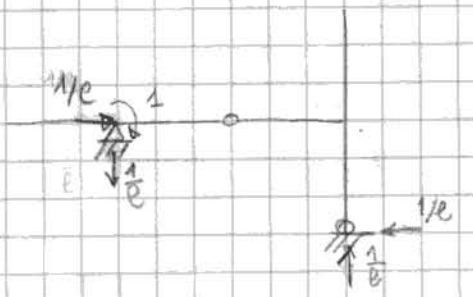


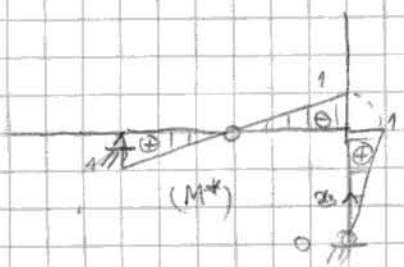


A.2)

PROGETTO: $W_x \geq \frac{2500 \cdot 100}{2400} = 104 \text{ cm}^3 \Rightarrow$ IPE 160 $\begin{cases} I_1 = 869 \text{ cm}^4 \\ A = 20,1 \text{ cm}^2 \end{cases}$

A.3) Rotazione in B. (si tiene conto le def. assiali)





$$1 \cdot \varphi_B = \frac{1}{EI_1}\left[\frac{2}{3}\ell \cdot 1 \cdot (2q\ell^2) + \int_0^{\ell}\left(\frac{x_3}{\ell}\right)\left(\frac{9}{8}q\ell^2 - \frac{25}{8}q\ell x_3 - q\frac{x_3^2}{2\ell}\right) dx_3\right]$$

$$= \frac{1}{EI_1}\left\{\frac{4}{3}q\ell^3 + \left[\frac{9}{8}q\frac{\ell^3}{2} - \frac{25}{8}q\frac{\ell^3}{3} - \frac{q}{2\ell}\frac{\ell^4}{4}\right]\right\}$$

$$= \frac{q\ell^3}{EI_1}\left[\frac{4}{3} + \frac{9}{16} - \frac{25}{24} - \frac{1}{8}\right] = -\frac{31}{16}\frac{q\ell^3}{EI_1}$$

$$= -\frac{31 \cdot 10 \cdot 100^3}{16 \cdot 2,1 \cdot 10^6 \cdot 869} = -0,01$$

$$= -0,6°$$

A.4) Cedimento vincolare

$$M_{10} + M_{11} X_1 = M_1$$

$$M_1 = 1 \cdot \Phi$$

$$M_M = \frac{l}{3EI_1} + \frac{9q}{EA}\frac{1}{e^2}$$

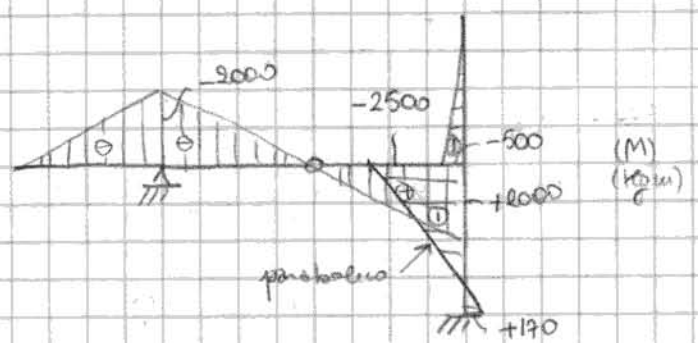
$$\frac{M_{11}^N}{M_{11}^M} = \frac{2}{EAl}\frac{3EJ1}{l} = \frac{6I_1}{Ae^2} = \frac{6 \cdot 869}{201 \cdot 100^2} = 2{,}6\% \rightarrow M_{11}^N \text{ è trascurabile rispetto a } M_{11}^M$$

$$M_{10} = \frac{3}{8} q e^3 \quad (M_{10}^N \text{ si ritiene trascurabile come } M_{11}^N)$$

$$X_1 = -\frac{M_{10}}{M_{11}} + \frac{M_1}{M_{11}} = -\frac{9}{8} q e^2 + \frac{3EI_1\Phi}{l} = -M25 + \frac{3 \cdot 2{,}1 \cdot 10^5 \cdot 869 \cdot 0{,}1 \cdot \pi/180}{100} \cdot 10^{-2} \text{ kgm}$$

$$= -M25 + 955 \text{ kgm} = -170 \text{ kgm}$$

Diagrammi finali punto A.4:



$$T_F = -2000 - 170 = -2170 \text{ kg}$$

$$T_E = -3000 - 170 = -3170 \text{ kg}$$