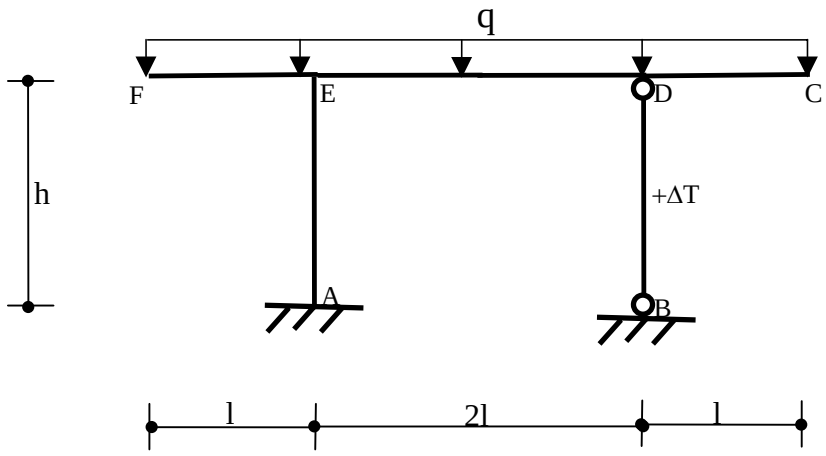
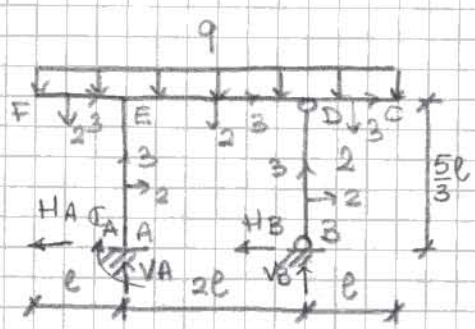
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 06/03/2008



$l = 2$ m, $h=5/3\ l$, $q = 2.5$ t/m, $\sigma_{AMM} = 2400$ kg/cm^2
$E = 2.1\ 10^6$ kg/cm^2, $\Delta T = 10°C$, $\alpha = 10^{-5}°C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del solo carico q e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). Trascurare le deformazioni assiali.
2. Progettare la travatura.
3. Calcolare la rotazione del nodo D della trave orizzontale.
4. Risolvere nuovamente la travatura considerando anche il carico termico (uniforme) nel tratto BD e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).



Eq.me della scomposizione in D:

$(D\curvearrowright)_{DB} \quad +H_B h = 0$

Eq.mi cardinali della Statica:

$(\leftarrow) \quad +H_A = 0$

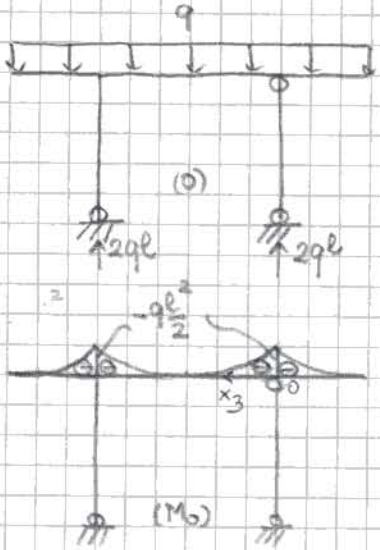
$(\uparrow) \quad V_A + V_B = 4q\ell$

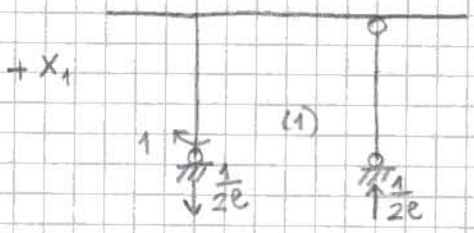
$(D\curvearrowright) \quad -V_A 2\ell - \mathcal{C}_A + 4q\ell^2 = 0$

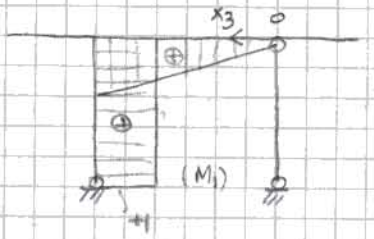
} 2 eq.mi 3 incognite

La travatura è una volta iperstatica.

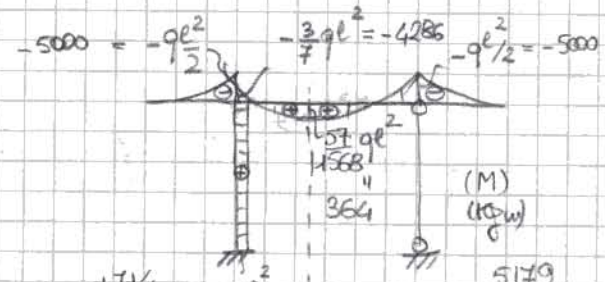
Incognita iperstatica: $X_1 = \mathcal{C}_A$.

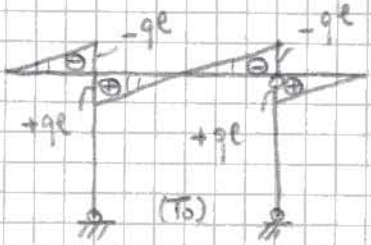


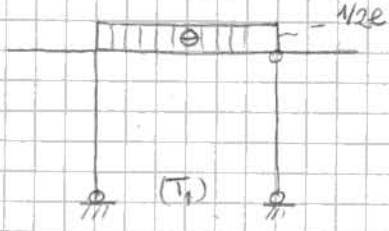


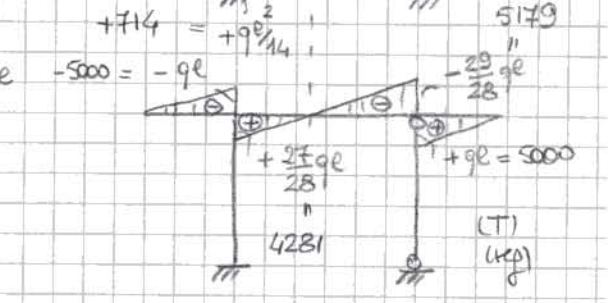


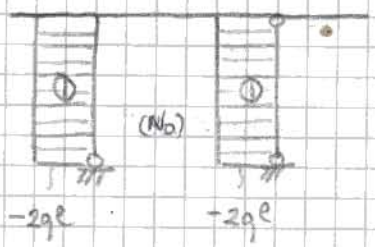
DIAGRAMMI FINALI

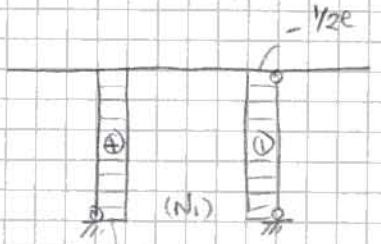


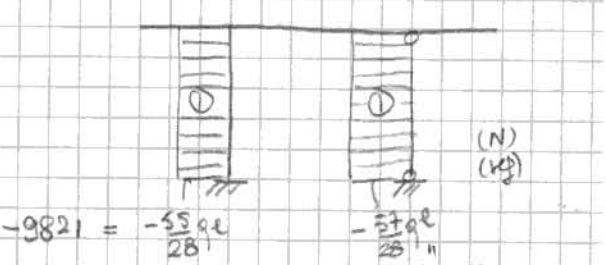












$$E I_1 M_{10} = \int_{ED} M_0 M_1 \, dx_3 = \int_0^{2\ell} \left(-\frac{q\ell^2}{2} + q\ell x_3 - q\frac{x_3^2}{2}\right)\left(\frac{x_3}{2\ell}\right) dx_3 = \frac{q}{2\ell}\int_0^{2\ell} \left(-\frac{\ell^2}{2}x_3 + \ell x_3^2 - \frac{x_3^3}{2}\right) dx_3$$

$$= \frac{q}{2\ell}\left[-\frac{\ell^2}{4}x_3^2 + \frac{\ell}{3}x_3^3 - \frac{x_3^4}{8}\right]_0^{2\ell}$$

$$= \frac{q}{2\ell}\ell^4\left[-\frac{1}{4}4 + \frac{1}{3}8 - \frac{2^4}{8}\right]$$

$$= \frac{q\ell^3}{2}\left[-3 + \frac{8}{3}\right] = -\frac{q\ell^3}{6}$$

$$E I_1 M_{11} = \int_{AE} M_1^2 + \int_{ED} M_1^2 = \frac{5}{3}\ell + 2\ell\frac{1}{3} = \frac{7}{3}\ell$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{ql^3}{6}\cdot\frac{3}{7l} = \frac{ql^2}{14} = 714 \text{ kgm}$$

$$ql - \frac{ql^2}{14}\cdot\frac{1}{2l} = \frac{27}{28}ql$$

$$-ql - \frac{ql}{28} = -\frac{29}{28}ql$$

$$\frac{3}{7}l \quad \frac{27}{28}ql$$

$$\bar{M} = -\frac{3}{7}ql^2 + \left(\frac{27}{28}\right)ql^2 - \frac{1}{2}\left(\frac{27}{28}\right)ql^2$$

$$= -\frac{3}{7}ql^2 + \frac{1}{2}\left(\frac{27}{28}\right)^2 ql^2 = \frac{57}{1568}ql^2$$

$$-2ql + \frac{ql}{28} = -\frac{55}{28}ql$$

$$-2ql - \frac{ql}{28} = -\frac{57}{28}ql$$

PROGETTO:

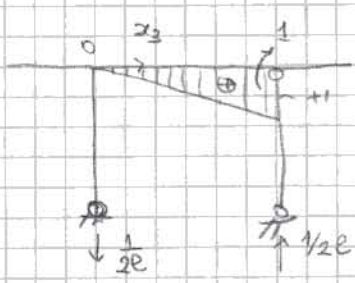
$$W_x \geq \frac{5000\cdot 106}{2690} = 208 \text{ cm}^3$$

IPE 220

$I_1 = 2772 \text{ cm}^4$

$A = 33{,}4 \text{ cm}^2$

ROTAZIONE



(M*)

$$1\cdot\varphi_0 = \frac{1}{EI_1}\int_0^{2l}\left(-\frac{3}{7}ql^2 + \frac{27}{28}qlx_3 - q\frac{x_3^2}{2}\right)\left(\frac{x_3}{2l}\right)dx_3$$

$$= \frac{q}{2lEI_1}\int_0^{2l}\left[-\frac{3}{7}l^2x_3 + \frac{27}{28}lx_3^2 - \frac{x_3^3}{2}\right]dx_3$$

$$= \frac{q}{2lEI_1}\left[-\frac{3}{7}l^2\cdot 4l^2 + \frac{27}{28}l\cdot\frac{8}{3}l^3 - \frac{1}{8}16l^4\right]$$

$$= \frac{ql^3}{EI_1}\left[-\frac{6}{7} + \frac{18}{7} - 2\right] = \frac{ql^3}{EI_1}\left[\frac{6}{7} - 1\right] = -\frac{ql^3}{7EI_1}$$

$$= -\frac{95\cdot 200^3}{7\cdot 2{,}1\cdot 10^6\cdot 2772}$$

$$= 0{,}005 \simeq 0{,}28°$$

CARICO TERMICO

$$\eta_{10} + \eta_{1T} + \eta_{11}X_1 = 0$$

$$\eta_{1T} = \int_B^D \varepsilon_T N_1\,dx_3 = \varepsilon_T\int_B^D N_1\,dx_3 = \varepsilon_T\left(-\frac{1}{2l}\right)h = -\frac{5}{6}\alpha\Delta T$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1T}}{\eta_{11}} = \frac{ql^2}{14} + \frac{5}{6}\frac{\alpha\Delta T EI_1\cdot\cancel{3}}{\cancel{2}\,7l} = \frac{ql^2}{14} + \frac{5}{14}\frac{EI_1\alpha\Delta T}{l} = 714 + 10{,}4 \text{ kgm}$$

Il diagramma delle caratteristiche di sollecitazione non cambia in modo significativo.