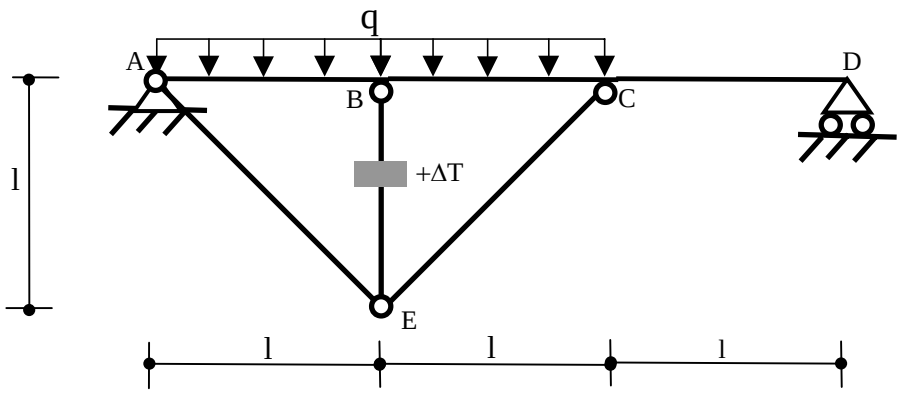
LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA

PROVA SCRITTA DI STATICA

FERRARA, 18/03/2009



$l = 1$ m, $q = 2$ t/m

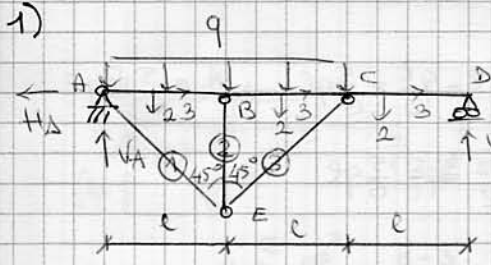
$\sigma_{AMM} = 2400$ kg/cm^2, $E = 2.1\ 10^6$ kg/cm^2

$\Delta T = 20$ °C, $\alpha = 10^{-5}$ °C^{-1}

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del solo carico q e disegnare i diagrammi delle caratteristiche di sollecitazione (N, T, M). Considerare trascurabili le deformazioni assiali.
2. Progettare la travatura.
3. Calcolare la rotazione del nodo B.
4. Risolvere nuovamente la travatura considerando anche un riscaldamento uniforme dell'asta BE. Disegnare i nuovi diagrammi delle caratteristiche di sollecitazione (N, T, M) comprensivi sia di q che di ΔT.

1)

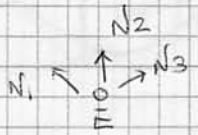


$(\rightarrow)\ +H_A = 0$

$(A\uparrow)\ V_D 3\ell = 2q\ell^2 \rightarrow V_D = \frac{2}{3}q\ell$

$(\uparrow)\ V_A = 2q\ell - \frac{2}{3}q\ell = \frac{4}{3}q\ell$

Esternamente la struttura è isostatica

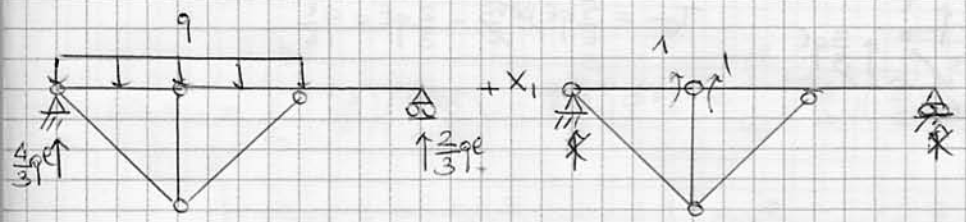


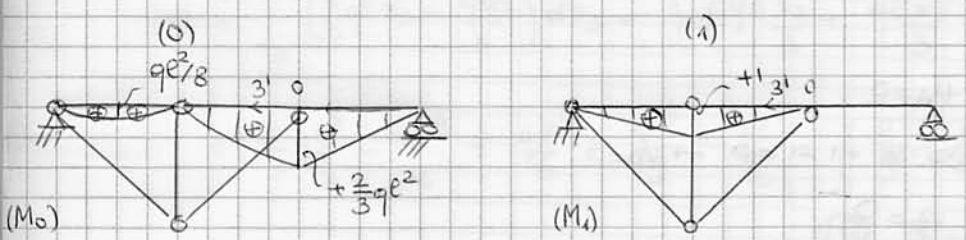
$(\rightarrow)\ N_1\frac{\sqrt{2}}{2} = N_3\frac{\sqrt{2}}{2}$

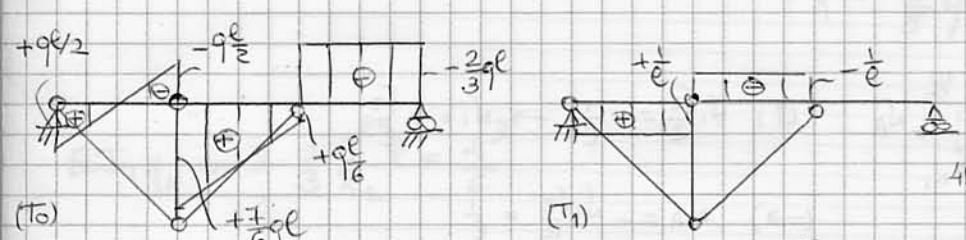
$(\uparrow)\ N_2 = -2N_1\frac{\sqrt{2}}{2}$

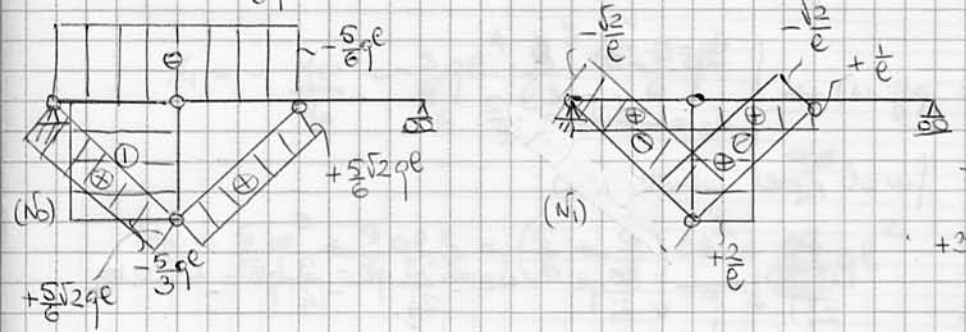
Internamente la struttura è una volta iperstatica

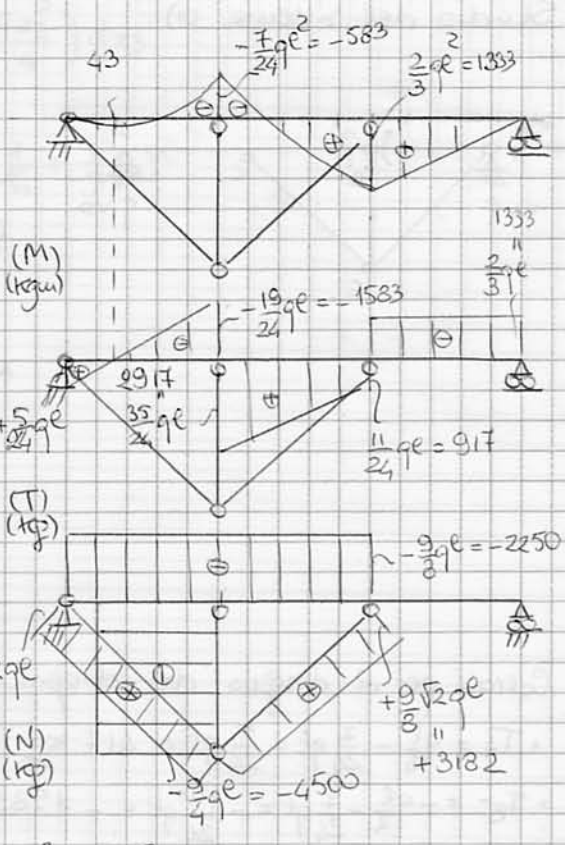
Incognita iperstatica: $X_1 = M_B$.











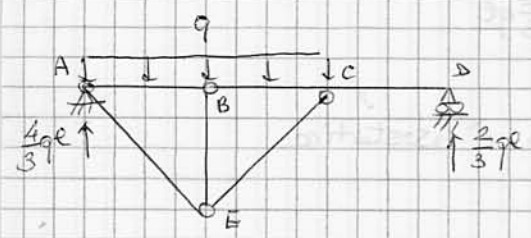
$$EI_1 M_{10} = q\frac{\ell^3}{24} + \int_0^\ell \left(\frac{x_3'}{\ell}\right)\left(\frac{2}{3}q\ell^2 - q\frac{\ell}{6}x_3' - q\frac{x_3'^2}{2}\right)dx_3' = q\frac{\ell^3}{24} + \int_0^\ell \left(\frac{2}{3}q\ell x_3' - q\frac{x_3'^2}{6} - q\frac{x_3'^3}{2\ell}\right)dx_3'$$

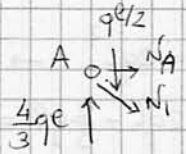
$$= q\frac{\ell^3}{24} + q\frac{\ell^3}{3} - q\frac{\ell^3}{18} - q\frac{\ell^3}{8} = \frac{(3+24-4-9)}{72}q\ell^3 = \frac{14}{72}q\ell^3 = \frac{7}{36}q\ell^3$$

$$EI_1 M_{11} = 2\cdot\frac{1}{3}\ell\cdot\ell = \frac{2}{3}\ell$$

$$M_{10} + M_{11}X_1 = 0 \rightarrow X_1 = -\frac{M_{10}}{M_{11}} = -\frac{7}{36}q\ell^3\cdot\frac{3}{2\ell} = -\frac{7}{24}q\ell^2 = -583\ \text{kgm}$$

Studio del sistema (0):

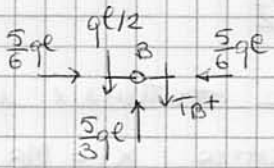




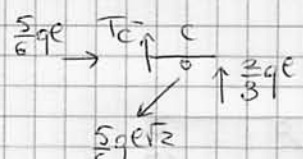
$(\uparrow)\ N_1\frac{\sqrt{2}}{2} = \frac{4}{3}q\ell - q\frac{\ell}{2} = \frac{5}{6}q\ell \rightarrow N_1 = \frac{5}{6}\sqrt{2}\,q\ell$

$(\rightarrow)\ N_A = -N_1\frac{\sqrt{2}}{2} = -\frac{5}{6}q\ell$

da pag 1 $\Rightarrow$ $\begin{cases} N_3 = \frac{5}{6}\sqrt{2}\,q\ell \\ N_2 = -N_1\sqrt{2} = -\frac{5}{3}q\ell \end{cases}$

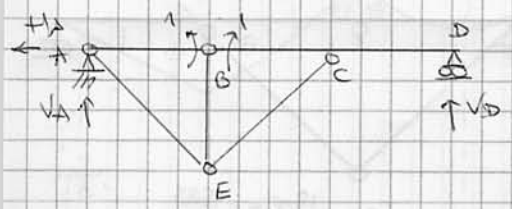


$T_{B^+} = \left(\frac{5}{3} - \frac{1}{2}\right)q\ell = \frac{7}{6}q\ell$



$T_{C^-} = \frac{5}{6}q\ell\frac{\sqrt{2}\sqrt{2}}{2} - \frac{2}{3}q\ell = q\frac{\ell}{6}$

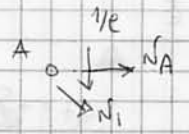
Studio del sistema (1):



$+1_A = 0$

$(A\uparrow)\ V_D \cdot 3\ell + 1 - 1 = 0 \rightarrow V_D = 0$

$(\uparrow)\ V_A = 0$



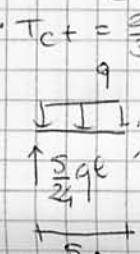
$(\uparrow)\ N_1\frac{\sqrt{2}}{2} = -\frac{1}{\ell} \rightarrow N_1 = -\frac{\sqrt{2}}{\ell}$

$(\rightarrow)\ N_A = -N_1\frac{\sqrt{2}}{2} = \frac{1}{\ell}$

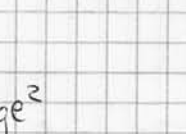
da pag (1) $\Rightarrow$ $\begin{cases} N_3 = -\sqrt{2}/\ell \\ N_2 = -N_1\sqrt{2} = +\frac{2}{\ell} \end{cases}$

Calcoli per le sforzo dei diagrammi finali del punto (1):

- $T_A = q\frac{\ell}{2} - \frac{7}{24}q\ell = \frac{5}{24}q\ell = 417$ kg
- $T_{B^-} = -q\frac{\ell}{2} - \frac{7}{24}q\ell = -\frac{19}{24}q\ell = -1583$ kg
- $T_{B^+} = \frac{7}{6}q\ell + \frac{7}{24}q\ell = \frac{35}{24}q\ell = 2917$ kg
- $T_{C^-} = q\frac{\ell}{6} + \frac{7}{24}q\ell = \frac{11}{24}q\ell = 917$ kg
- $M_C = \frac{2}{3}q\ell^2 = 1333$ kgm
- $T_{C^+} = \frac{2}{3}q\ell = 1333$ kg

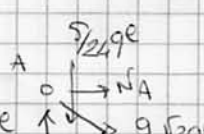


$M_{max} = \frac{1}{2}\frac{25}{24\cdot 24}q\ell^2 = \frac{25}{1152}q\ell^2 = 43$ kgm



$N_2 = -\frac{54}{24}q\ell = -\frac{9}{4}q\ell = -4500$ kg

da pag (1) $\Rightarrow$ $\begin{cases} N_3 = -N_2\frac{\sqrt{2}}{2} = +\frac{9}{8}\sqrt{2}\,q\ell = +3182 \text{ kg} \\ N_1 = N_3 \end{cases}$



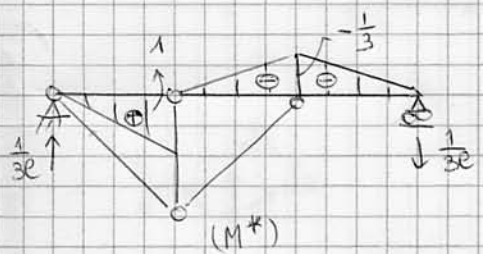
$(\uparrow)\ \frac{5}{24}q\ell + \frac{9}{8}\sqrt{2}\frac{\sqrt{2}}{2}q\ell = \frac{32}{24}q\ell$ ok

$(\rightarrow)\ N_A = -\frac{9}{8}\sqrt{2}q\ell\frac{\sqrt{2}}{2} = -\frac{9}{8}q\ell = -2250$ kg

2) Progetto: $M_{max} = \frac{2}{3} q\ell^2$ (punto C)

$$W_1 \geq \frac{1333 \cdot 100}{2400} = 55{,}5 \text{ cm}^3 \rightarrow \text{IPE 140} \begin{cases} I_1 = 541 \text{ cm}^4 \\ A = 16{,}4 \text{ cm}^2 \end{cases}$$

3) Rotazione in B:



$$1 \cdot \varphi_B = \frac{1}{EI_1}\left[\int_0^\ell \left(\frac{x_3}{\ell}\right)\left(\frac{5}{24} q\ell x_3 - q\frac{x_3^2}{2}\right) dx_3 + \int_0^\ell \left(-\frac{x_3}{3\ell}\right)\left(-\frac{7}{24} q\ell^2 + \frac{35}{24} q\ell x_3 - q\frac{x_3^2}{2}\right) dx_3 + \frac{1}{3}\left(-\frac{1}{3}\right)\frac{2}{3} q\ell^2 \cdot \ell\right]$$

$$= \frac{1}{EI_1}\left[-\frac{q\ell^3}{18} - \frac{31}{144} q\ell^3 - \frac{2}{27} q\ell^3\right]$$

$$= -\frac{149}{432}\frac{q\ell^3}{EI_1} = -\frac{149}{432}\frac{20 \cdot 100^3}{2{,}1 \cdot 10^6 \cdot 541} = 0{,}34°$$

4) Carico termico: $\quad \gamma_{1t} + \gamma_{11} X_1 = -\gamma_{10}$

$$\gamma_{1t} = \int_{BD} N_1 \varepsilon_t = \frac{2}{\ell} \ell \varepsilon_t = 2\varepsilon_t = 2\alpha\Delta T$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} - \frac{\gamma_{1T}}{\gamma_{11}} = -\frac{7}{24} q\ell^2 - \frac{2\alpha\Delta T \, 3EI_1}{2\ell} = -583 - \frac{3 \cdot 2{,}1 \cdot 10^6 \cdot 541 \cdot 10^{-5} \cdot 2 \cdot 10^2}{100}$$

$$= -583 - 68 = -651 \text{ kgm}$$

Il diagramma non si discosta significativamente da quello calcolato al punto (1).