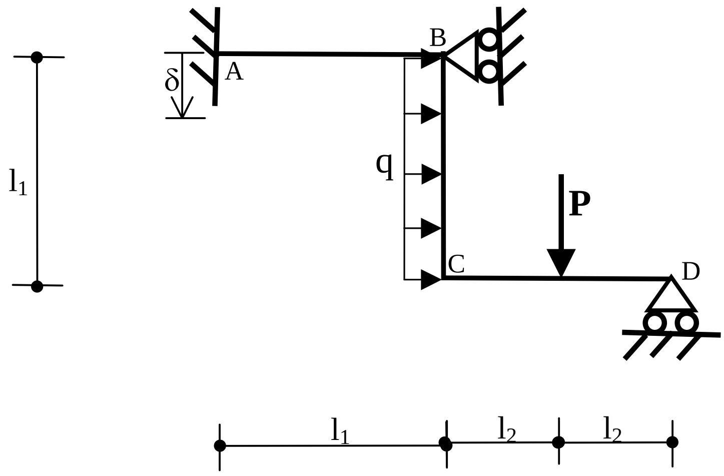
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 12/12/2008

C

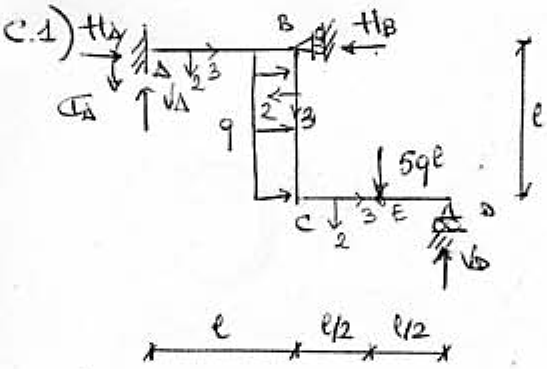


l_1 = 1 m, l_2 = 0.5 m, q = 1 t/m, P = 5 t,
σ_{AMM} = 2400 kg/cm^2 , E = 2.1 10^6 kg/cm^2
δ = 1 cm

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q, P e disegnare i diagrammi delle caratteristiche di sollecitazione (N, T, M). Considerare trascurabili le deformazioni assiali.
2. Progettare la travatura.
3. Calcolare la rotazione del nodo D.
4. Risolvere nuovamente la travatura considerando anche il cedimento verticale δ del vincolo in A. Disegnare i nuovi diagrammi delle caratteristiche di sollecitazione (N, T, M) comprensivi sia di q, P che di δ.

C.1)



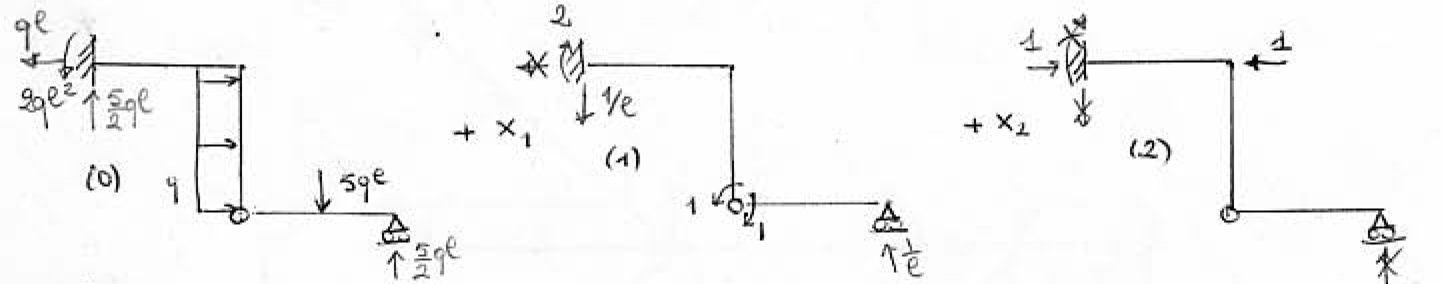
$(\rightarrow)\quad H_A = H_B - q\ell$

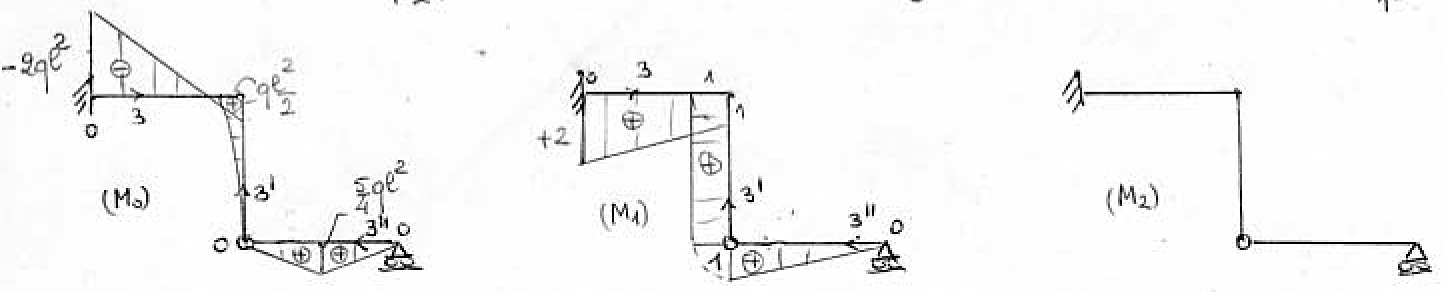
$(\uparrow)\quad V_A + V_D = 5q\ell$

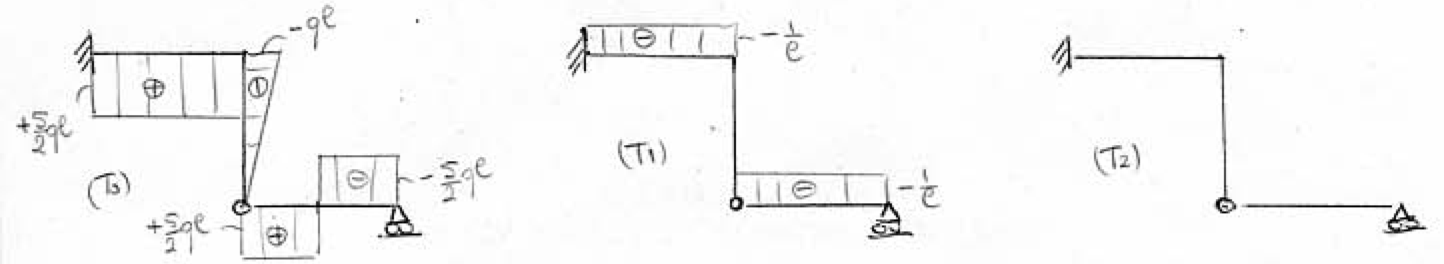
$(A\circlearrowleft)\quad C_A + V_D 2\ell = 5q\ell\,\frac{3}{2}\ell - q\frac{\ell^2}{2}$

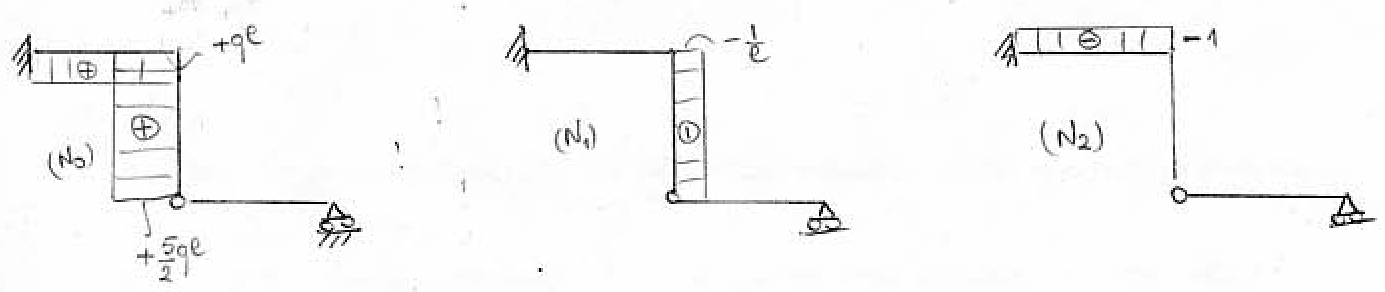
Travatura due volte iperstatica - Incognite $\quad X_1 = M_C$

$X_2 = H_B$









Sistema (0)

$(C\circlearrowleft)_{ABC} + C_A - \frac{5}{2}q\ell^2 + q\ell^2 - q\frac{\ell^2}{2} = 0$

$\hookrightarrow C_A = 2q\ell^2$

$$EJ_1\eta_{10} = \int_0^\ell \left(-2q\ell^2 + \frac{5}{2}q\ell x_3\right)\left(2 - \frac{x_3}{\ell}\right)dx_3 + \int_0^\ell \left(q\frac{x_3'^2}{2}\right)\cdot 1\,dx_3' + \int_0^{\ell/2}\left(\frac{5}{2}q\ell x_3''\right)\left(\frac{x_3''}{\ell}\right)dx_3'' + \int_{\ell/2}^{\ell}\left(\frac{5}{2}q\ell^2 - \frac{5}{2}q\ell x_3''\right)\left(\frac{x_3''}{\ell}\right)$$

$$= -\frac{4}{3}q\ell^3 + q\frac{\ell^3}{6} + \frac{5}{48}q\ell^3 + \frac{5}{24}q\ell^3 = -\frac{41}{48}q\ell^3$$

$$EI_1\,\eta_{11} = \ell + \frac{\ell}{3} + \int_0^{\ell} \left(2 - \frac{x_3}{\ell}\right)^2 dx_3 = \frac{4}{3}\ell + \frac{7}{3}\ell = \frac{11}{3}\ell$$

Nell'ipotesi di trascurare le deformazioni assiali, i coefficienti η_{12}, η_{20} e η_{22} sono nulli. Non è pertanto possibile calcolare X_2, a meno di rimuovere l'ipotesi di trascurabilità.

Si procede così. Si calcola $X_1 = -\frac{\eta_{10}}{\eta_{11}}$, si progetta la struttura a flessione e scelta la sezione si ripete il calcolo di X_1 e X_2 considerando anche le deformazioni assiali.

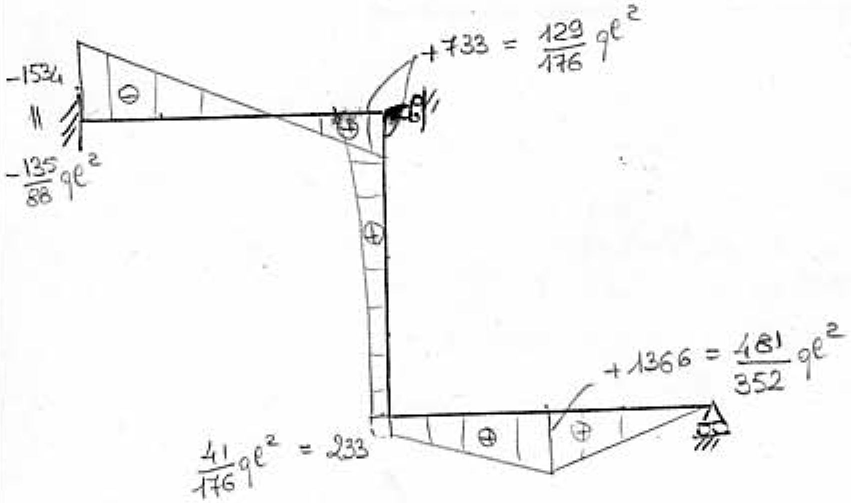
$$X_1 = +\frac{41}{48} q\ell^{\cancel{3}\,2} \cdot \frac{3}{11\cancel{\ell}} = \frac{41}{176} q\ell^2 = 233 \text{ kgm}$$

$$M_B = q\ell^2\left(\frac{1}{2} + \frac{41}{176}\right) = \frac{129}{176} q\ell^2 = 733 \text{ kgm}$$

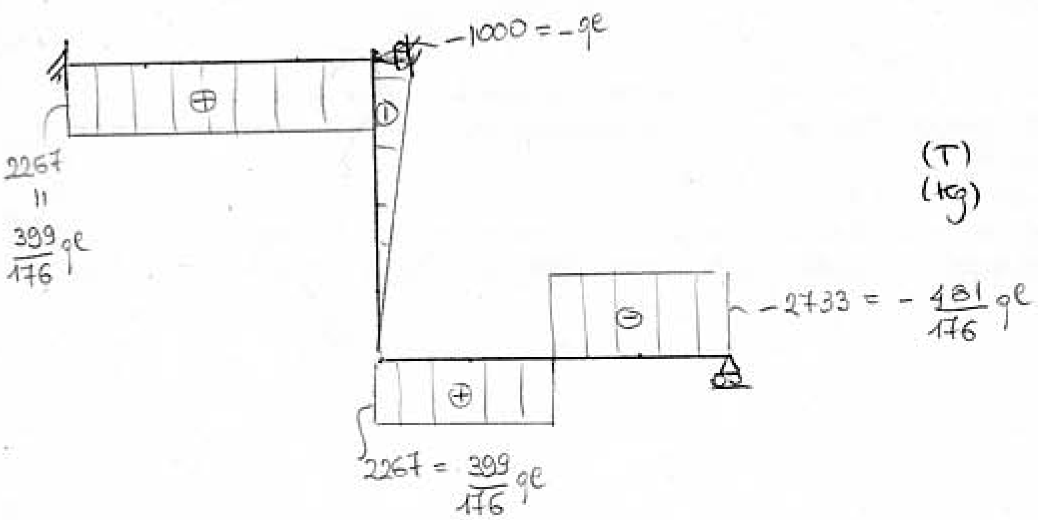
$$M_A = q\ell^2\left(-2 + 2\cdot\frac{41}{176}\right) = -\frac{135}{88} q\ell^2 = -1534 \text{ kgm}$$

$$M_E = q\ell^2\left(\frac{5}{4} + \frac{1}{2}\cdot\frac{41}{176}\right) = \frac{481}{352} q\ell^2 = 1366 \text{ kgm}$$

$$T_A = 1534 + 733 = 2267 \text{ kg} = \frac{399}{176} q\ell$$



(M)
(kgm)



(T)
(kg)

Progetto: $W_1 \geq \frac{1534 \cdot 100}{2400} = 64 \ cm^3 \rightarrow IPE\ 140$ $\qquad I_1 = 541\ cm^4$, $A = 16.4\ cm^2$

Calcolo di X_2:

$$\eta_{10} = -\frac{41}{48}\frac{q\ell^3}{EI_1} - \frac{5q\ell}{2EA}$$

$$\eta_{11} = \frac{11\ell}{3EI_1} + \frac{1}{\ell EA}$$

$$\eta_{12} = 0$$

$$\eta_{20} = -\frac{q\ell^2}{EA}$$

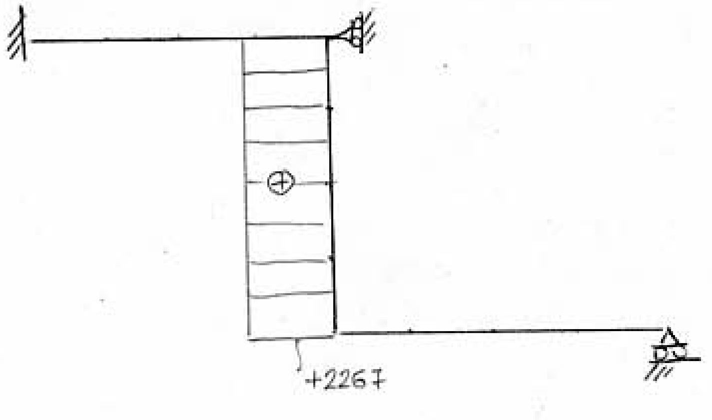
$$\eta_{22} = \frac{\ell}{EA}$$

$$\begin{cases} -\frac{41}{48}\frac{q\ell^3}{EI_1} - \frac{5q\ell}{2EA} + X_1\left(\frac{11\ell}{3EI_1} + \frac{1}{\ell EA}\right) = 0 \\ -\frac{q\ell^2}{\cancel{EA}} + \frac{\ell X_2}{\cancel{EA}} = 0 \end{cases}$$

$$\frac{\eta_{11}^N}{\eta_{11}^M} = \frac{1}{\ell \cancel{E} A}\frac{3\cancel{E}I_1}{11\ell} = \frac{3I_1}{11A\ell^2} \cong 0{,}1\%$$

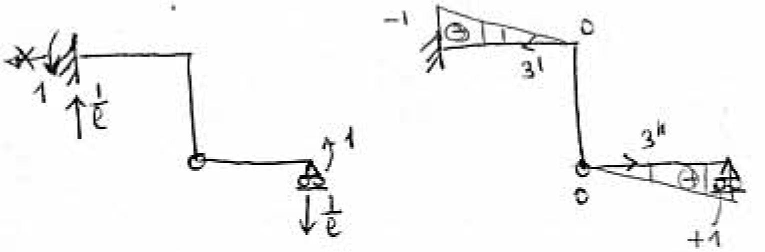
$$\begin{cases} X_1 = \dfrac{\frac{41}{48}\frac{q\ell^3}{EI_1} - \frac{5q\ell}{2EA}}{\frac{11\ell}{3EI_1} + \frac{1}{\ell EA}} \cong \frac{41}{176}q\ell^2 \\ X_2 = q\ell \end{cases}$$

Grafico dello sforzo normale:



(N)
(kg)

C.3) Rotazione in D



$$1 \cdot \varphi_D = \frac{1}{EI_1}\int_0^\ell \left(-\frac{x_3'}{\ell}\right)\left(\frac{129}{176}q\ell^2 - \frac{399}{176}q\ell x_3'\right)dx_3'$$

$$+ \frac{1}{EI_1}\int_0^{\ell/2}\left(\frac{x_3''}{\ell}\right)\left(\frac{41}{176}q\ell^2 + \frac{399}{176}q\ell x_3''\right)dx_3''$$

$$+ \frac{1}{EI_1}\int_{\ell/2}^{\ell}\left(\frac{x_3''}{\ell}\right)\left(\frac{481}{176}q\ell^2 - \frac{481}{176}q\ell x_3''\right)dx_3''$$

$$\rightarrow \varphi_D = \frac{1}{EI_1}\left(\frac{137}{352}q\ell^3 + \frac{87}{704}q\ell^3 + \frac{481}{2112}q\ell^3\right) = \frac{391}{528}\frac{q\ell^3}{EI_1} = \frac{391 \cdot 10 \cdot \cancel{100}^3}{528 \cdot 2{,}1 \cdot \cancel{10^6} \cdot 541} = 0{,}37°$$

c.4) Cedimento

$$\varphi_1 = \varphi_{10} + \varphi_{11} X_1$$

$$\varphi_1 = \frac{\delta}{\ell}$$

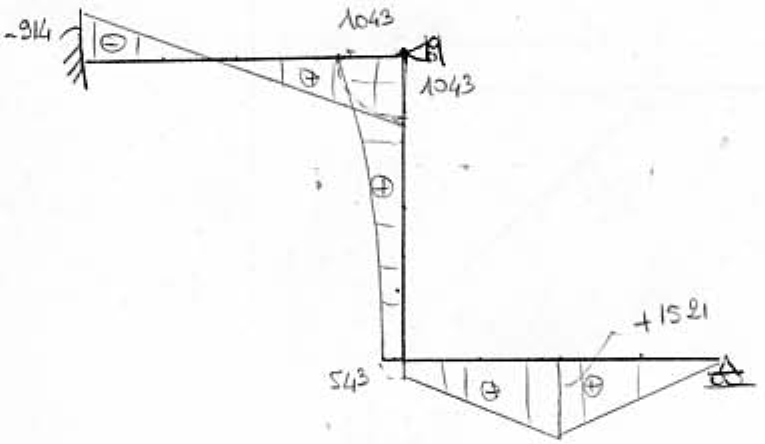
$$X_1 = \frac{41}{176} q\ell^2 + \frac{3EJ\delta}{11\ell^2} = 233 + \frac{3 \cdot 2,1 \cdot 10^{6} \cdot 541 \cdot 1}{11 \cdot \cancel{100^2} \cdot \cancel{10}} = 233 + 310 = 543 \text{ kgm}$$

Diagrammi (comprensivi sia di q, P che di δ):

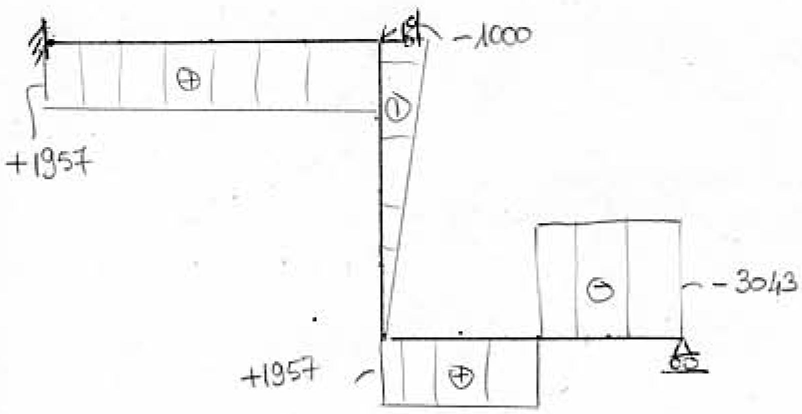
$M_A = -3000 + 2 \cdot 543 = -1914$ kgm

$M_B = 500 + 543 = 1043$ kgm

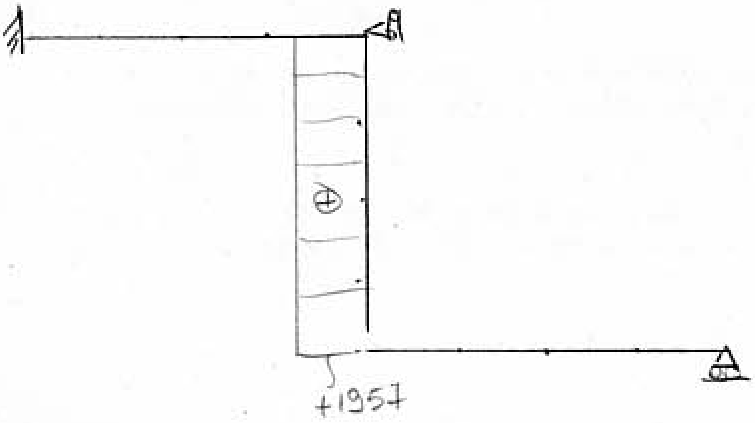
$M_E = 1250 + \frac{543}{2} = 1521$ kgm



(M)
(kgm)



(T)
(kg)



(N)
(kg)