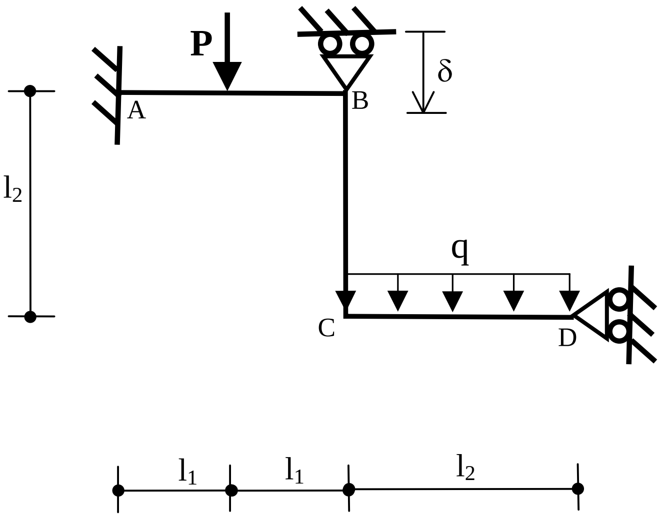
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 12/12/2008

B

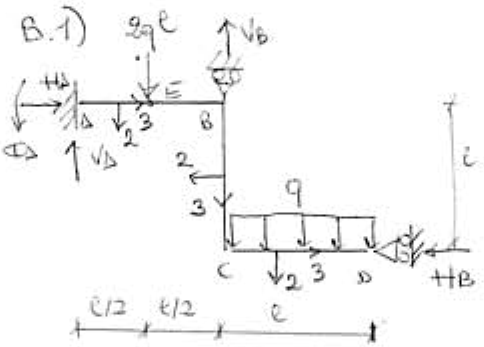


$l_1 = 0.5$ m, $l_2 = 1$ m, $q = 1.5$ t/m, $P = 3$ t,
$\sigma_{AMM} = 2400$ kg/cm^2 , $E = 2.1\ 10^6$ kg/cm^2
$\delta = 1$ cm

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q, P e disegnare i diagrammi delle caratteristiche di sollecitazione (N, T, M). Considerare trascurabili le deformazioni assiali.
2. Progettare la travatura.
3. Calcolare la rotazione del nodo B.
4. Risolvere nuovamente la travatura considerando anche il cedimento verticale δ del vincolo in B. Disegnare i nuovi diagrammi delle caratteristiche di sollecitazione (N, T, M) comprensivi sia di q, P che di δ.

B.1)

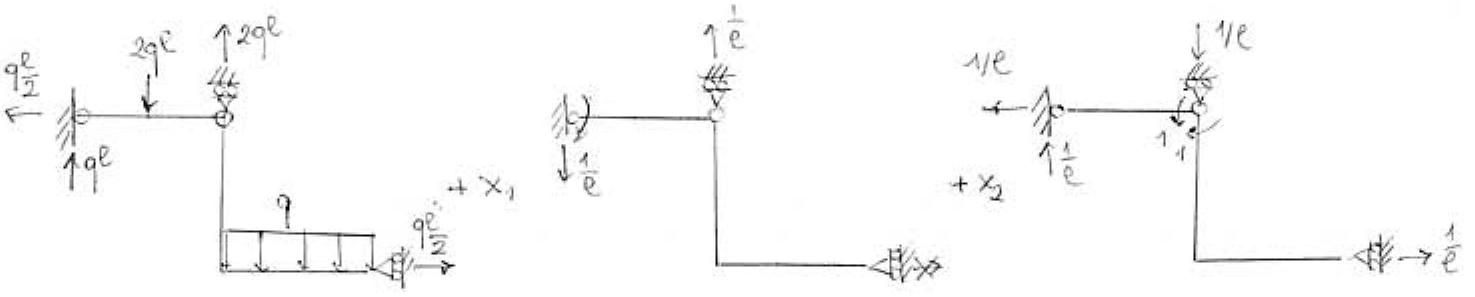


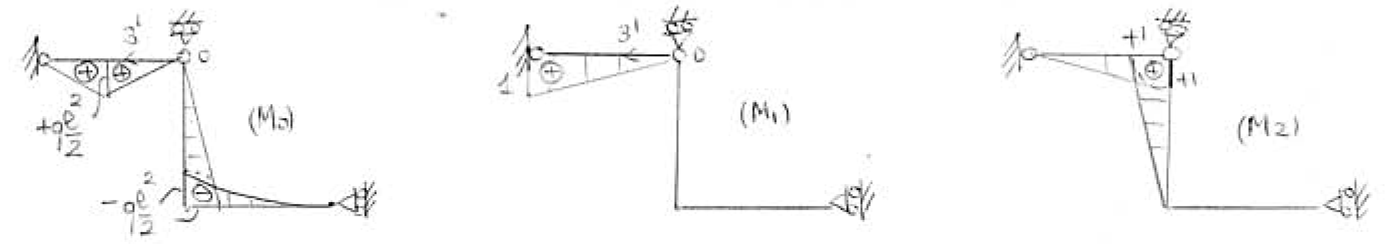
(→) $H_A = H_B$

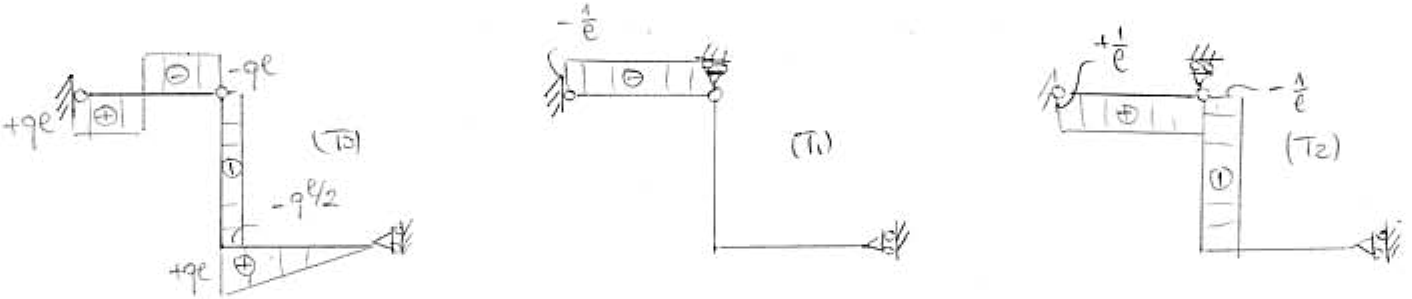
(↑) $V_A + V_B = q\ell + 2q\ell$

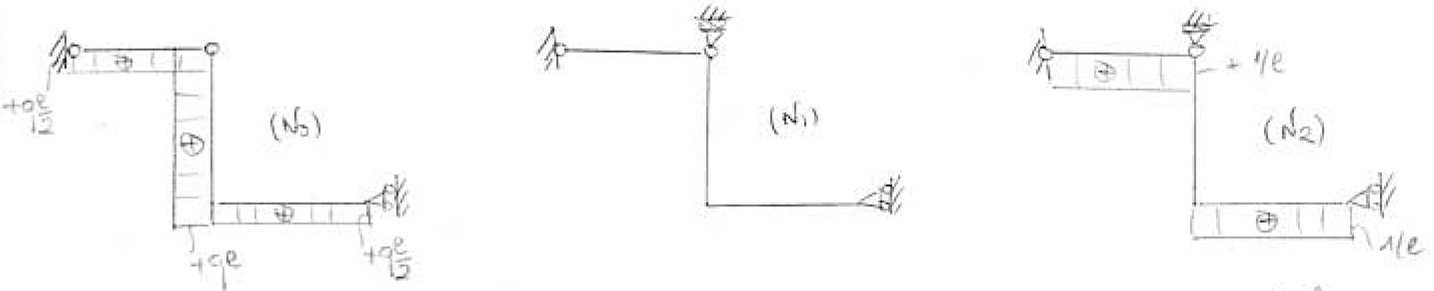
(A↺) $C_A + V_B\ell - H_B\ell = 2q\ell\frac{\ell}{2} + q\ell\frac{3}{2}\ell$

Travatura due volte iperstatica. Incognite iperstatiche: $X_1 = M_A$
$X_2 = M_B$









$$EI_1\eta_{10} = \int_0^{\ell/2}(q\ell x_3')\left(\frac{x_3'}{\ell}\right)dx_3' + \int_{\ell/2}^{\ell}(q\ell^2 - q\ell x_3')\left(\frac{x_3'}{\ell}\right)dx_3' = \frac{q}{3}\frac{\ell^3}{8} + q\left[\ell\frac{x_3'^2}{2} - \frac{x_3'^3}{3}\right]_{\ell/2}^{\ell}$$

$$= \frac{q\ell^3}{24} + q\left[\frac{\ell^3}{2} - \frac{\ell^3}{3} - \frac{\ell^3}{8} + \frac{\ell^3}{24}\right]$$

$$= \frac{q\ell^3}{24} + q\ell^3\frac{(12-8-3+1)}{24}$$

$$= \frac{q\ell^3}{24} + \frac{q\ell^3}{12} = \frac{3}{24}q\ell^3 = \frac{q\ell^3}{8}$$

$$EJ_1 y_{20} = q\frac{\ell^3}{8} - \frac{1}{6}\ell \cdot 1 \cdot q\frac{\ell^2}{2} = q\ell^3\left(\frac{1}{8} - \frac{1}{12}\right) = \frac{q\ell^3}{24}$$

$$EJ_1 y_{11} = \frac{\ell}{3}$$

$$EJ_1 y_{12} = \frac{\ell}{6}$$

$$EJ_1 y_{22} = \frac{2}{3}\ell$$

$$\begin{bmatrix} \ell/3 & \ell/6 \\ \ell/6 & \frac{2}{3}\ell \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\frac{q\ell^3}{24}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\frac{q\ell^2}{4}\begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\frac{q\ell^2}{28}\begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= -\frac{q\ell^2}{28}\begin{bmatrix} 12-1 \\ -3+2 \end{bmatrix} = \begin{bmatrix} -\frac{11}{28}q\ell^2 \\ \frac{q\ell^2}{28} \end{bmatrix}$$

$\det = 8 - 1 = 7$

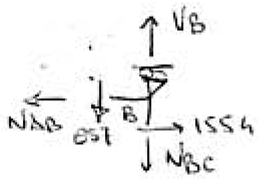
$$\begin{cases} X_1 = -\frac{11}{28}q\ell^2 = -589 \text{ kgm} \\ X_2 = \frac{q\ell^2}{28} = 54 \text{ kgm} \end{cases}$$

$$M_E = q\frac{\ell^2}{2} + \frac{X_1}{2} + \frac{X_2}{2} = q\frac{\ell^2}{2} - \frac{11}{56}q\ell^2 + \frac{1}{56}q\ell^2 = q\ell^2\left(\frac{1}{2} - \frac{\cancel{10}^{\,5}}{\cancel{56}_{\,28}}\right) = \frac{9}{28}q\ell^2 = 482 \text{ kgm}$$

$$T_A = q\ell - \frac{X_1}{\ell} + \frac{X_2}{\ell} = 1500 + 589 + 54 = 2143 \text{ kg}$$

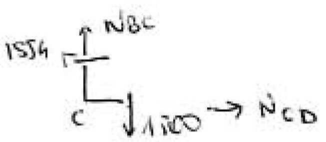
$$T_B^- = -q\ell - \frac{X_1}{\ell} + \frac{X_2}{\ell} = -1500 + 589 + 54 = -857 \text{ kg}$$

$$T_B^+ = -q\frac{\ell}{2} - \frac{X_2}{\ell} = -1500 - 54 = -1554 \text{ kg} = -\frac{q\ell\,14}{2\cdot 14}\,\frac{q\ell}{28} = -\frac{15}{28}q\ell$$



$N_{AB} = +1554$ kg

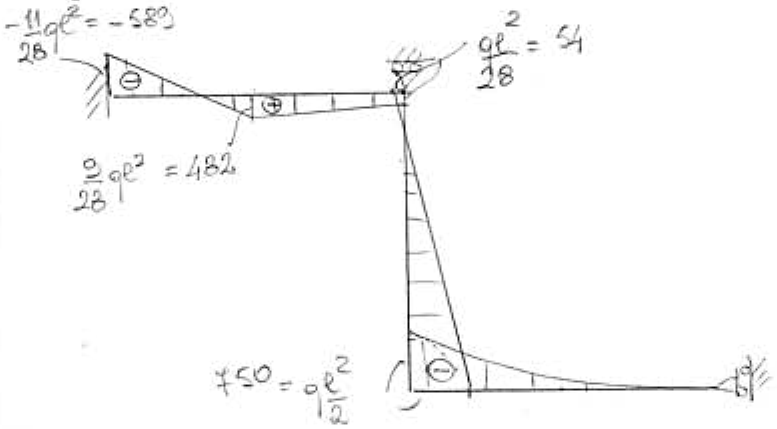
$V_B - N_{BC} =$ kg



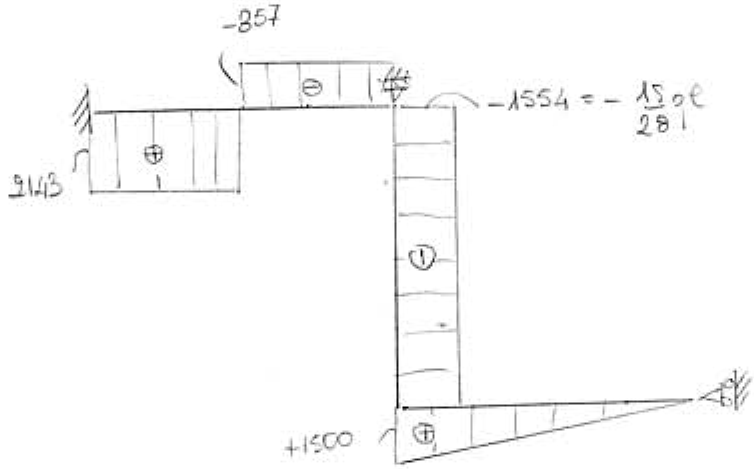
$N_{BC} = 1500$ kg

$N_{CD} = 1554$ kg

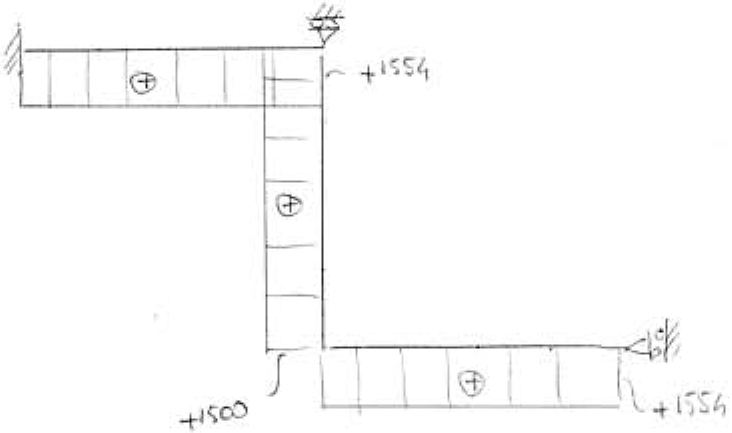
Diagrammi:



(M)
(kg m)



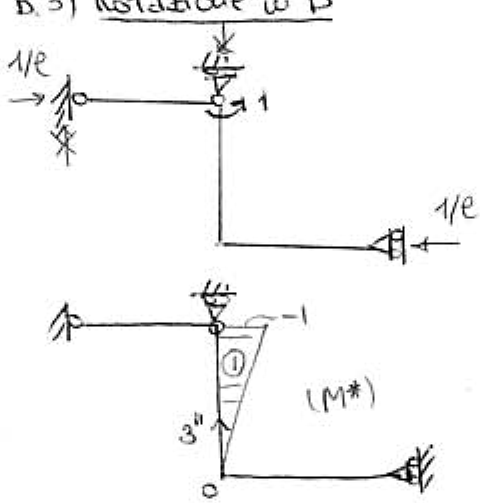
(T)
(kg)



(N)
(kg)

B.2) Progetto: $W_1 \geq \frac{750\cdot 1\cancel{0}0}{24\cancel{0}0} = 31,2\ cm^3 \rightarrow$ IPE 80 $\quad I_1 = 80\ cm^4$, $A = 7,6\ cm^2$

B.3) Rotazione in B



$$1\cdot\varphi_B = \frac{1}{EI_1}\int_0^\ell \left(-\frac{x_3'}{\ell}\right)\left(-q\frac{\ell^2}{2}+\frac{15}{28}q\ell x_3'\right)dx_3'$$

$$= \frac{1}{EI_1}\int_0^\ell \left(-\frac{15}{28}q x_3'^2 + q\frac{\ell}{2}x_3'\right)dx_3'$$

$$= \frac{1}{EI_1}\left[-\frac{15}{28}q\frac{\ell^3}{3} + q\frac{\ell^3}{4}\right] = \frac{1}{EI_1}\frac{(-15+21)}{84}q\ell^3 = \frac{q\ell^3}{14EI_1}$$

$$= \frac{15\cdot 10\cancel{0}^3}{14\cdot 2,1\cdot 10^{\cancel{6}}\cdot 80} = 0,36°$$

B.4) Cedimento

$$\begin{cases} y_1 = y_{10} + y_{11} X_1 + y_{12} X_2 \\ y_2 = y_{20} + y_{21} X_1 + y_{22} X_2 \end{cases} \qquad \begin{aligned} y_1 &= -\delta/\ell \\ y_2 &= \delta/\ell \end{aligned}$$

$$\begin{bmatrix} \ell/3 & \ell/6 \\ \ell/6 & \frac{2}{3}\ell \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\frac{q\ell^3}{24} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \frac{EI_1\delta}{\ell} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = -\frac{q\ell^2}{4} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \frac{6EI_1\delta}{\ell^2} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

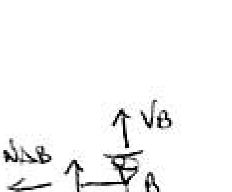
$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -\frac{11}{28} q\ell^2 \\ \frac{q\ell^2}{28} \end{bmatrix} + \frac{6}{7} \frac{EI_1\delta}{\ell^2} \underbrace{\begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{\begin{bmatrix} -5 \\ 3 \end{bmatrix}} = \begin{bmatrix} -\frac{11}{28} q\ell^2 - \frac{30}{7} \frac{EI_1\delta}{\ell^2} \\ \frac{q\ell^2}{28} + \frac{18}{7} \frac{EI_1\delta}{\ell^2} \end{bmatrix}$$

$$\begin{cases} X_1 = -589 - \frac{30}{7} \frac{2{,}1 \cdot \cancel{10^6} \cdot 80 \cdot 1}{\cancel{100^2}} \frac{1}{\cancel{100}} = -589 - 720 = -1309 \text{ kgm} \\ X_2 = 54 + \frac{18}{7} \frac{2{,}1 \cdot \cancel{10^6} \cdot 80 \cdot 1}{\cancel{100^2}} \frac{1}{\cancel{100}} = 54 + 432 = 486 \text{ kgm} \end{cases}$$

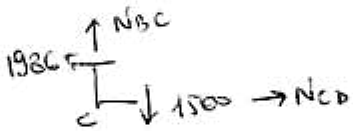
$M_E = q\frac{\ell^2}{2} + \frac{X_1}{2} + \frac{X_2}{2} = 338$ kgm

$T_A = 1500 + 1309 + 486 = 3295$ kg

$T_B = -1500 + 1309 + 486 = 295$



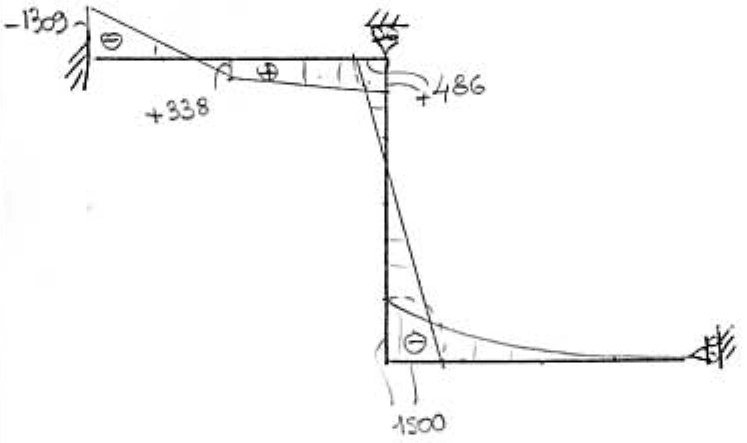
$N_{DB} = 1986$ kg



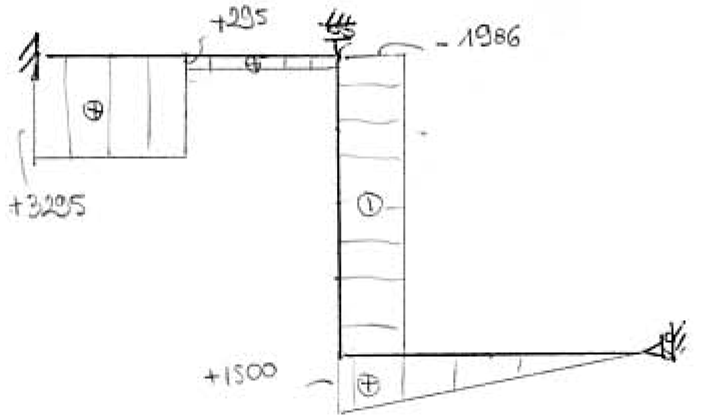
$N_{CD} = 1986$ kg

$N_{BC} = 1500$ kg

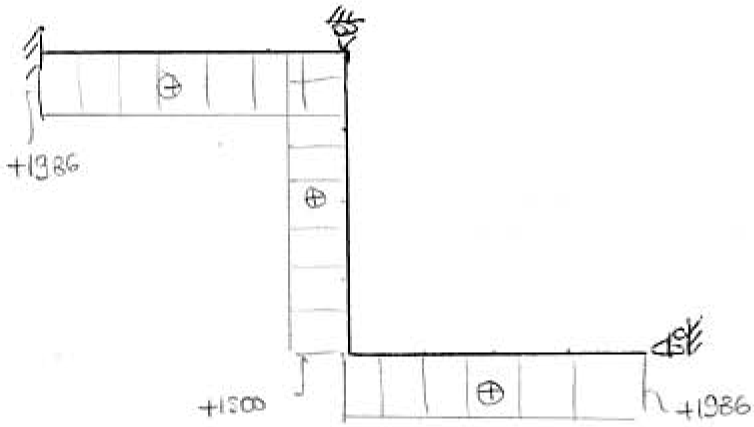
Diagrammi (completi di q, P, δ):



(M)
(kgm)



(T)
(kg)



(N)
(kg)