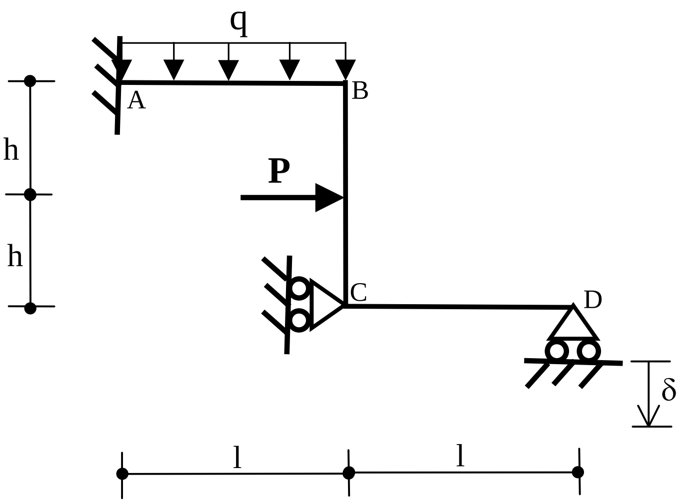
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 12/12/2008

A

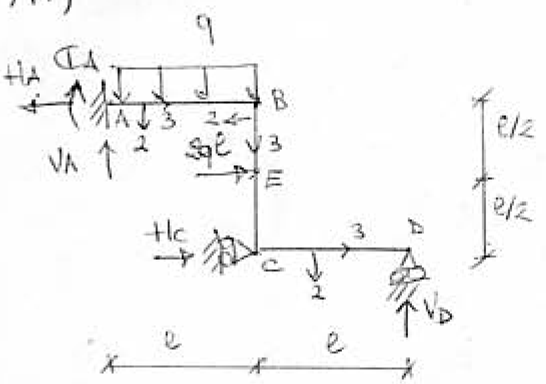


l = 1 m, h = 0.5 m, q = 2 t/m, P = 4 t,
σ_{AMM} = 2400 kg/cm^2, E = 2.1 10^6 kg/cm^2
δ = 1 cm

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q, P e disegnare i diagrammi delle caratteristiche di sollecitazione (N, T, M). Considerare trascurabili le deformazioni assiali.
2. Progettare la travatura.
3. Calcolare la rotazione del nodo C.
4. Risolvere nuovamente la travatura considerando anche il cedimento verticale δ del vincolo in D. Disegnare i nuovi diagrammi delle caratteristiche di sollecitazione (N, T, M) comprensivi sia di q, P che di δ.

A1)



(→) $H_A = H_C + 2q\ell$

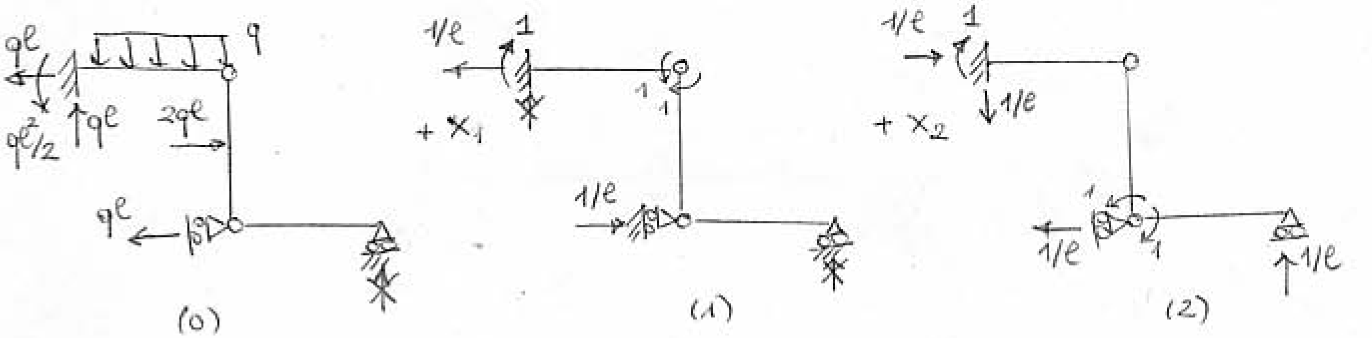
(↑) $V_A + V_D = q\ell$

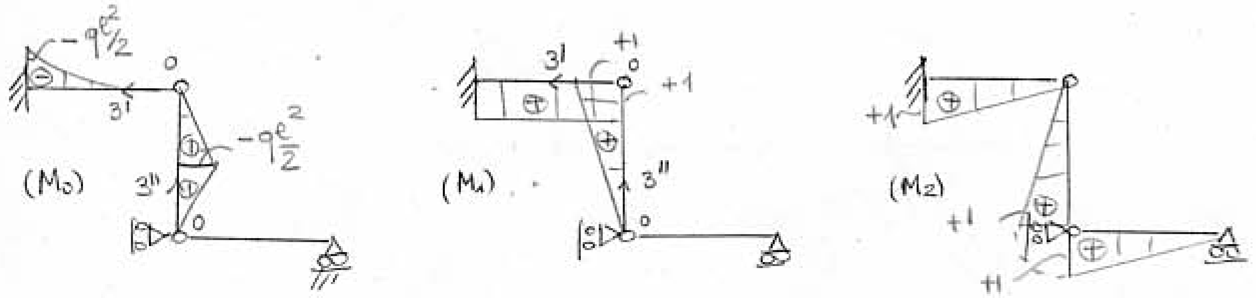
(A↺) $V_D 2\ell + H_C \ell - \mathcal{C}_A = q\frac{\ell^2}{2} - \cancel{2}q\ell\frac{\ell}{\cancel{2}}$

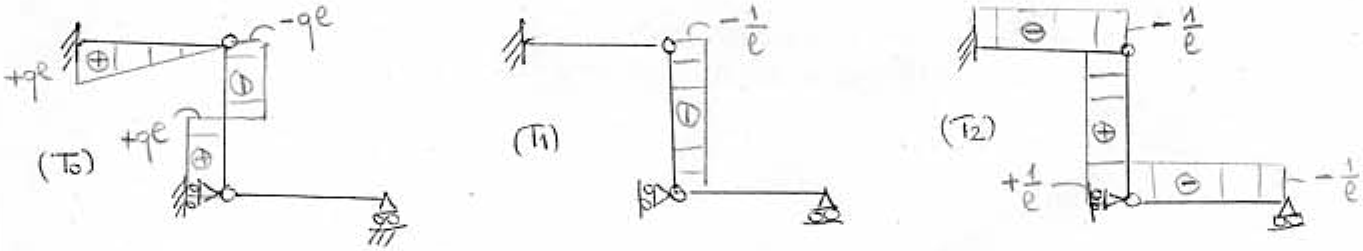
La struttura è due volte iperstatica.

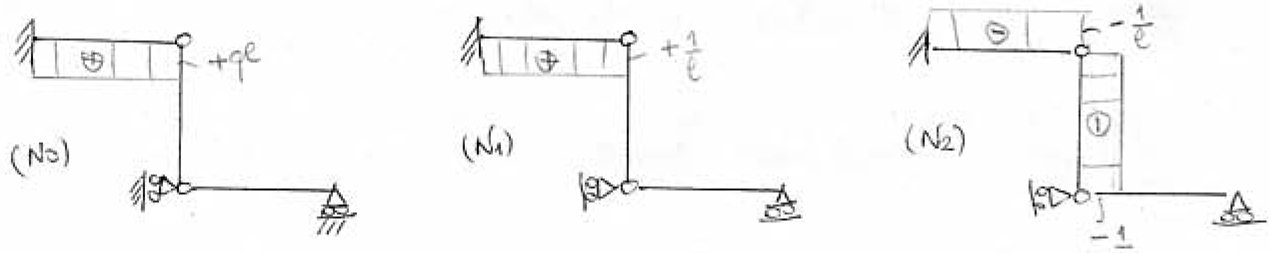
Incognite iperstatiche: $X_1 = M_B$

$X_2 = M_C$









$$EI_1 M_{10} = \int_0^\ell (1)\left(-\frac{q}{2}x_3'^2\right)dx_3' + \int_0^{\ell/2}\left(\frac{x_3''}{\cancel{\ell}}\right)\left(-q\cancel{\ell}x_3''\right)dx_3'' + \int_{\ell/2}^{\ell}\left(\frac{x_3''}{\cancel{\ell}}\right)\left(-q\ell^{\cancel{2}}+q\cancel{\ell}x_3''\right)dx_3''$$

$$= -\frac{q}{2}\frac{\ell^3}{3} - q\frac{1}{3}\frac{\ell^3}{8} - q\int_{\ell/2}^{\ell}\left(\ell x_3'' - x_3''^2\right)dx_3'' = -\frac{q\ell^3}{6} - \frac{q\ell^3}{24} - q\left[\ell\frac{x_3''^2}{2} - \frac{x_3''^3}{3}\right]_{\ell/2}^{\ell}$$

$$= -\frac{q\ell^3}{6} - \frac{q\ell^3}{24} - q\ell^3\left[\frac{1}{2} - \frac{1}{3} - \frac{1}{8} + \frac{1}{24}\right] = -q\ell^3\,\frac{4+1+12-8-3+1}{24} = -\frac{7}{24}q\ell^3$$

$$EI_1\eta_{20} = \int_0^\ell \left(\frac{x_3'}{\ell}\right)\left(-q\frac{x_3'^2}{2}\right)dx_3' + \int_0^{\ell/2}\left(-q\cancel{\ell}x_3''\right)\left(\frac{x_3''}{\cancel{\ell}}\right)dx_3'' + \int_{\ell/2}^{\ell}\left(-q\ell^2+q\ell x_3''\right)\left(\frac{x_3''}{\ell}\right)dx_3''$$

$$= -\frac{q}{2\ell}\frac{\ell^4}{4} - \frac{q}{3}\frac{\ell^3}{8} - q\int_{\ell/2}^{\ell}\left(\ell x_3'' - x_3''^2\right)dx_3'' = -q\frac{\ell^3}{8} - \frac{q\ell^3}{24} - q\left[\ell\frac{x_3''^2}{2} - \frac{x_3''^3}{3}\right]_{\ell/2}^{\ell}$$

$$= -q\frac{\ell^3}{8} - q\frac{\ell^3}{24} - q\ell^3\left[\frac{1}{2} - \frac{1}{3} - \frac{1}{2}\frac{1}{4} + \frac{1}{3}\frac{1}{8}\right] = -q\frac{\ell^3}{8} - q\frac{\ell^3}{24} - q\ell^3\,\frac{12-8-3+1}{24}$$

$$= -q\frac{\ell^3}{8} - q\frac{\ell^3}{24} - q\frac{\ell^3}{12} = -q\ell^3\frac{(3+1+2)}{24} = -q\frac{\ell^3}{4}$$

$$EI_1\eta_{11} = \ell + \frac{\ell}{3} = \frac{4}{3}\ell$$

$$EI_1\eta_{12} = \frac{\ell}{2} + \frac{1}{6}\ell = \frac{4}{6}\ell = \frac{2}{3}\ell$$

$$EI_1\eta_{22} = 3\cdot\frac{\ell}{3} = \ell$$

$$\begin{bmatrix} \frac{4}{\cancel{3}}\cancel{\ell} & \frac{2}{\cancel{3}}\cancel{\ell} \\ \frac{2}{\cancel{3}}\cancel{\ell} & 3\cancel{\ell} \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = q\frac{\ell^{\cancel{3}\,2}}{\cancel{24}\,8}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

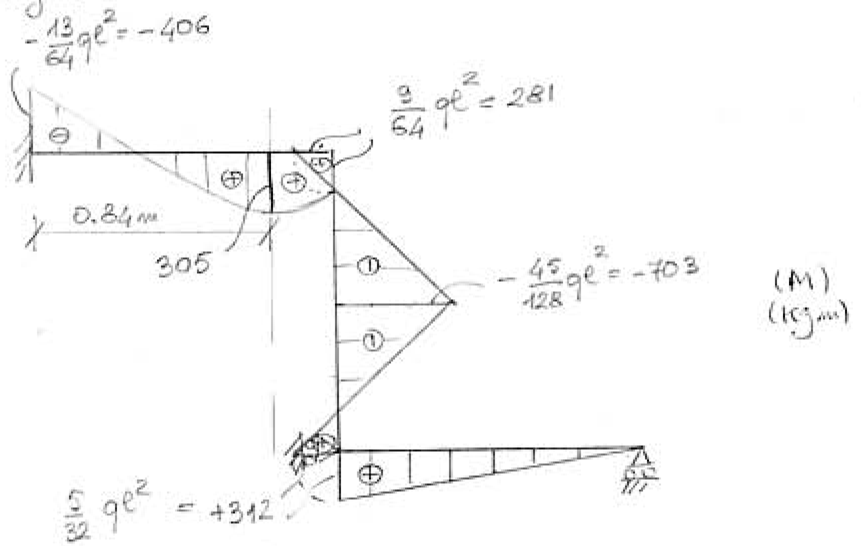
$$\begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = q\frac{\ell^2}{8}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

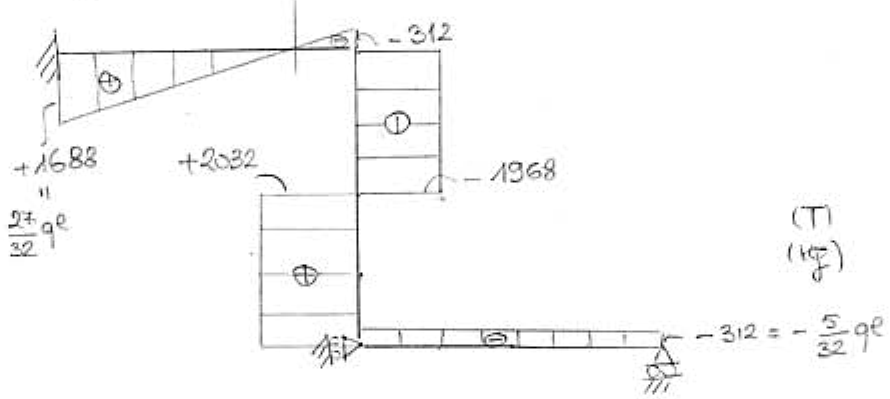
$\det = 12 - 4 = 8$

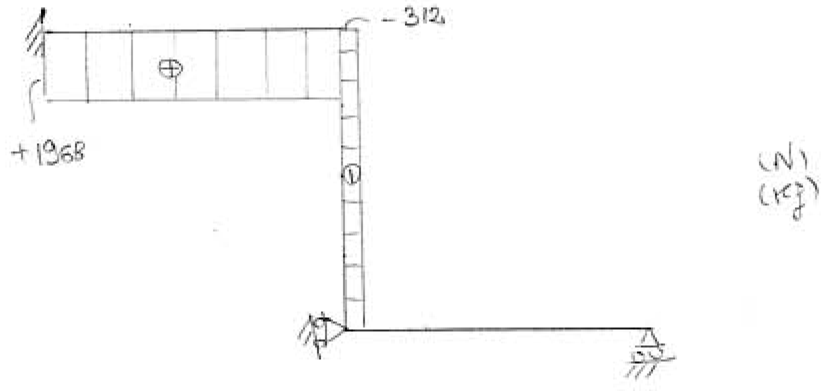
$$X_1 = \frac{1}{8}q\frac{\ell^2}{8}\det\begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix} = \frac{q\ell^2}{64}(21-12) = \frac{9}{64}q\ell^2 = 281\ \text{kgm}$$

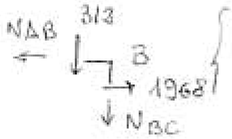
$$X_2 = \frac{1}{8}q\frac{\ell^2}{8}\det\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} = \frac{q\ell^2}{64}(24-14) = \frac{10}{64}q\ell^2 = \frac{5}{32}q\ell^2 = 312\ \text{kgm}$$

Diagrammi:







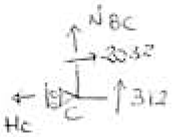


$N_{AB} = 1968$ kg
$N_{BC} = -312$ kg

$T_A = q\ell - \frac{5}{32}q\ell = \frac{27}{32}q\ell = 1688$ kg

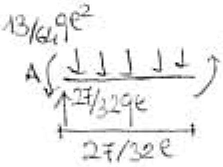
$T_B^- = -\frac{5}{32}q\ell = -312$ kg

$M_A = -q\frac{\ell^2}{2} + \frac{9}{64}q\ell^2 + \frac{5}{32}q\ell^2 = \frac{(-32+9+10)q\ell^2}{64} = -\frac{13}{64}q\ell^2 = -406$ kgm



$N_{BC} = -312$ kg
$H_C = 2032$ kg

$M_E = -q\frac{\ell^2}{2} + \frac{9}{128}q\ell^2 + \frac{5}{64}q\ell^2 = \frac{-64+9+10}{128}q\ell^2 = -\frac{45}{128}q\ell^2 = -703$ kgm



$\overline{M} = -\frac{13}{64}q\ell^2 + \frac{1}{2}\left(\frac{27}{32}\right)^2 q\ell^2 = 305$ kgm

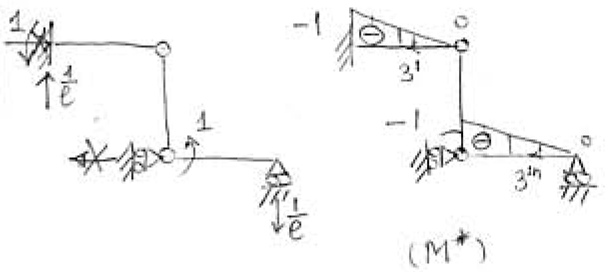
$T_B^+ = -\left(\frac{281+703}{1/2}\right) = -1968$ kg

$T_C^- = \left(\frac{703+312}{1/2}\right) = 2032$ kg

$T_C^+ = -312$ kg

A) Progetto : $W_1 \geq \frac{M_1}{\sigma_{amm}} = \frac{703 \cdot \cancel{100}}{24\cancel{0}} = 29,3\ cm^3 \longrightarrow$ IPE 100 $(W_1 = 34\ cm^3)$ $\quad I_1 = 171\ cm^4$, $A = 10,32\ cm^2$

A.3) Rotazione in C



$$1 \cdot \varphi_C = \frac{1}{EI_1}\int_0^\ell \left(-\frac{x_3'}{\ell}\right)\left(\frac{9}{64}q\ell^2 + \frac{5}{32}q\ell x_3' - q\frac{x_3'^2}{2}\right)dx_3'$$

$$+ \frac{1}{EI_1}\,\frac{\ell}{3}\,(-1)\left(\frac{5}{32}q\ell^2\right)$$

$$= \frac{q}{EI_1}\int_0^\ell \left(\frac{x_3'^3}{2\ell} - \frac{5}{32}x_3'^2 - \frac{9}{64}\ell x_3'\right)dx_3' - \frac{5}{96}\frac{q\ell^3}{EI_1}$$

$$= \frac{q}{EI_1}\left[\frac{\ell^3}{8} - \frac{5}{96}\ell^3 - \frac{9}{128}\ell^3\right] - \frac{5}{96}\frac{q\ell^3}{EI_1}$$

$$= \frac{q\ell^3}{EI_1}\left[\frac{1}{8} - \frac{10}{96} - \frac{9}{128}\right] = \frac{48-40-27}{384}\frac{q\ell^3}{EI_1}$$

$$= -\frac{19}{384}\frac{q\ell^3}{EI_1}$$

$$= -\frac{19 \cdot 20 \cdot \cancel{100^3}}{384 \cdot 2,1 \cdot \cancel{10^6} \cdot 171}$$

$$= -0,157^\circ$$

A.4) Cedimento in D

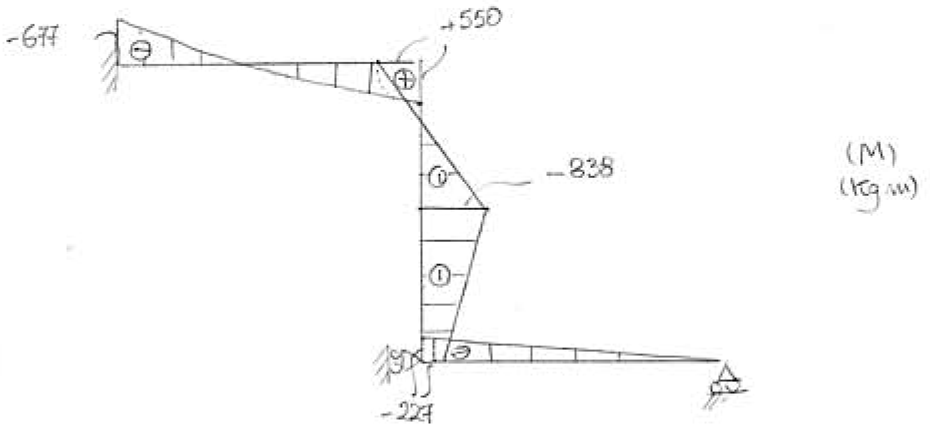
$$\begin{cases} \eta_1 = \eta_{10} + \eta_{11}X_1 + \eta_{12}X_2 & \eta_1 = 0 \\ \eta_2 = \eta_{20} + \eta_{21}X_1 + \eta_{22}X_2 & \eta_2 = -\frac{\delta}{\ell} \end{cases}$$

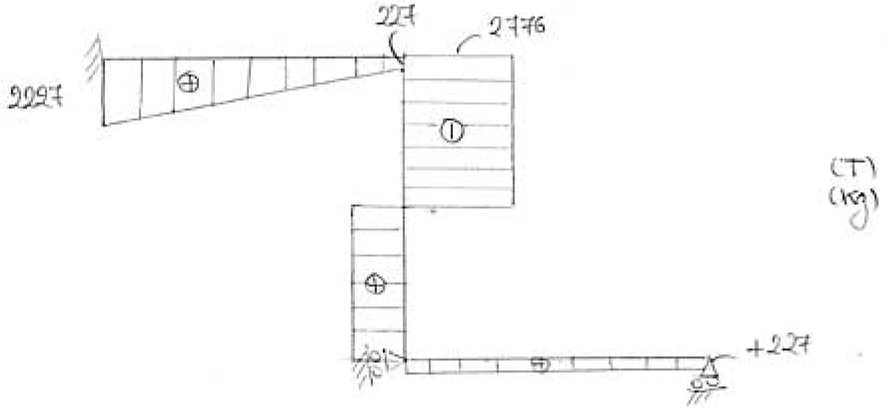
$$\begin{bmatrix} \frac{4}{\cancel{3}}\cancel{\ell} & \frac{2}{\cancel{3}}\cancel{\ell} \\ \frac{2}{\cancel{3}}\cancel{\ell} & 3\cancel{\ell} \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \frac{q\ell^{\cancel{3}\,2}}{\cancel{24}\,8}\begin{bmatrix} 7 \\ 6 \end{bmatrix} + \frac{3\delta EI_1}{\ell^2}\begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}$$

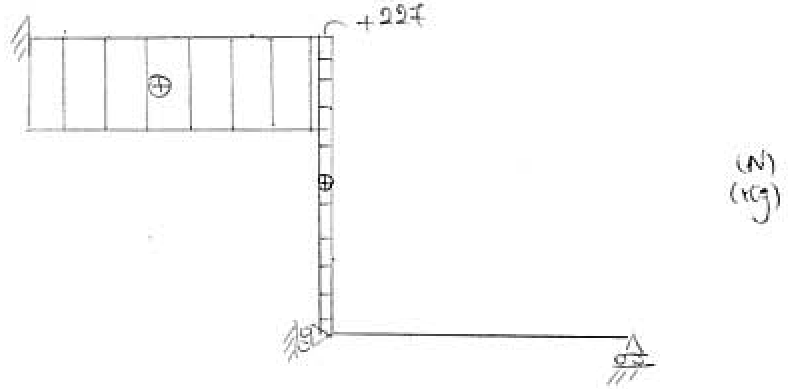
$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \frac{9}{64}q\ell^2 \\ \frac{5}{32}q\ell^2 \end{bmatrix} + \frac{3EI_1\delta}{8\ell^2}\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{9}{64}q\ell^2 \\ \frac{5}{32}q\ell^2 \end{bmatrix} + \frac{3EI_1\delta}{8\ell^2}\begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$\begin{cases} X_1 = \frac{9}{64}q\ell^2 + \frac{3}{4}\frac{EI_1\delta}{\ell^2} = 281 + \frac{3 \cdot 2,1 \cdot 10^6 \cdot 171 \cdot 1}{4 \cdot 100^2}\,\frac{1}{100} = 281 + 269 = 550\ kgm \\ X_2 = \frac{5}{32}q\ell^2 - \frac{3}{2}\frac{EI_1\delta}{\ell^2} = 312 - \frac{3 \cdot 2,1 \cdot 10^6 \cdot 171 \cdot 1}{2 \cdot 100^2}\cdot\frac{1}{100} = 312 - 539 = -227\ kgm \end{cases}$$

Diagrammi (complessivi sia di q, P che di δ):





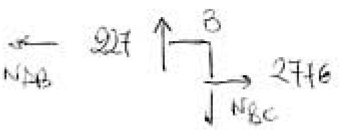


$M_D = -q\frac{\ell^2}{2} + X_1 + X_2 = -1000 + 550 - 227 = -677$ kg m

$M_E = -q\frac{\ell^2}{12} + \frac{X_1}{2} + \frac{X_2}{2} = -1000 + 275 - 113 = -838$ kg m

$T_A = 2000 + 227 = 2227$ kg

$T_B = 227$ kg



$N_{BC} = 227$ kg

$N_{AB} = 2776$ kg