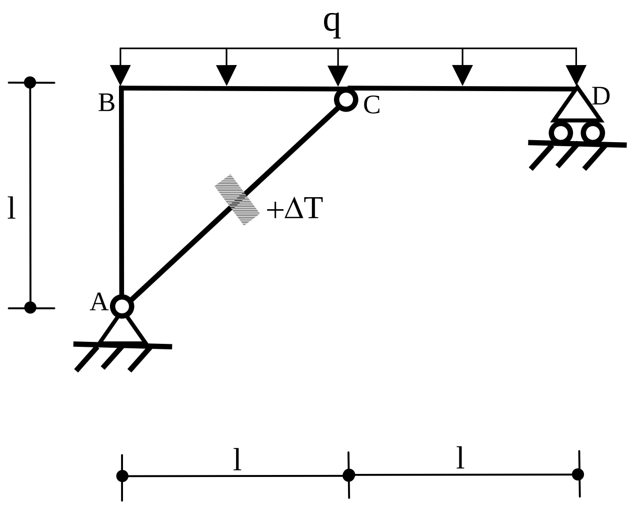
**A**

**CORSO DI LAUREA IN INGEGNERIA MECCANICA**
**UNIVERSITÀ DEGLI STUDI DI FERRARA**
**PROVA SCRITTA DI STATICA**
**FERRARA, 09/01/2009**




$l = 2$ m, $q = 2$ t/m,

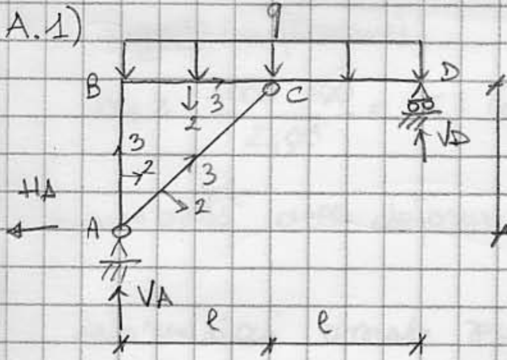
$\sigma_{AMM} = 2400$ kg/cm$^2$ , $E = 2.1\ 10^6$ kg/cm$^2$

$\Delta T = 20°C$, $\alpha = 10^{-5}\ °C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del solo carico q e disegnare i diagrammi delle caratteristiche di sollecitazione (N, T, M).
2. Progettare la travatura.
3. Calcolare la rotazione del nodo B.
4. Risolvere nuovamente la travatura considerando anche il riscaldamento della biella AC. Disegnare i nuovi diagrammi delle caratteristiche di sollecitazione (N, T, M) comprensivi sia di q che del carico termico.

A.1)

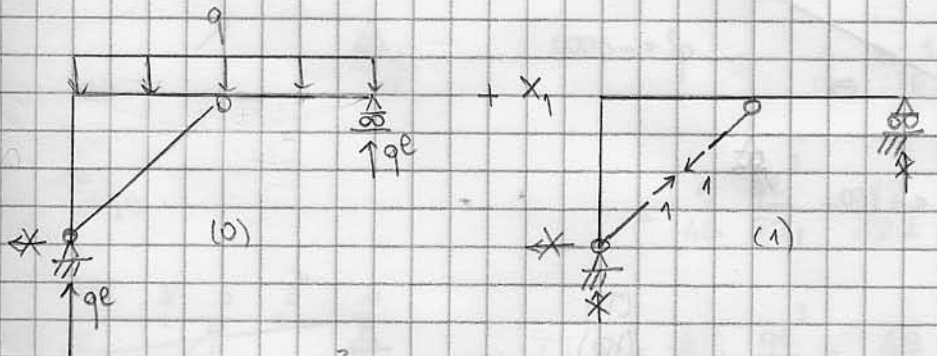


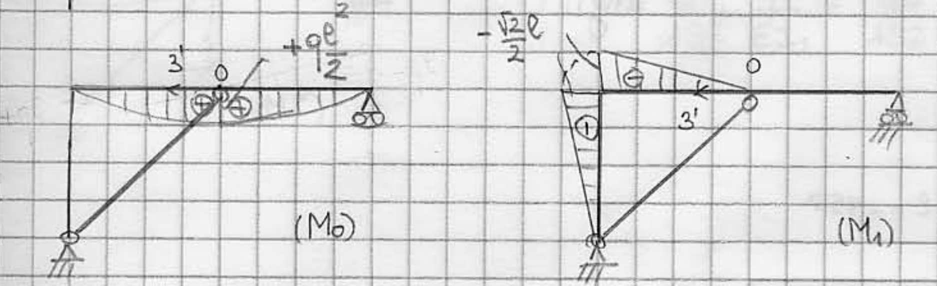

$$\begin{cases} H_A = 0 \\ V_A + V_D = 2ql \\ (A\circlearrowleft)\quad V_D\, 2l = 2ql^2 \end{cases}$$

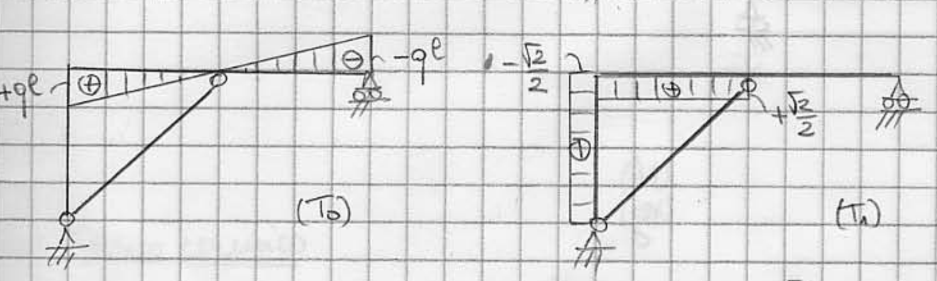
$$\begin{cases} H_A = 0 \\ V_D = V_A = ql \end{cases}$$

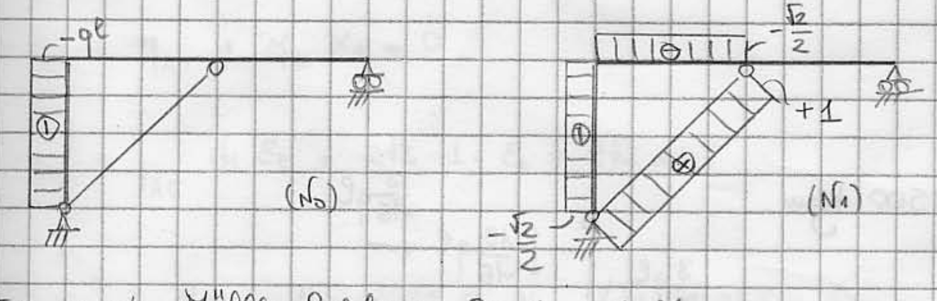
La travatura è isostatica per vincoli esterni.

Internamente è una volta iperstatica: $X_1 = N_{AC}$.



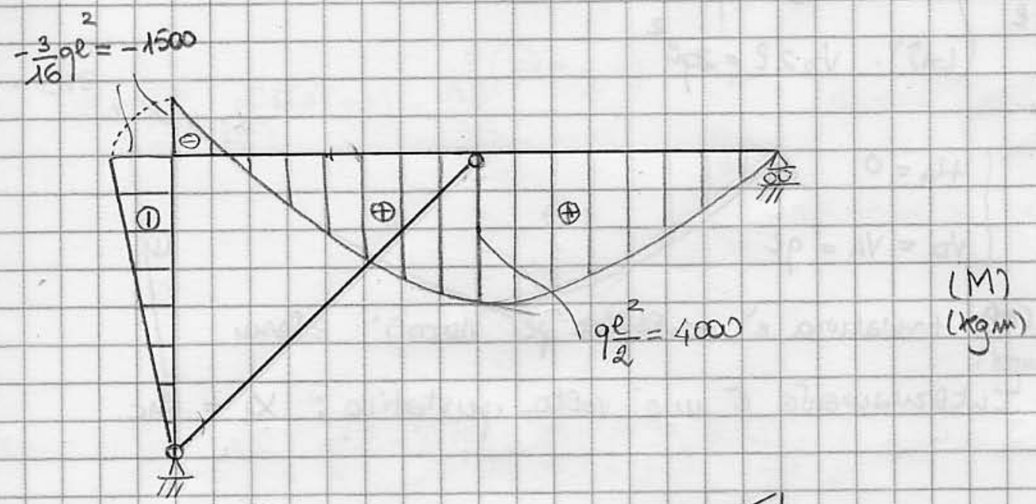

Eq.ne di Müller-Breslau: $0 = \eta_{10} + \eta_{11} X_1$

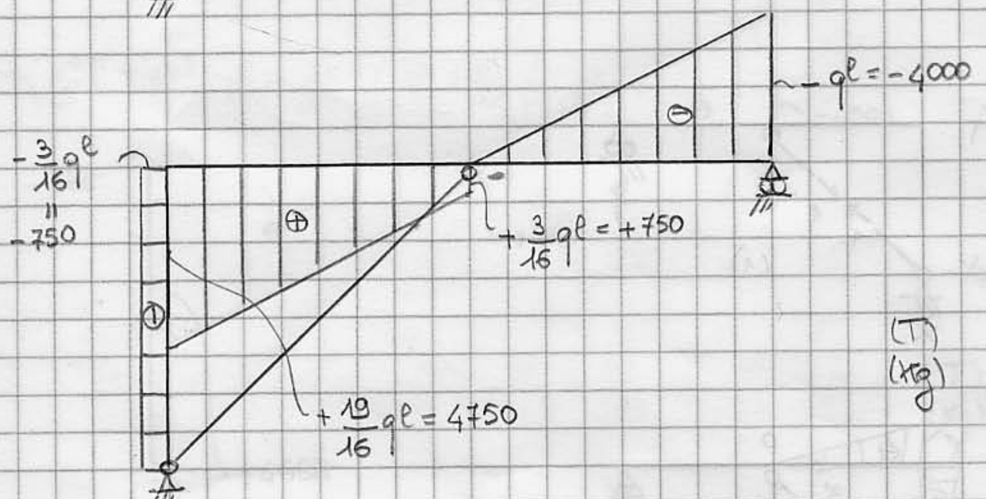
$$EI_1\, \eta_{10} = \int_0^l \left(q\frac{l^2}{2} - q\frac{x_{3'}^2}{2}\right)\left(-\frac{\sqrt{2}}{2}x_{3'}\right)dx_{3'} = +q\frac{\sqrt{2}}{4}\int_0^l \left(x_{3'}^3 - l^2 x_{3'}\right)dx_{3'} = +q\frac{\sqrt{2}}{4}\left[\frac{l^4}{4} - \frac{l^4}{2}\right] = -\frac{q\sqrt{2}}{16}l^4$$

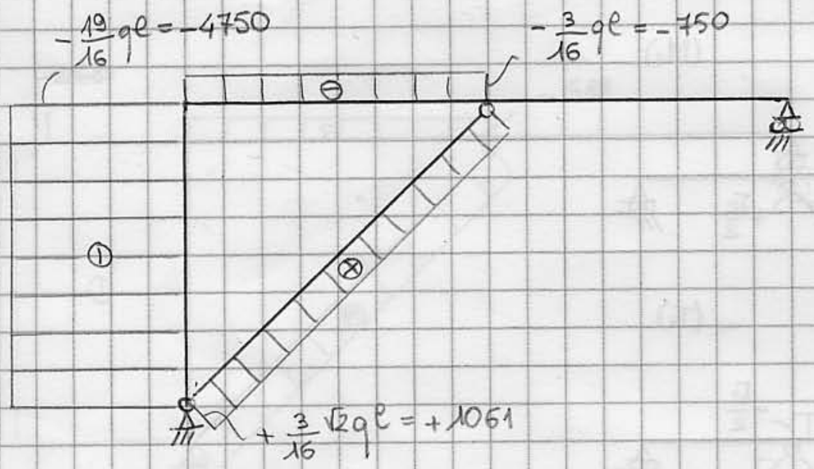
$$EI_1\, \eta_{11} = 2\cdot\frac{1}{3}l\left(-\frac{\sqrt{2}}{2}l\right)^2 = \frac{2}{3}\,\frac{1}{2}\,l^3 = \frac{l^3}{3}$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = q\frac{\sqrt{2}}{16}l^4\,\frac{3}{l^3} = \frac{3}{16}\sqrt{2}\,ql = 1061\ \text{Kg}$$

Diagrammi quotati:



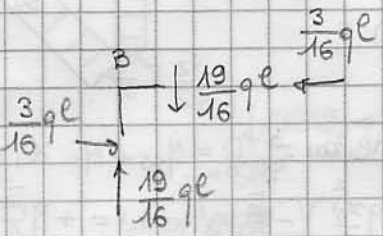




$M_B = -\frac{\sqrt{2}}{2}l\,\frac{3}{16}\sqrt{2}\,ql = -\frac{3}{16}ql^2 = -1500$ kgm

$T_A = -\frac{3}{16}ql = -750$ kg

$T_{B^+} = +ql + \frac{\sqrt{2}}{2}\,\frac{3}{16}\sqrt{2}\,ql = \frac{19}{16}ql = 4750$ kg

$T_{C^-} = +\frac{\sqrt{2}}{2}\,\frac{3}{16}\sqrt{2}\,ql = +750$ kg



## A2) Progetto (a flessione):

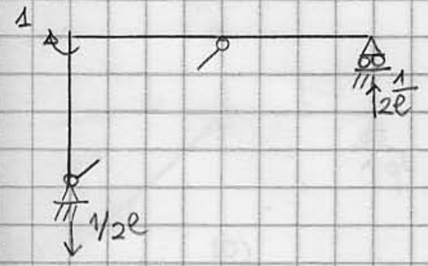
$$W_1 \geq \frac{4000 \cdot 100}{2400} = 167 \text{ cm}^3 \qquad \text{IPE } 200 \quad \begin{cases} A = 28{,}5 \text{ cm}^2 \\ I_1 = 1943 \text{ cm}^4 \end{cases}$$

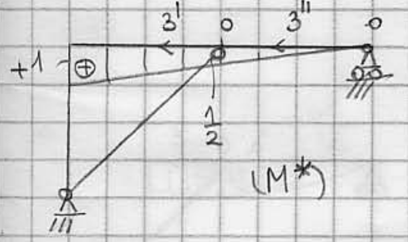
Trascurabilità delle deformazioni assiali:

$$\frac{\eta_{11}^N}{\eta_{11}^M} = \frac{\left(2 \cdot \frac{1}{2} + \sqrt{2}\right)\frac{\ell}{EA}}{\frac{\ell^3}{3EI_1}} = (1+\sqrt{2})\frac{3 I_1}{\ell^2 A} = 1{,}23\%$$

Le deformazioni assiali sono trascurabili.

## A3) Rotazione in B




$$1 \cdot \varphi_B = \int_S \frac{MM^*}{EI_1} = \frac{1}{EI_1}\int_0^\ell \left(\frac{x_3''}{2\ell}\right)\left(q\ell x_3'' - q\frac{x_3''^2}{2}\right) dx_3''$$

$$+ \frac{1}{EI_1}\int_0^\ell \left(\frac{1}{2} + \frac{x_3'}{2\ell}\right)\left(q\frac{\ell^2}{2} - \frac{3}{16}q\ell x_3' - q\frac{x_3'^2}{2}\right) dx_3'$$

$$= \frac{5}{48}\frac{q\ell^3}{EI_1} + \frac{29}{192}\frac{q\ell^3}{EI_1}$$

$$= \frac{49}{192}\frac{q\ell^3}{EI_1} = \frac{49}{192} \cdot \frac{20 \cdot 200^3}{2{,}1 \cdot 10^6 \cdot 1943} = 0{,}01 = 0{,}57^\circ$$

## A4) Carico termico

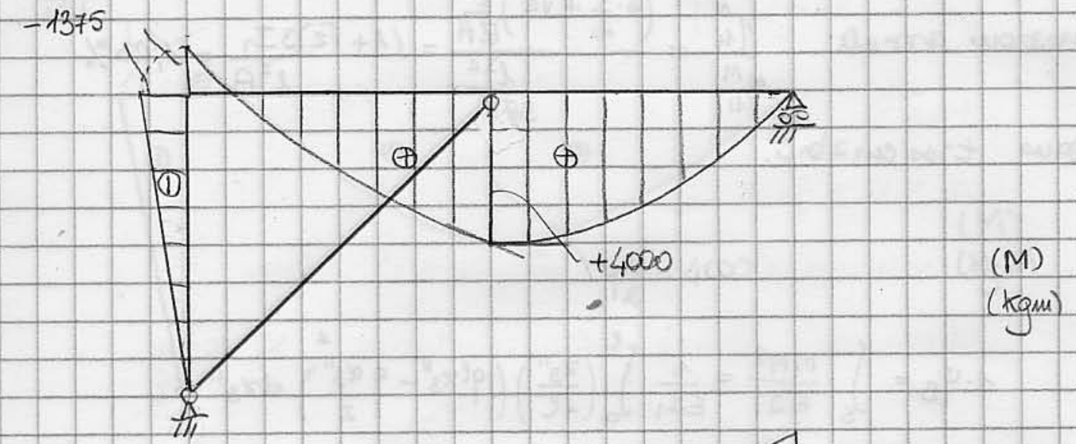
$$\eta_{1t} + \eta_{10} + \eta_{11} X_1 = 0$$

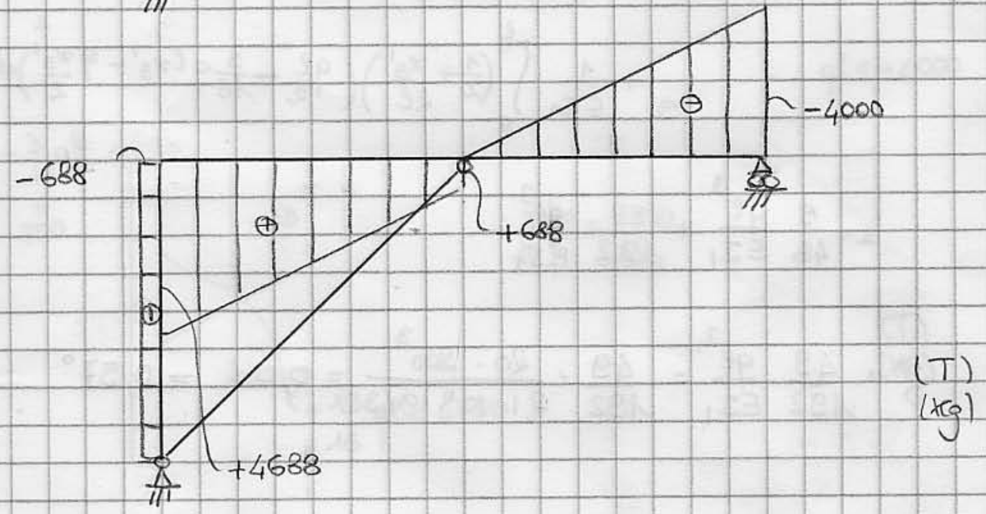
$$\eta_{1t} = \int_{AC} N_1 \varepsilon_t = \ell\sqrt{2} \cdot 1 \cdot \varepsilon_t = \ell\sqrt{2}\,\alpha\Delta T$$

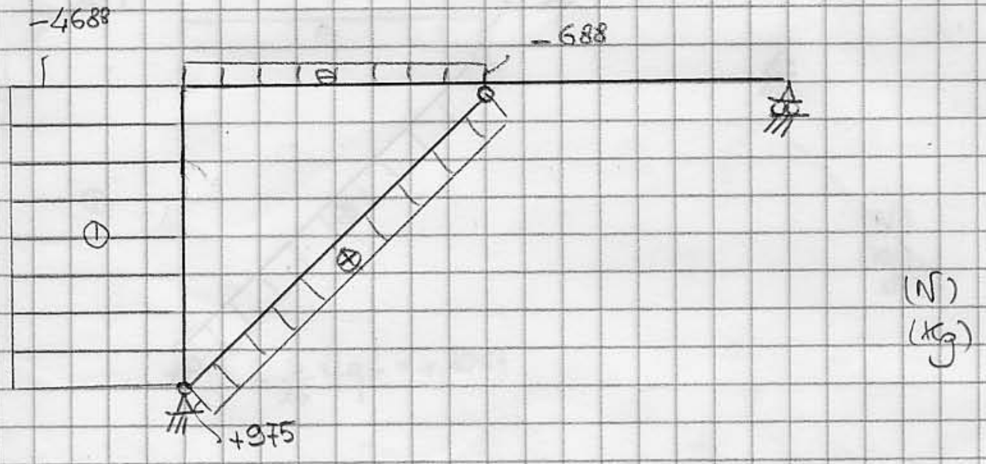
$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1t}}{\eta_{11}} = \frac{3}{16}\sqrt{2}\,q\ell - \frac{\ell\sqrt{2}\,\alpha\Delta T\, 3EI_1}{\ell^2} = 1061 - \frac{\sqrt{2}\cdot 10^{-5}\cdot 20 \cdot 3 \cdot 2{,}1 \cdot 10^6 \cdot 1943}{4 \cdot 10^4}$$

$$= 1061 - 86 = 975 \text{ Kg}$$

Diagrammi quotati complessivi di q e di $\Delta T$:




$M_B = X_1\left(-\frac{\sqrt{2}}{2}l\right) = -1,41 \cdot 975 = -1375$ kgm

$T_B^+ = ql + X_1\frac{\sqrt{2}}{2} = 4000 + 0,7 \cdot 975 = 4688$ kg

$T_C^- = X_1\frac{\sqrt{2}}{2} = 688$ kg