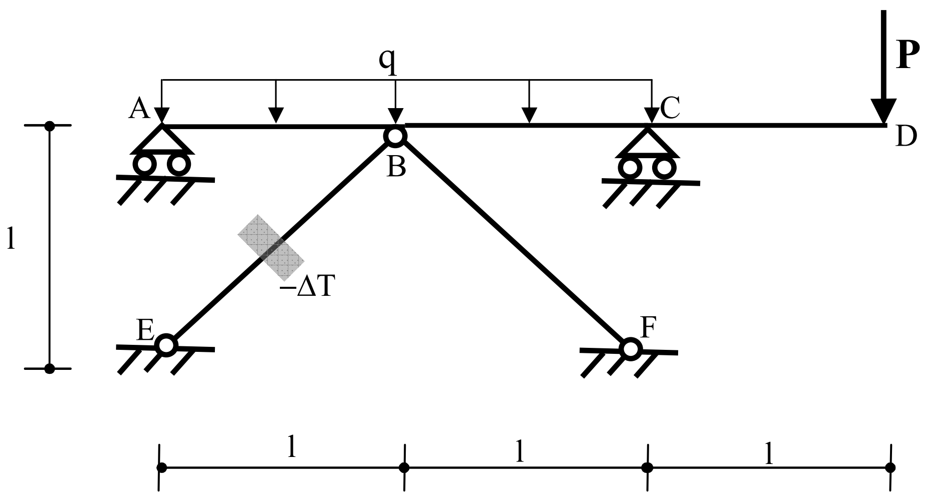
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 1/12/2008

D



$l = 1$ m, $q = 1.5$ t/m, $P = 3$ t,
$E = 2.1\ 10^6$ kg/cm^2, $\alpha = 10^{-5}$ °C^{-1}, $\Delta T = 20$ °C

La travatura iperstatica di figura è realizzata con profilati IPE 180 (H= 180 mm, A = 24 cm^2, I_1 = 1317 cm^4).

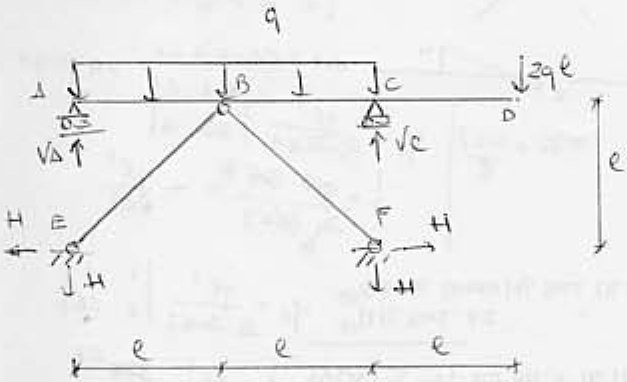
1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). Valutare l'effetto delle deformazioni assiali.
2. Calcolare lo spostamento verticale in D.
3. Risolvere nuovamente la travatura considerando anche il carico termico nel tratto BE e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q,P che di ΔT.

D1)

$$(\uparrow)\quad V_A + V_C - 2H = 4q\ell$$

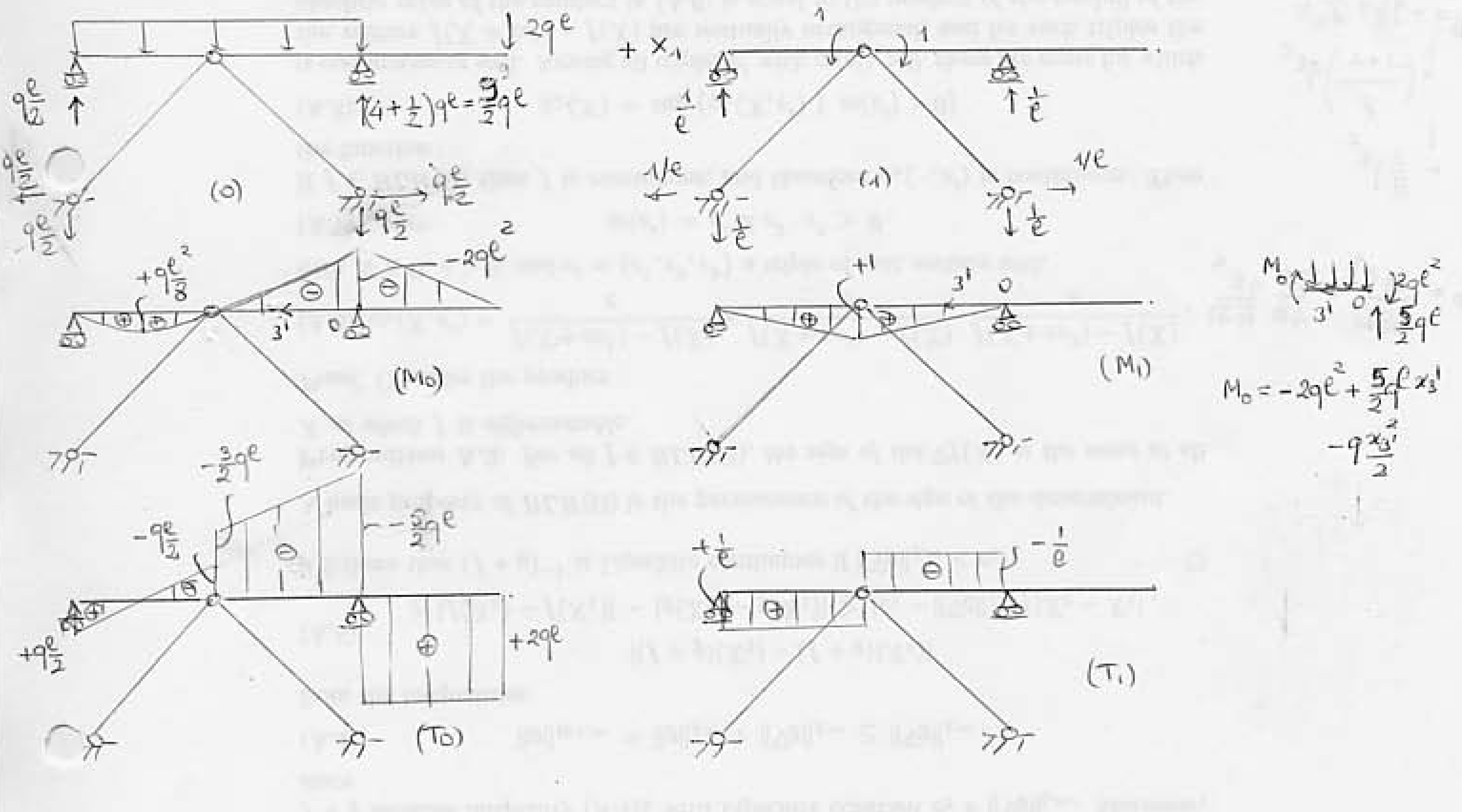
$$(B\circlearrowleft)\quad V_C\,\ell - V_A\,\ell = 2q\ell^2$$

$\ell = 1\,m$
$q = 1.5\,t/m$
$H = 18\,cm$
$A = 24\,cm^2$
$I_1 = 1317\,cm^4$



Struttura una volta iperstatica

Incognita iperstatica $X_1 = M_B$.



$$EI_1\eta_{10} = \frac{q\ell^3}{24} + \int_0^{\ell}\left(\frac{x_3'}{\ell}\right)\left(-2q\ell^2 + \frac{5}{2}q\ell x_3' - q\frac{x_3'^2}{2}\right)dx_3' = \frac{q\ell^3}{24} + \int_0^{\ell}\left(-2q\ell x_3' + \frac{5}{2}q x_3'^2 - q\frac{x_3'^3}{2\ell}\right)dx_3'$$

$$= \frac{q\ell^3}{24} + \left[-q\ell^3 + \frac{5q\ell^3}{6} - \frac{q\ell^3}{8}\right] = \frac{(1-24+20-3)}{24}q\ell^3 = -\frac{6}{24}q\ell^3 = -\frac{1}{4}q\ell^3$$

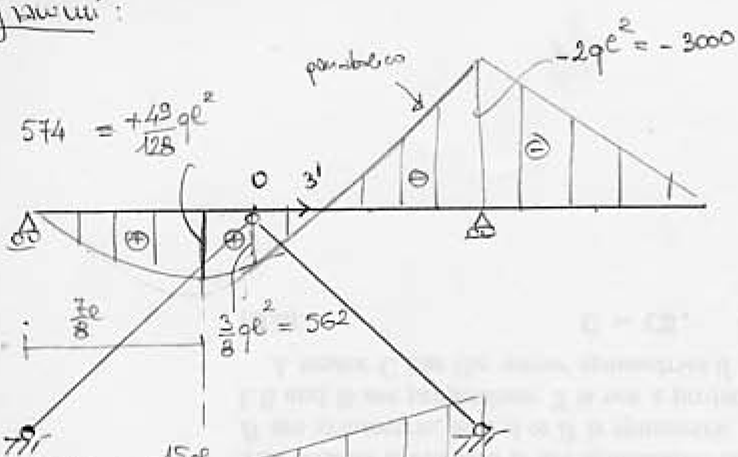
$$EI_1\eta_{11}^M = \frac{2}{3}\ell$$

$$EA\eta_{11}^N = 2\ell\sqrt{2}\,\frac{2}{\ell^2} \qquad \frac{\eta_{11}^N}{\eta_{11}^M} = 4{,}6\%$$

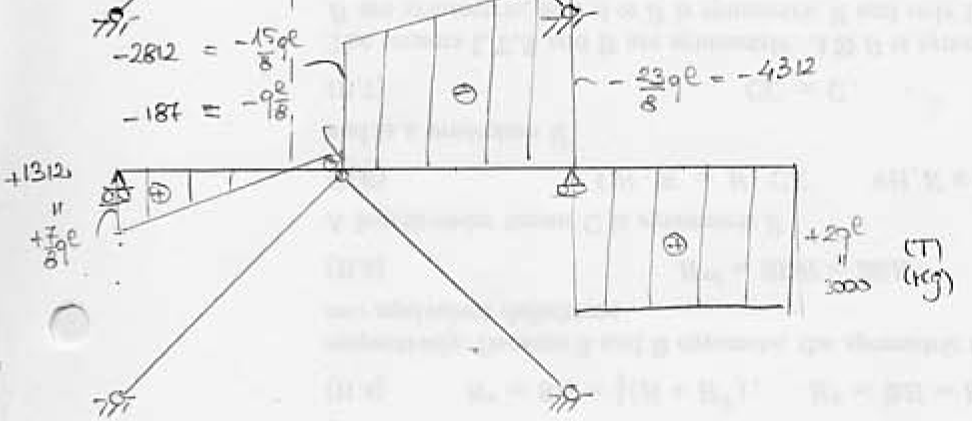
$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{1}{4}q\ell^3\,\frac{3}{2\ell} = \frac{3}{8}q\ell^2$$

(deformazioni assiali trascurabili)

Diagrammi:



$\frac{7}{8} q\ell$, $\frac{7\ell}{8}$: $\bar{M} = \frac{1}{2} \frac{49}{64} q\ell^2 = \frac{49}{128} q\ell^2 = 574$ kgm



$T_A = q\frac{\ell}{2} + \frac{3}{8} q\ell = \frac{7}{8} q\ell$

$T_{B^-} = -q\frac{\ell}{2} + \frac{3}{8} q\ell = -q\frac{\ell}{8}$

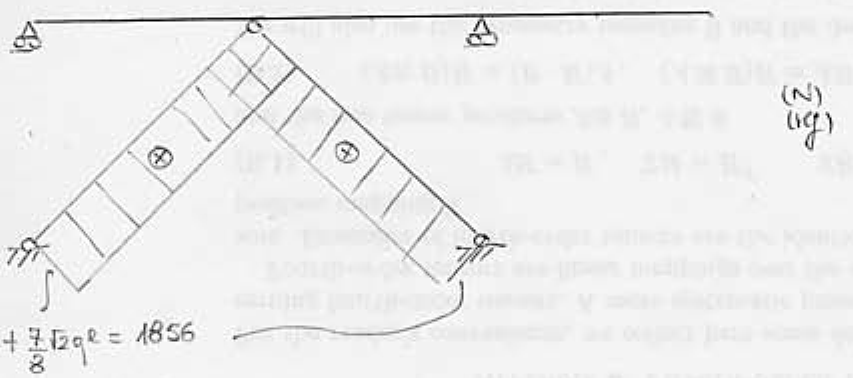
$T_{B^+} = -\frac{3}{2} q\ell - \frac{3}{8} q\ell = -\frac{15}{8} q\ell$

$T_{C^-} = -\frac{5}{2} q\ell - \frac{3}{8} q\ell = -\frac{23}{8} q\ell$

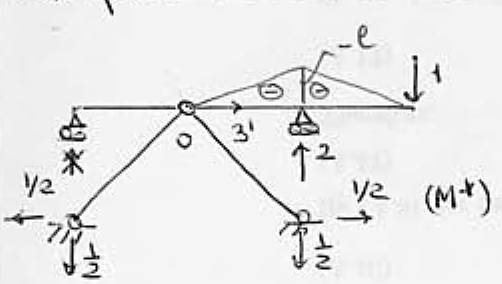
$\frac{q\ell}{8}$, $\frac{15}{8} q\ell$, B, N, N

$2N\frac{\sqrt{2}}{2} = \frac{14}{8} q\ell = \frac{7}{4} q\ell$

$\hookrightarrow N = \frac{7}{8}\sqrt{2} q\ell = 1856$ kg



D2) Spostamento verticale in D.



$$1 \cdot \delta_D = \frac{1}{EI_1}\left[\frac{1}{3}\ell(-\ell)(-2q\ell^2) + \int_0^\ell (-x_3')\left(\frac{3}{8}q\ell^2 - \frac{15}{8}q\ell x_3' - q\frac{x_3'^2}{2}\right)dx_3'\right]$$

$$= \frac{1}{EI_1}\left[\frac{2}{3}q\ell^4 + \frac{q\ell^4}{8} + \frac{15}{8}\frac{1}{3}q\ell^4 - \frac{3}{16}q\ell^4\right]$$

$$= \frac{59}{48}\frac{q\ell^4}{EI_1} = \frac{59 \cdot 15 \cdot 100^4}{48 \cdot 2{,}1 \cdot 10^6 \cdot 1317} = 0{,}67 \text{ cm}$$

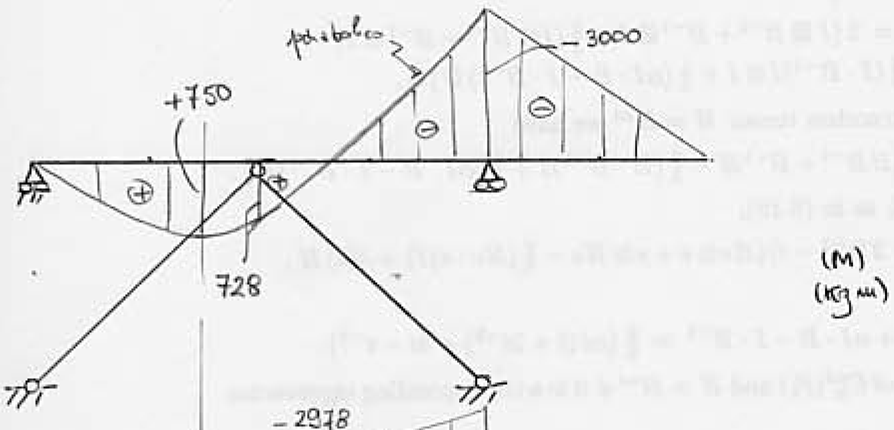
D3) Carico termico

$$\eta_{1T} = \int_{BE} N_1 \varepsilon_t = \varepsilon_t \int_{BE} N_1 = \varepsilon_t \cancel{\ell}\sqrt{2}\,\frac{\sqrt{2}}{\cancel{\ell}} = 2\varepsilon_t = -2\alpha\Delta T$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1T}}{\eta_{11}} = \frac{3}{8}q\ell^2 + \frac{\cancel{2}\alpha\Delta T}{\cancel{2}\ell}\,3EI_1 = 562 + \frac{\cancel{10^{5}}\cdot 2\cancel{0}\cdot 3\cdot 2{,}1\cdot 10^{6}\cdot 1317}{\cancel{100}}\,\frac{1}{100}$$

$$= 562 + 166 = 728 \text{ kgm}$$

Diagrammi complessivi sia di P, q che di ΔT:



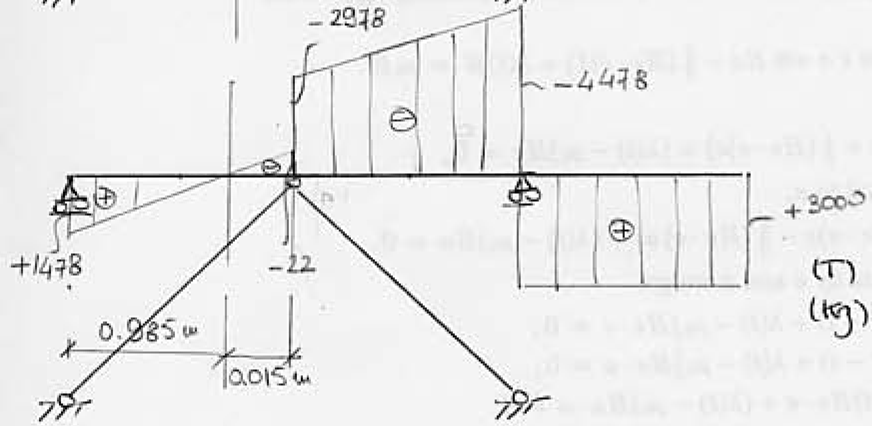
Calcoli

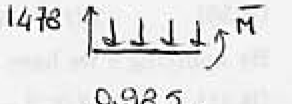
$T_A = 750 + 728 = 1478$ kg

$T_B^- = -750 + 728 = -22$ kg

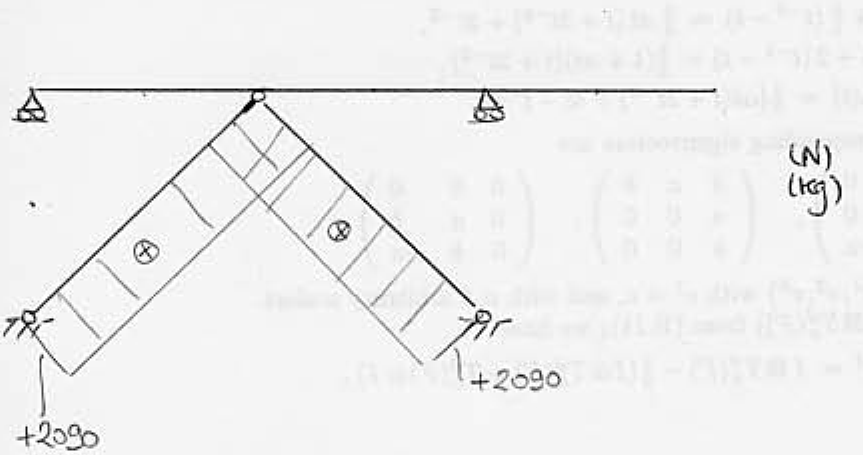
$T_B^+ = -2250 - 728 = -2978$ kg

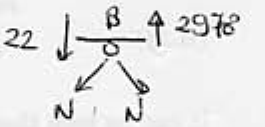
$T_C = -3750 - 728 = -4478$ kg





$$\bar{M} = 1478 - 1500\cdot\frac{0{,}985^2}{2} = 750 \text{ kgm}$$





$$2N\frac{\sqrt{2}}{2} = 2978 - 22 = 2956$$

$$N = \frac{2956}{2}\sqrt{2} = 2090 \text{ kg}$$