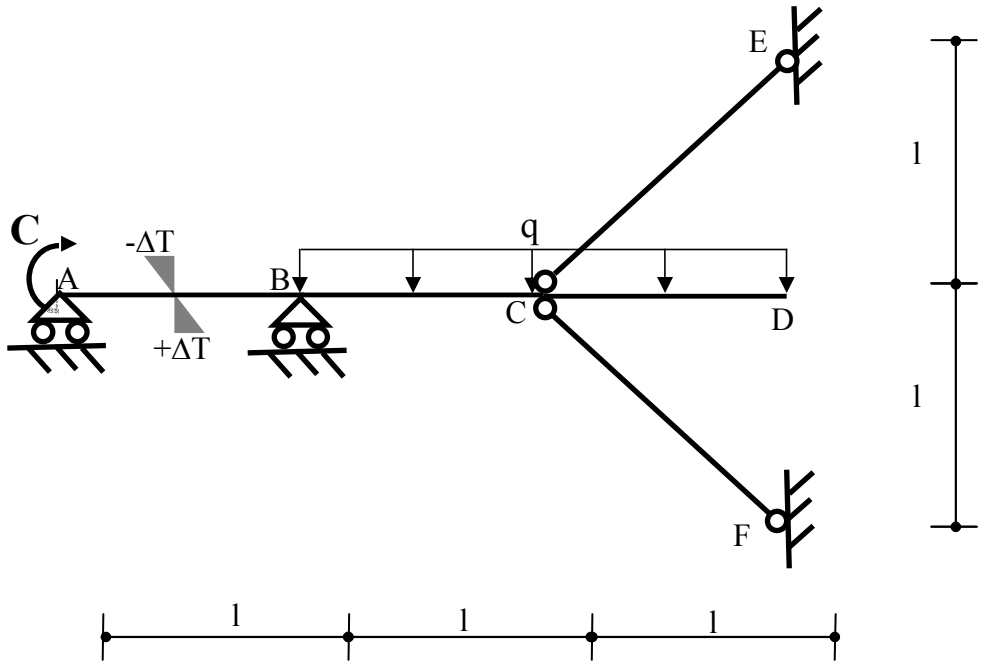
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 1/12/2008

C

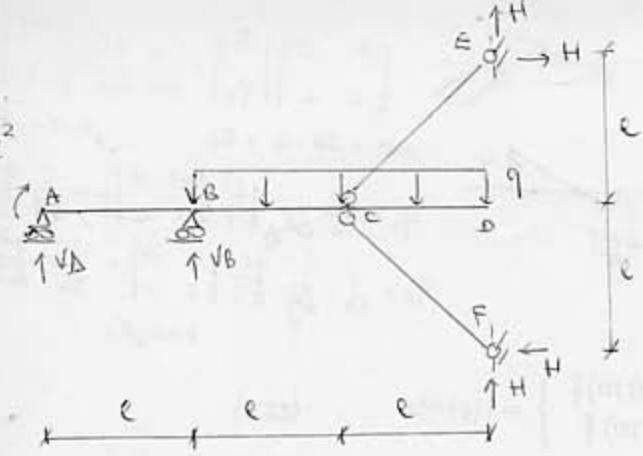


l = 1 m, q = 2 t/m, C = 2 tm,
E = 2.1 10^6 kg/cm^2, α= 10^{-5} °C^{-1}, ΔT = 20 °C

La travatura iperstatica di figura è realizzata con profilati IPE 160 (H= 160 mm, A = 20 cm^2, I_1 = 869 cm^4).

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). Valutare l'effetto delle deformazioni assiali.
2. Calcolare la rotazione del nodo B.
3. Risolvere nuovamente la travatura considerando anche il carico termico nel tratto AB e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q,C che di ΔT.

C1)



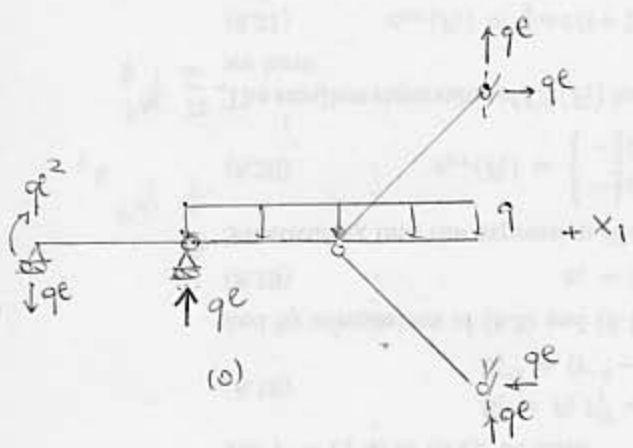
$(\uparrow)\quad V_A + V_B + 2H = 2q\ell$

$(C\uparrow)\quad V_B\,2\ell + V_B\,\ell = -q\ell^2$

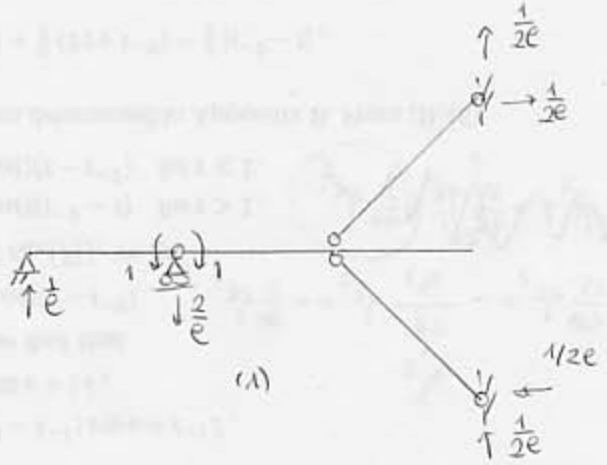
$q = 2\ t/m$
$\ell = 1m$
$H = 16\ cm$
$A = 20\ cm^2$
$I_1 = 869\ cm^4$

Struttura una volta iperstatica

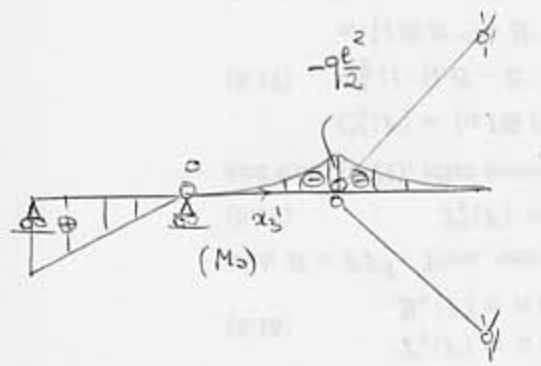
Incognita iperstatica: $X_1 = M_B$.



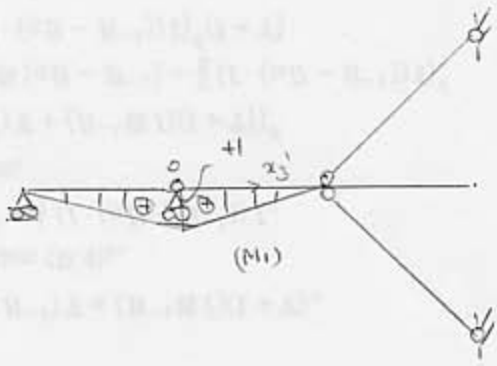
(0)



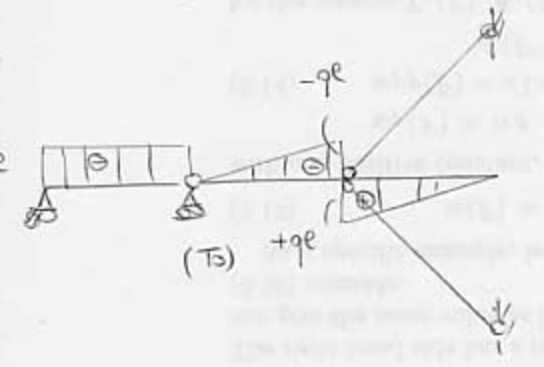
(1)



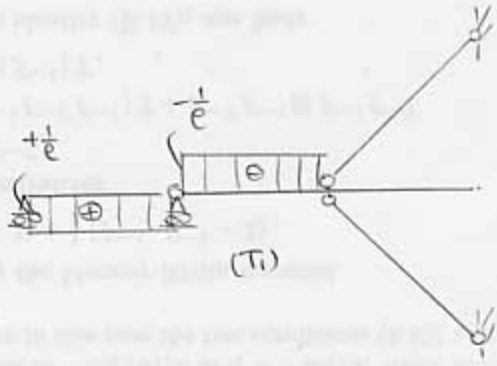
(M_0)



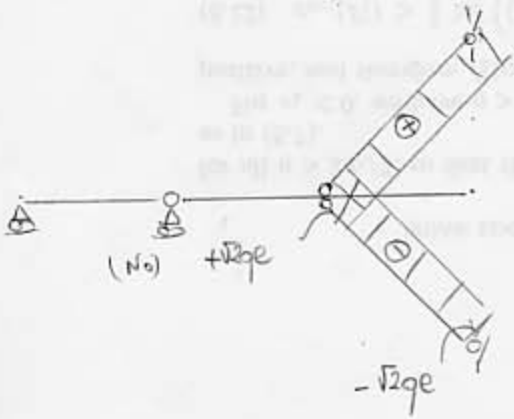
(M_1)



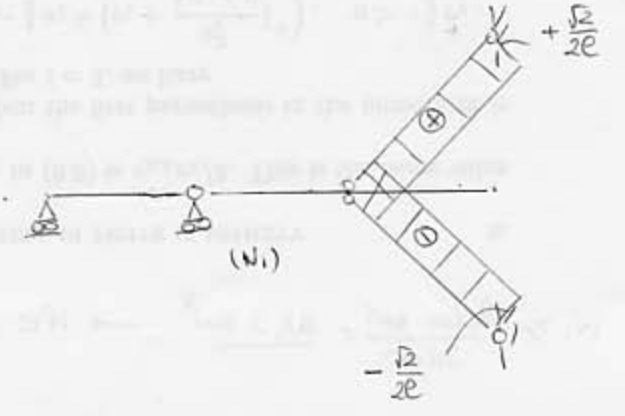
(T_0)



(T_1)



(N_0)



(N_1)

$$EI_1\,\eta_{10} = \frac{1}{6}\ell\, q\ell^2 + \int_0^\ell \left(-q\frac{x_3'^2}{2}\right)\left(1-\frac{x_3'}{\ell}\right)dx_3' = q\frac{\ell^3}{6} - \frac{q}{2}\int_0^\ell \left(x_3'^2 - \frac{x_3'^3}{\ell}\right)dx_3'$$

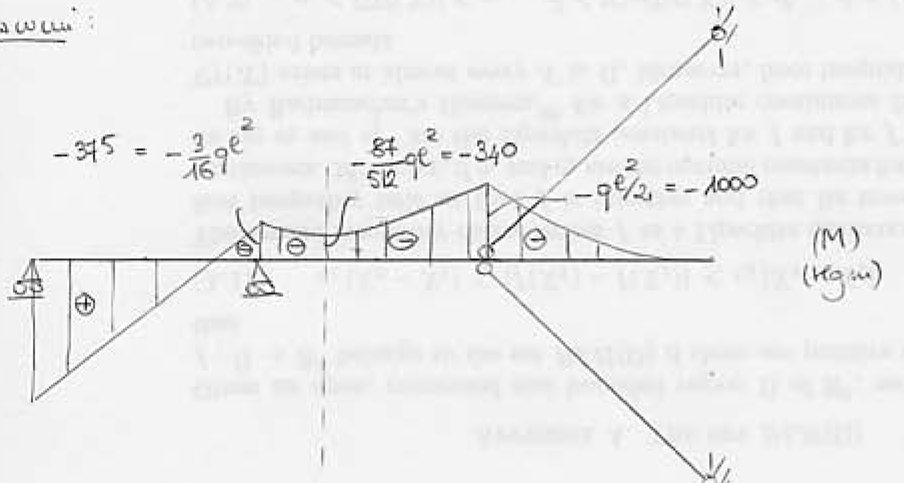
$$= q\frac{\ell^3}{6} - \frac{q}{2}\left[\frac{\ell^3}{3} - \frac{\ell^3}{4}\right] = q\ell^3\left(1/6 - \frac{1}{24}\right) = \frac{(4-1)}{24}q\ell^3 = \frac{3}{24}q\ell^3 = q\ell^3/8$$

$$EI_1\,\eta_{11}^M = \frac{2}{3}\ell \,, \quad EA\,\eta_{11}^N = 2\ell\sqrt{2}\,\frac{1}{2\ell^2} \qquad \frac{\eta_{11}^N}{\eta_{11}^M} = \frac{\sqrt{2}}{4A\ell}\,\frac{3EI_1}{2\ell} = 1\text{‰}$$

deformazioni assiali trascurabili

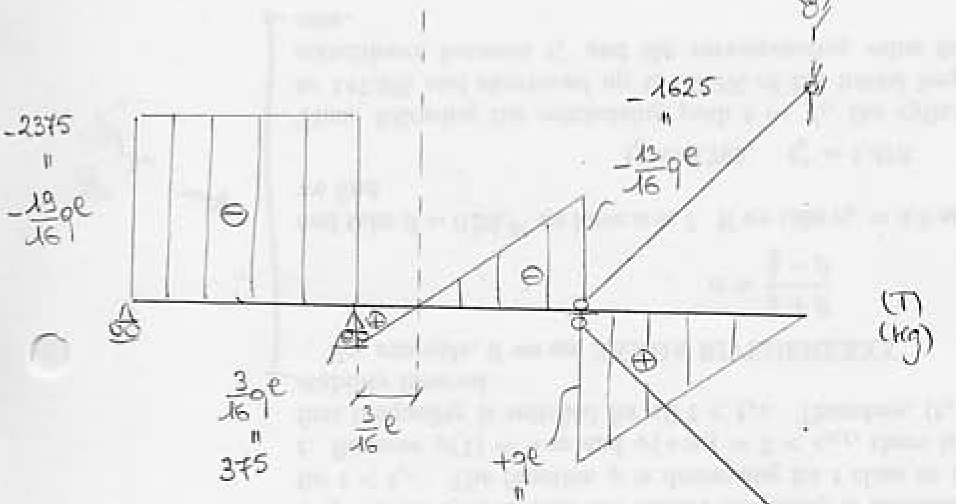
$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = -\frac{q\ell^3}{8}\cdot\frac{3}{2\ell} = -\frac{3}{16}q\ell^2$$

Diagrammi:

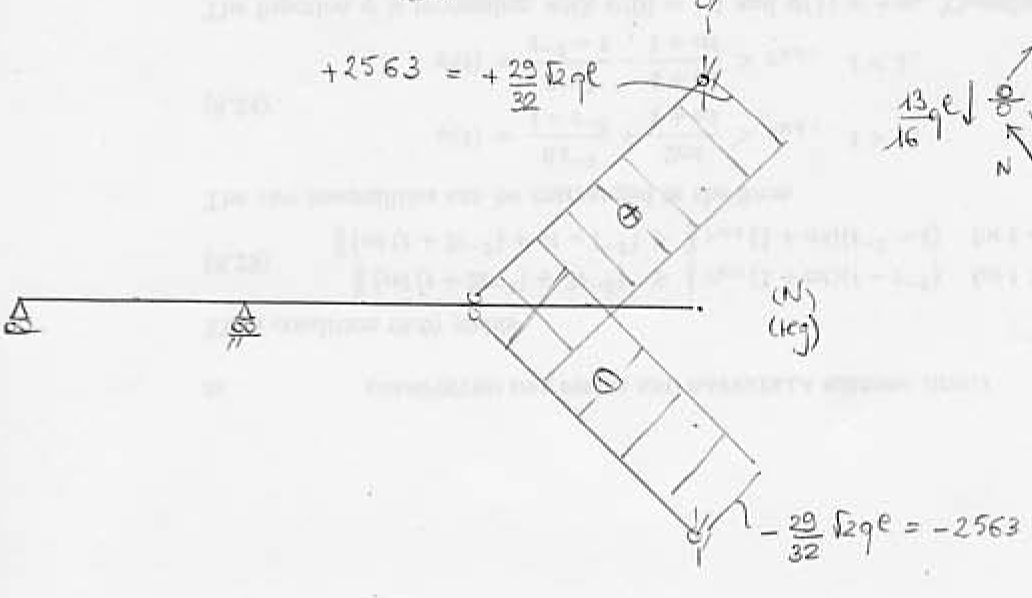


$$T_A = -q\ell - \frac{3}{16}q\ell = -\frac{19}{16}q\ell$$

$$T_C^- = -q\ell + \frac{3}{16}q\ell = -\frac{13}{16}q\ell$$



$$\overline{M} = -\frac{3}{16}q\ell^2 + \frac{1}{2}\frac{9}{256}q\ell^2 = \frac{(-96+9)}{512}q\ell^2 = -\frac{87}{512}q\ell^2 = -340\text{ kgm}$$

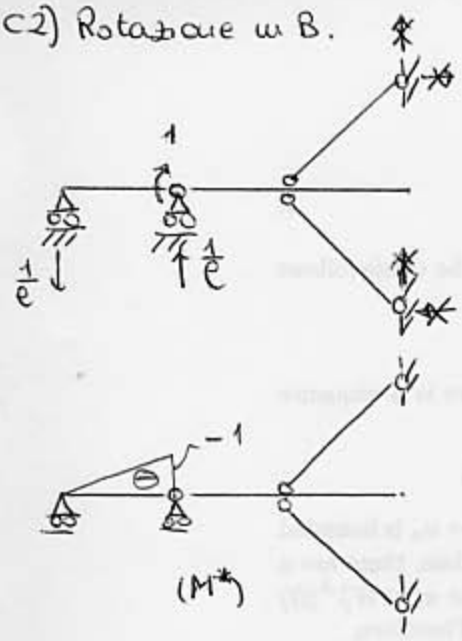


$$\frac{13}{16}q\ell \downarrow \ \frac{q}{8}\downarrow q\ell \quad N$$

$$2N\frac{\sqrt{2}}{2} = \frac{29}{16}q\ell$$

$$N = \frac{29}{32}\sqrt{2}q\ell = 2563\text{ kg}$$

c2) Rotazione in B.



$$1 \cdot \varphi_B = \frac{1}{EI_1} \int_0^\ell \left(-\frac{x_3}{\ell}\right)\left(q\ell^2 - \frac{19}{16} q\ell x_3\right) dx_3$$

$$= \frac{1}{EI_1} \int_0^\ell \left(\frac{19}{16} q x_3^2 - q\ell x_3\right) dx_3$$

$$= \frac{1}{EI_1}\left[\frac{19}{16} q\frac{\ell^3}{3} - q\frac{\ell^3}{2}\right] = -\frac{5}{48EI_1} q\ell^3$$

$$= 0{,}05°$$

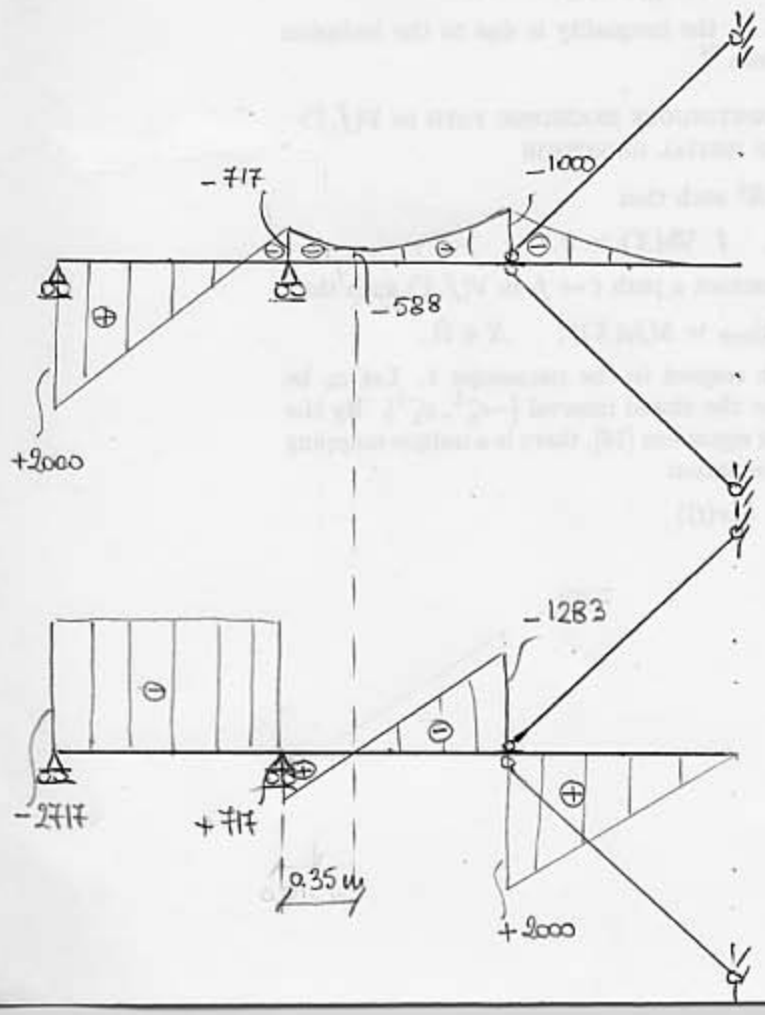
c3) Carico termico

$$\eta_{1T} = \int_{AB} M_1 \chi_t \, dx_3 = \chi_t \int_{AB} M_1 \, dx_3 = \chi_t \cdot \frac{\ell}{2} = +\frac{\alpha \Delta T}{H} \frac{\ell}{2}$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1T}}{\eta_{11}} = -\frac{3}{16} q\ell^2 - \frac{3\alpha\Delta T EI_1 \ell}{2H\ell} = -375 - \frac{3 \cdot 10^{-5} \cdot 40 \cdot 2{,}1 \cdot 10^6 \cdot 869}{2 \cdot 16} \frac{1}{100}$$

$$= -375 - 342 = -717 \text{ kg m}$$

Diagramma comprensivo di q, e di ΔT:

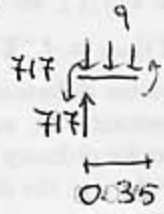


Calcol.

$$T_A = -2000 - 717 = -2717 \text{ kg}$$

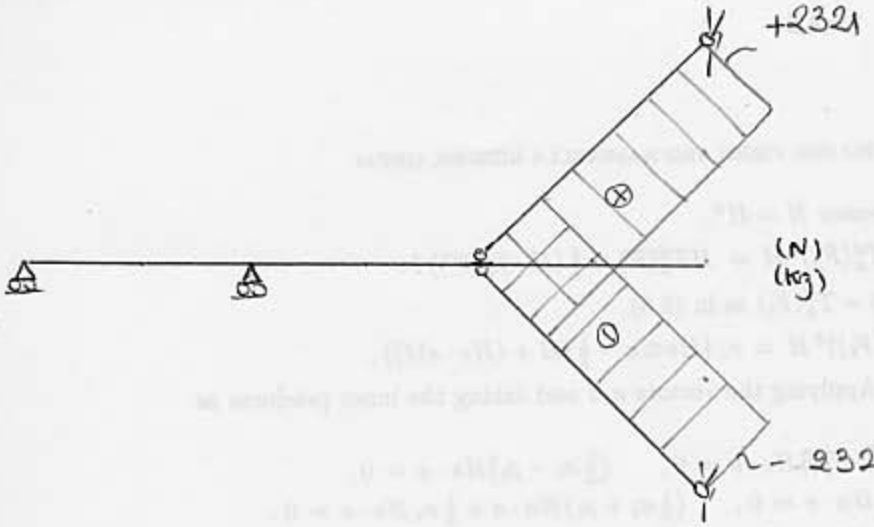
$$T_B^+ = +717 \text{ kg}$$

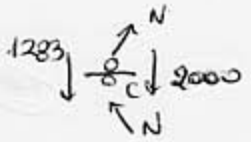
$$T_C^- = -2000 + 717 = -1283 \text{ kg}$$



$$\bar{M} = -717 + 717 \cdot 0{,}35 - \frac{2000}{2} \cdot 0{,}35^2$$

$$= -588 \text{ kg m}$$





$$2N\frac{\sqrt{2}}{2} = 2000 + 1283$$

$$\hookrightarrow N = \frac{3283\sqrt{2}}{2} = 2321 \text{ kg}$$