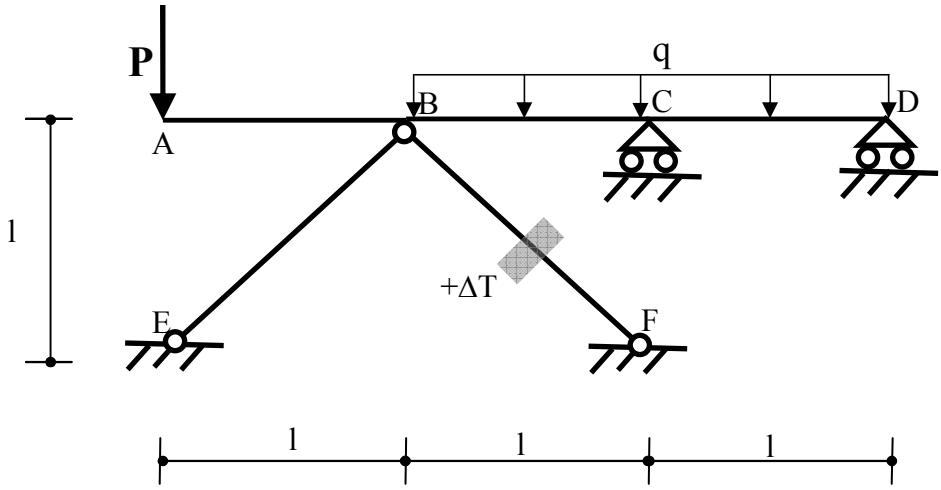
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 1/12/2008

B

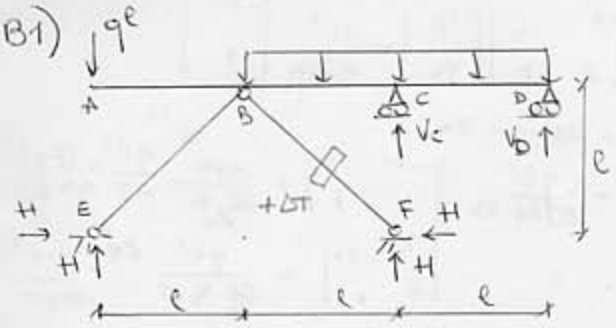


l = 1 m, q = 1.5 t/m, P = 1.5 t,
E = 2.1 10^6 kg/cm^2, α= 10^{-5} °C^{-1}, ΔT = 20 °C

La travatura iperstatica di figura è realizzata con profilati IPE 140 (H= 140 mm, A = 16.4 cm^2, I_1 = 541 cm^4).

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). Valutare l'effetto delle deformazioni assiali.
2. Calcolare lo spostamento verticale in A.
3. Risolvere nuovamente la travatura considerando anche il carico termico nel tratto BF e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q,P che di ΔT.

B1)



$$\begin{cases} 2H + V_C + V_D = 2q\ell \\ (B\uparrow)\quad V_C\ell + V_D 2\ell = 2q\ell^2 - q\ell^2 = q\ell^2 \end{cases}$$

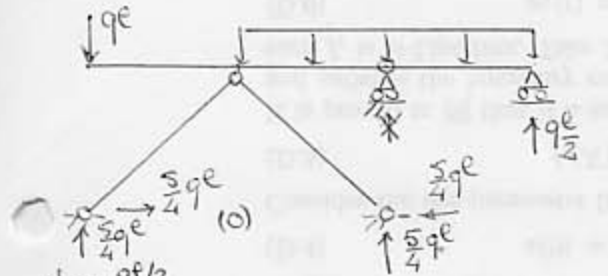
Struttura una volta iperstatica

Incognita iperstatica: $X_1 = M_C$.

$\ell = 1\,m$

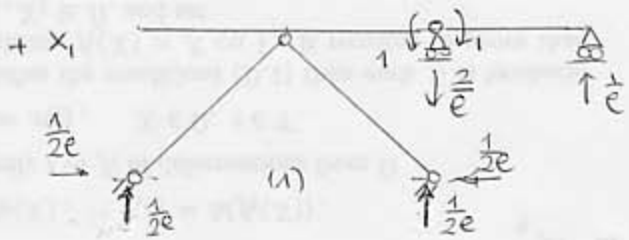
$q = 1500\ kg/m$

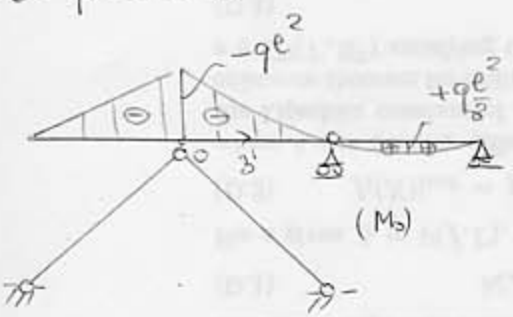
$H = 14\,cm$, $A = 16{,}4\,cm^2$, $I_1 = 541\,cm^4$

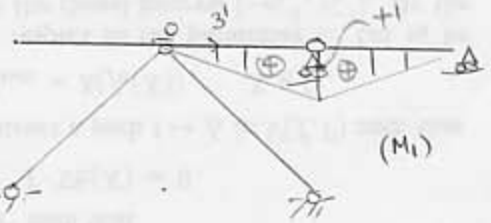


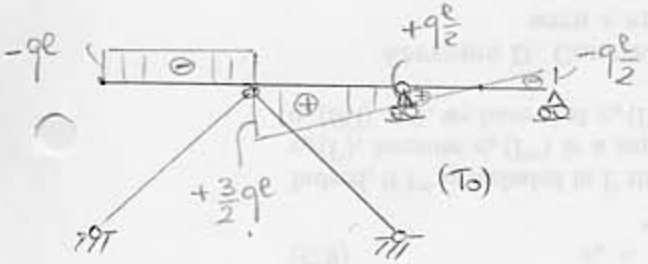
$V_D = q\ell/2$

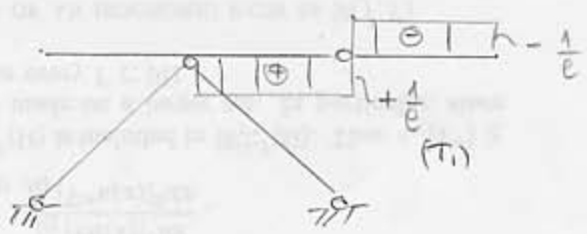
$\curvearrowleft V_C = q\ell - 2V_D = 0$

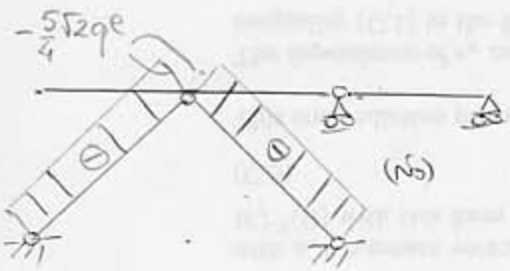


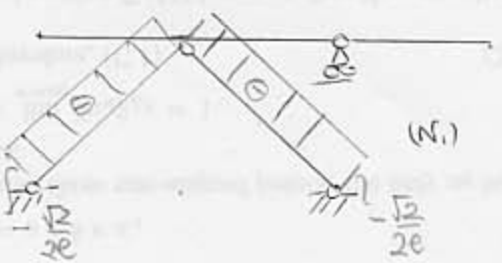


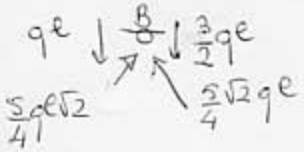












$$EI_1 y_{10} = \frac{qe^3}{24} + \int_0^e \left(\frac{x_3'}{e}\right)\left(-qe^2 + \frac{3}{2}qe\,x_3' - q\frac{x_3'^2}{2}\right)dx_3'$$

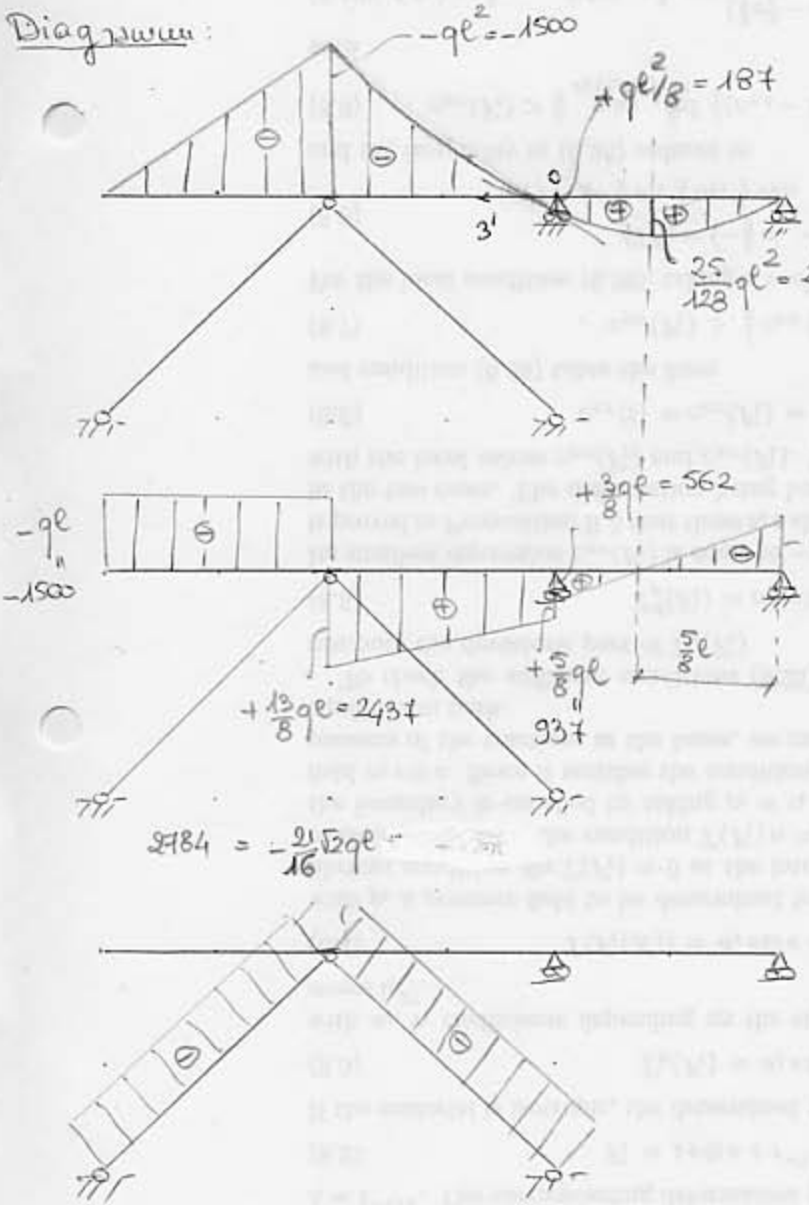
$$= \frac{qe^3}{24} + \int_0^e \left(-qe\,x_3' + \frac{3}{2}q\,x_3'^2 - q\frac{x_3'^3}{2e}\right)dx_3' = \frac{qe^3}{24} + \left[-q\frac{e^2}{2} + \frac{q}{2}e^3 - \frac{q\cdot^3 e^4}{8\cdot 3}\right]$$

$$= -\frac{2}{24}qe^3 = -\frac{1}{12}qe^3$$

$EI_1 y_{11}^M = \frac{2}{3}e$, $EA\, y_{11}^N = 2\cdot e\sqrt{2}\left(\frac{\sqrt{2}}{2e}\right)^2 = \frac{2\sqrt{2}}{2e^2}$ $\quad \frac{y_{11}^N}{y_{11}^M} = \frac{\sqrt{2}}{2Ae} \cdot \frac{3EI_1}{2e} = 1{,}4\,\%$. Deformation axial vernachlässigbar

$$X_1 = -\frac{y_{10}}{y_{11}} = \frac{qe^3}{12}\cdot\frac{2}{2e} = \frac{qe^2}{8}$$

Diagramm:

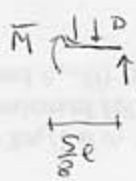


$T_{B+} = -\frac{13}{4\cdot 2}qe + q\frac{e}{8} = \frac{13}{8}qe$

$T_{C-} = q\frac{e}{2\cdot 4} + q\frac{e}{8} = \frac{5}{8}qe$

$T_{C+} = q\frac{e}{2} - q\frac{e}{8} = \frac{3}{8}qe$

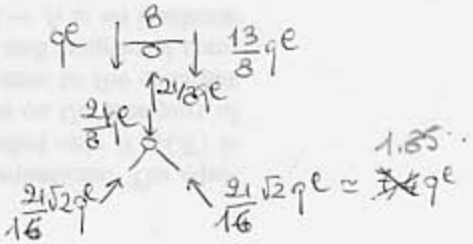
$T_D = -q\frac{e}{2} - q\frac{e}{8} = -\frac{5}{8}qe$

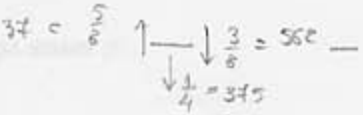


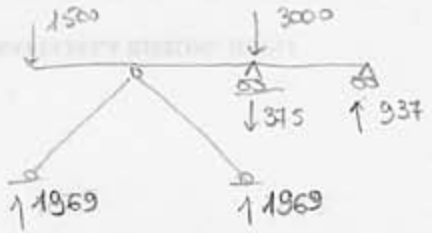
$\bar{M} = \frac{1}{2}\frac{25}{64}qe^2$

$= \frac{25}{128}qe^2$

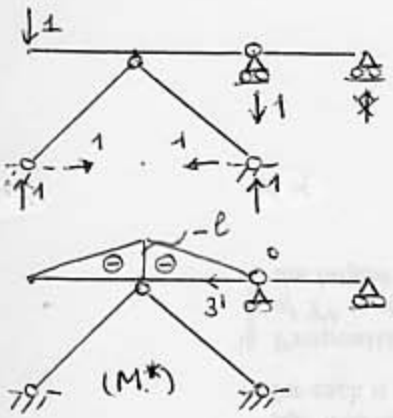
$\approx 0{,}2\,qe^2$







B2) Spostamento verticale in A.



$$1 \cdot \delta_A = \frac{1}{EI_1} \int_{AC} M M^* \, dx_3$$

$$= \frac{1}{EI_1} \left[\frac{1}{3} \ell (-\ell)(-q\ell^2) + \int_0^\ell (-x_3')\left(q\frac{\ell^2}{8} - \frac{5}{8} q \ell x_3' - q\frac{x_3'^2}{2}\right) dx_3' \right]$$

$$= \frac{1}{EI_1} \left[q\frac{\ell^4}{3} + \int_0^\ell \left(q\frac{x_3'^3}{2} + \frac{5}{8} q \ell x_3'^2 - q\frac{\ell^2}{8} x_3' \right) dx_3' \right]$$

$$= \frac{1}{EI_1} \left[q\frac{\ell^4}{3} + q\frac{\ell^4}{8} + \frac{5}{24} q\ell^4 - \frac{q\ell^4}{16} \right] = \frac{29}{48} \frac{q\ell^4}{EI_1}$$

$$= \frac{29 \cdot 15 \cdot 100^4}{48 \cdot 2{,}1 \cdot 10^6 \cdot 541}$$

$$= 0{,}8 \text{ cm}$$

B3) Carico termico

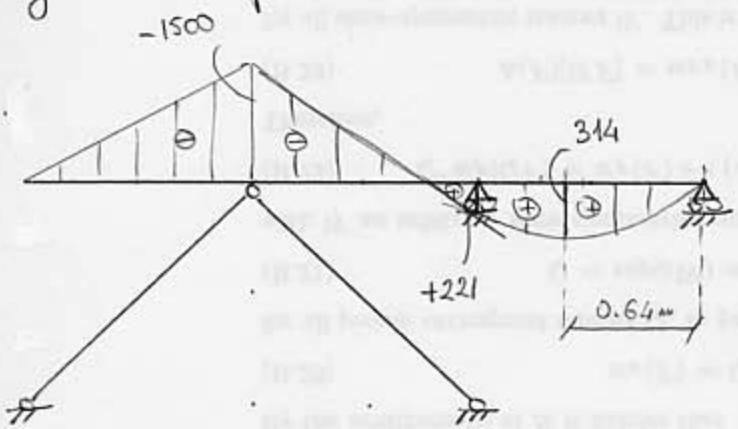
$$N_{1T} = \int_{BF} N_1 \varepsilon_t = \cancel{EA}\left(-\frac{\cancel{\delta x}}{\cancel{\delta x}}\right) \alpha \Delta T = -\alpha \Delta T$$

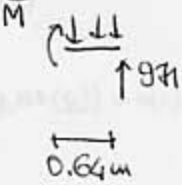
$$N_{1T} + N_{10} + N_{11} X_1 = 0$$

$$\hookrightarrow X_1 = -\frac{N_{10}}{N_{11}} - \frac{N_{1T}}{N_{11}} = +\frac{q\ell^2}{8} + \frac{\alpha \Delta T \, 3EI_1}{2\ell} = 187 + \frac{\cancel{10^6} \cdot \cancel{20} \cdot 3 \cdot 2{,}1 \cdot \cancel{10^6} \cdot 541}{\cancel{20}} \frac{1}{100}$$

$$= (187 + 34) \text{ kgm} = 221 \text{ kgm}$$

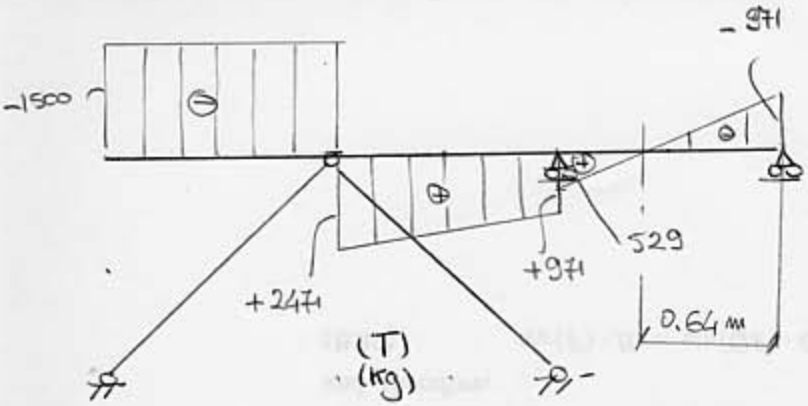
Diagrammi compresa sia di q, P che di ΔT:





$$\bar{M} = 971 \cdot 0{,}64 - 1500 \cdot \frac{0{,}64^2}{2}$$

$$= 314 \text{ kgm}$$

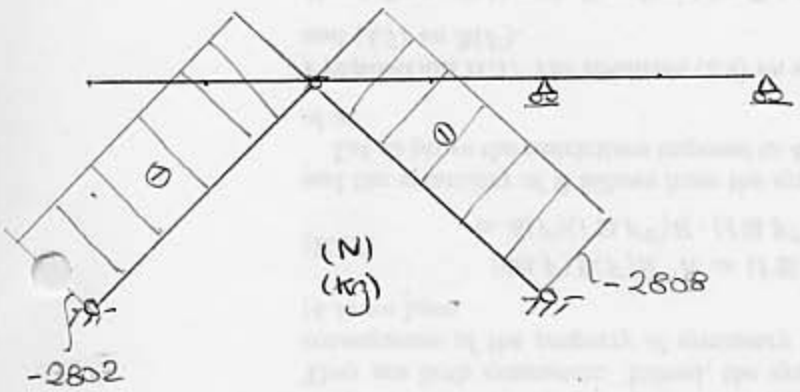


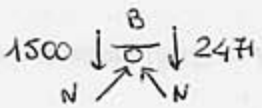
$T_{B+} = 2250 + 221 = 2471$ kg

$T_{C-} = 750 + 221 = 971$ kg

$T_{C+} = 750 - 221 = 529$ kg

$T_D = -750 - 221 = -971$ kg





$2N\frac{\sqrt{2}}{2} = 1500 + 2471 = 3971$

$N = \frac{3971}{2}\sqrt{2} = 2808$ kg (compressione)