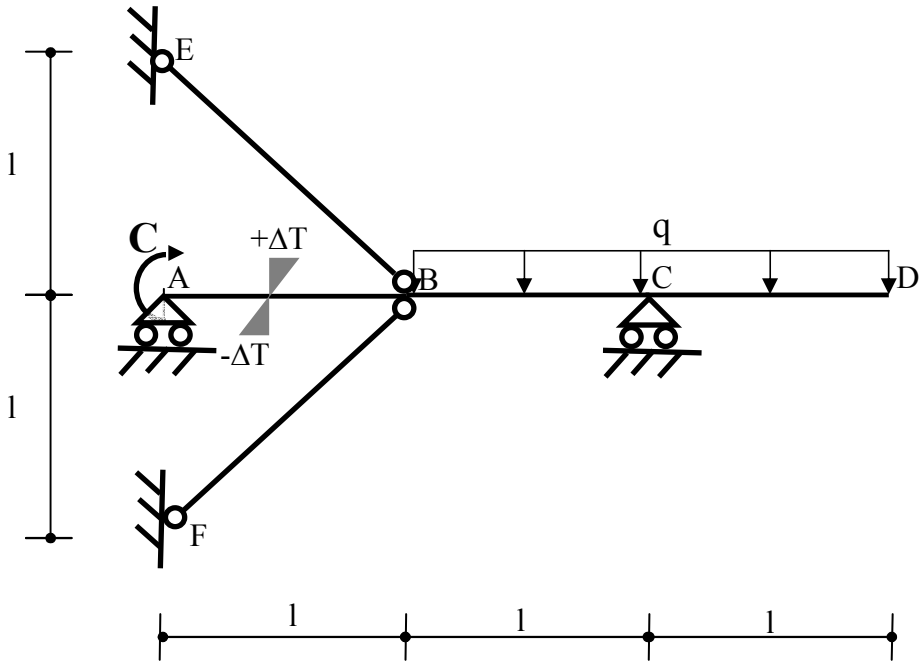
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 1/12/2008

A

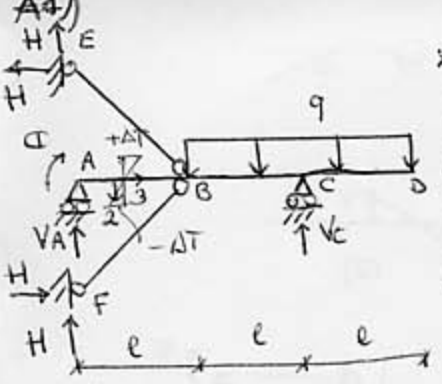


l = 1 m, q = 2 t/m, C = 4 tm,
E = 2.1 10^6 kg/cm^2, α= 10^{-5} °C^{-1}, ΔT = 20 °C

La travatura iperstatica di figura è realizzata con profilati IPE 200 (H= 200 mm, A = 28.4 cm^2, I_1 = 1943 cm^4).

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). Valutare l'effetto delle deformazioni assiali.
2. Calcolare la rotazione del nodo B.
3. Risolvere nuovamente la travatura considerando anche il carico termico nel tratto AB e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q,C che di ΔT.

A4)



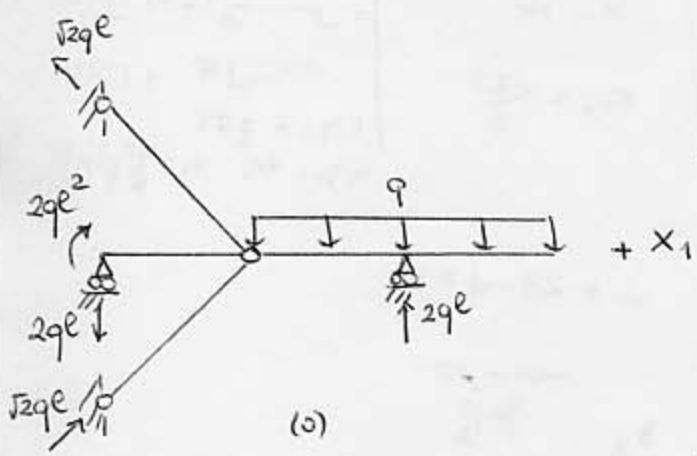
(↑) $V_A + V_C + 2H = 2q\ell$

(BT) $V_C\,\ell - V_A\,\ell = \mathcal{C} + 2q\ell^2$

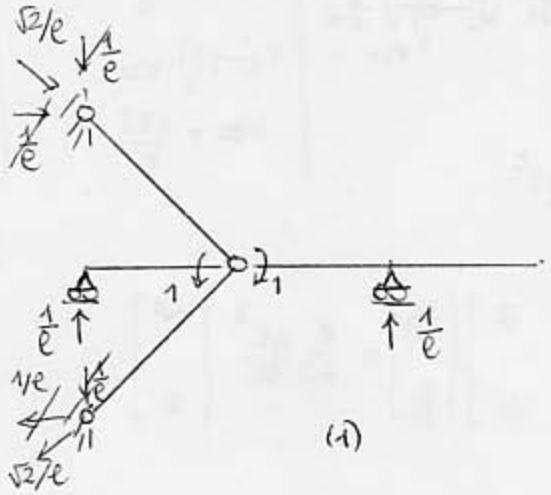
Struttura una volta iperstatica.

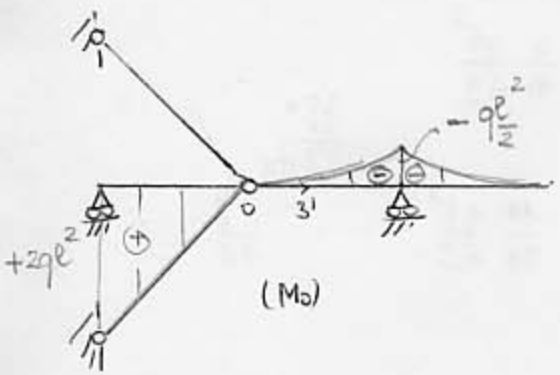
Incognita iperstatica $X_1 = M_B$

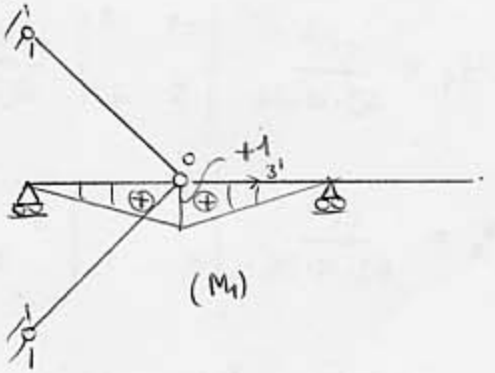
$q = 2\,t/m$
$\ell = 1\,m$
$\mathcal{C} = 2q\ell^2$
$H = 20\,cm$
$I_1 = 1943\,cm^4$
$A = 28{,}4\,cm^2$

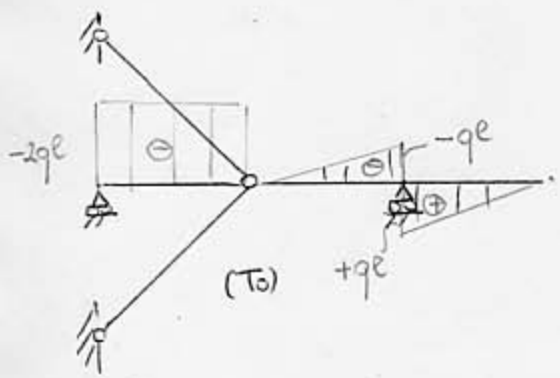


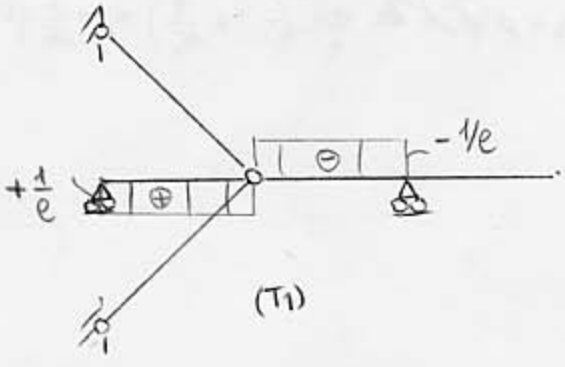
+ X_1

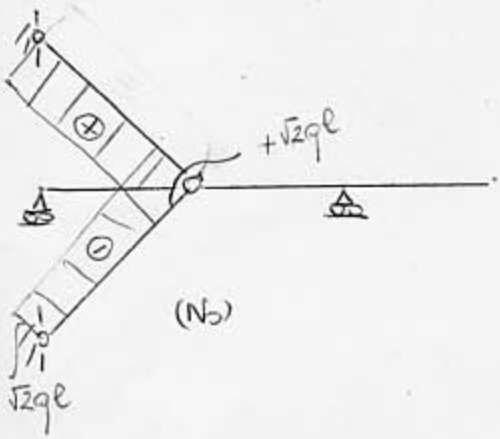


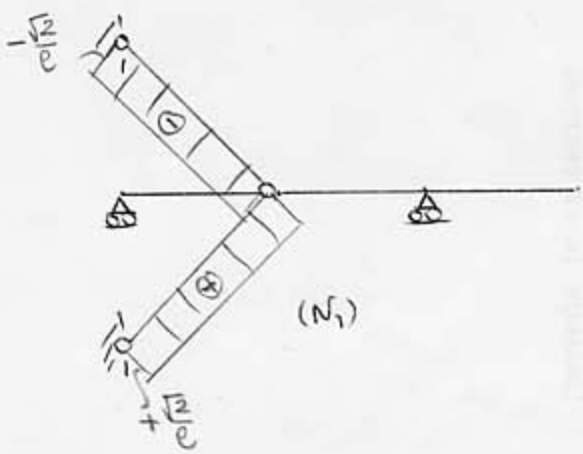




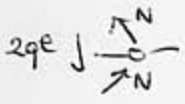








Calcoli



$\nearrow N\frac{\sqrt{2}}{2} = 2q\ell$

$\hookrightarrow N = \sqrt{2}\,q\ell$

$EI_1\,\delta_{11}^{M} = 2\cdot\frac{1}{3}\ell\cdot 1 = \frac{2}{3}\ell$

$EA\,\delta_{11}^{N} = 2\cdot\sqrt{2}\cdot\frac{\sqrt{2}}{\ell^2} = \frac{4\sqrt{2}}{\ell}$

$\frac{\delta_{11}^{N}}{\delta_{11}^{M}} = \frac{4\sqrt{2}}{\ell A}\cdot\frac{3I_1}{2\ell} = \frac{4\sqrt{2}\cdot 3\cdot 1943}{100^2\cdot 2\cdot 28.4} = 1{,}45\%$ Le deformazioni assiali si ritengono trascurabili.

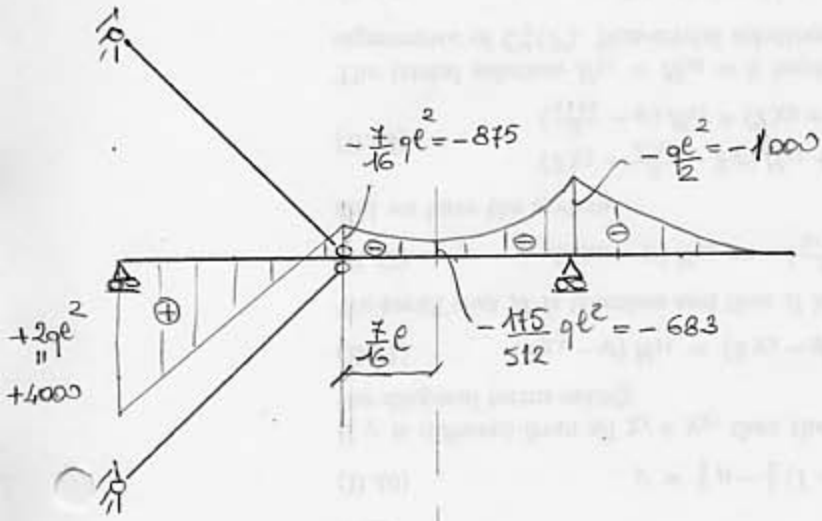
$EI_1\,\delta_{10} = \frac{1}{6}\ell\,2q\ell^2\cdot 1 + \int_0^{\ell}\left(-q\frac{x_3'^2}{2}\right)\left(1-\frac{x_3'}{\ell}\right)dx_3' = q\frac{\ell^3}{3} - \frac{q}{2}\int_0^{\ell}\left(x_3'^2 - \frac{x_3'^3}{\ell}\right)dx_3' = q\frac{\ell^3}{3} - \frac{q}{2}\left[\frac{\ell^3}{3} - \frac{\ell^3}{4}\right]$

$= \frac{7}{24}q\ell^3$

Equazione di congruenza: $(\Delta_{4B} = 0)$

$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{7}{24}\frac{3}{2}q\ell^2 = -\frac{7}{16}q\ell^2$

Diagrammi



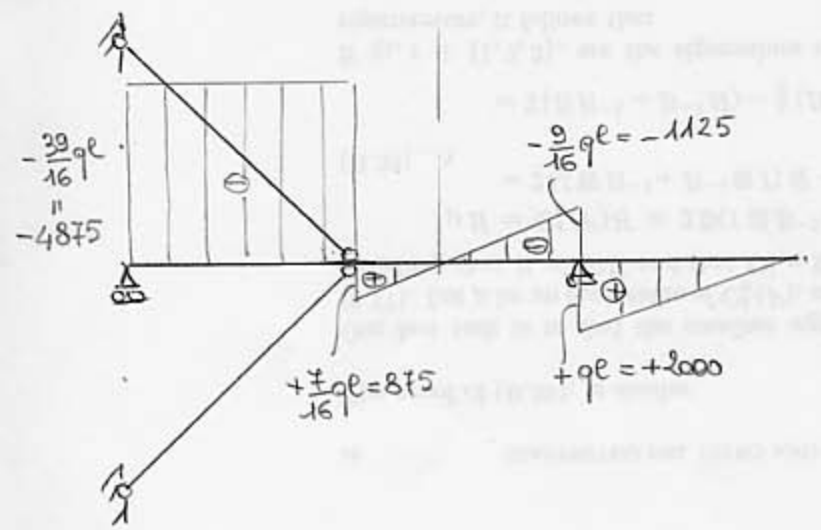
Calcoli

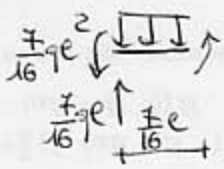
$T_A = -2q\ell - \frac{7}{16}q\ell = -\frac{39}{16}q\ell$

$T_{B^+} = +\frac{7}{16}q\ell$

$T_{C^-} = -q\ell + \frac{7}{16}q\ell = -\frac{9}{16}q\ell$

$T_{C^+} = q\ell$

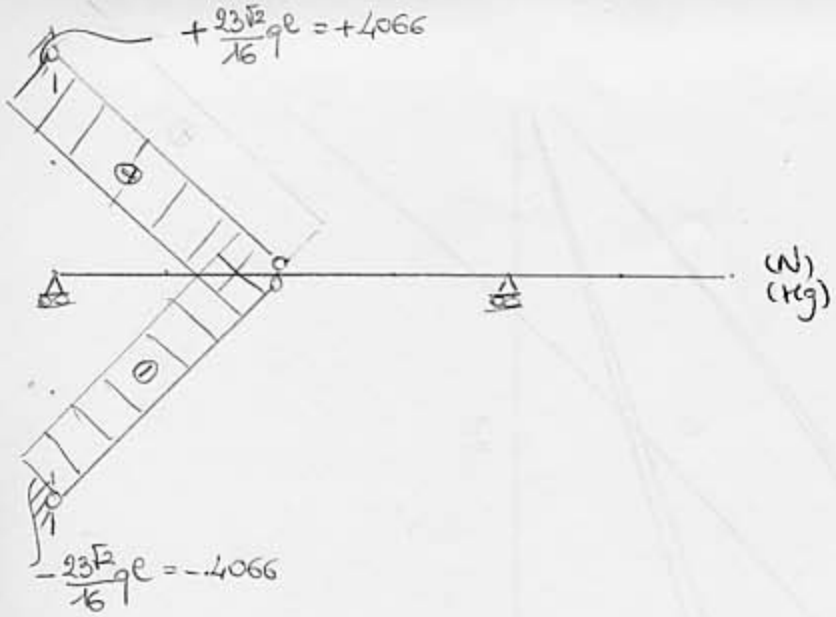


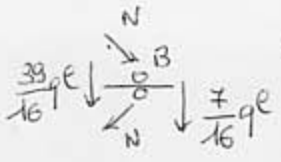


$\bar{M} = -\frac{7}{16}q\ell^2 + \frac{49}{256}q\ell^2 - \frac{1}{2}\frac{49}{256}q\ell^2$

$= \left(\frac{49}{512} - \frac{7}{16}\right)q\ell^2$

$= -\frac{175}{512}q\ell^2 = -683\ kgm$





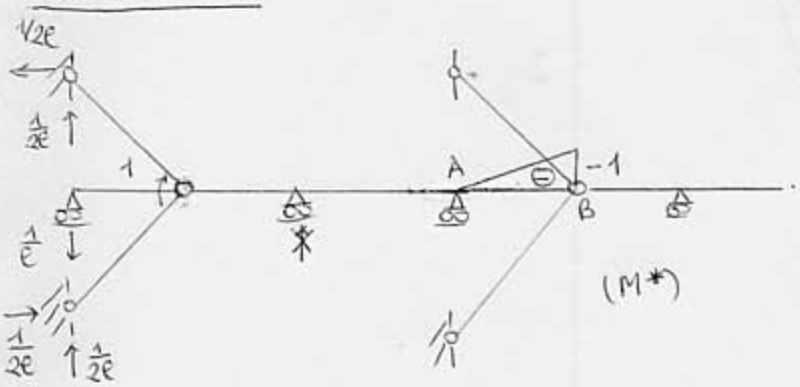
$$\not{N}\frac{\sqrt{2}}{2} = -\frac{\not{4}}{16}ql$$

$$\hookrightarrow N = -\frac{23\sqrt{2}}{16}ql$$

$$\simeq -2{,}03\,ql$$

A2)

Rotazione in B



$$1\cdot\varphi_B = \frac{1}{EI_1}\int_{AB} M M^*\,dx_3$$

$$= \frac{1}{EI_1}\int_0^{\ell}\left(-\frac{x_3}{\ell}\right)\left(2q\ell^2 - \frac{39}{16}q\ell x_3\right)dx_3$$

$$= -\frac{1}{\ell EI_1}\int_0^{\ell}\left(2q\ell^2 x_3 - \frac{39}{16}q\ell x_3^2\right)dx_3$$

$$= -\frac{1}{\not{\ell}EI_1}\left[\not{2}q\ell^{\not{2}}\frac{\ell^2}{\not{2}} - \frac{\overset{13}{\not{39}}}{\not{8}\cdot 16}q\not{\ell}\ell^3\right]$$

$$= -\frac{q\ell^3}{EI_1}\left(1 - \frac{13}{16}\right) = -\frac{3q\ell^3}{16EI_1}$$

$$= -0{,}05°$$

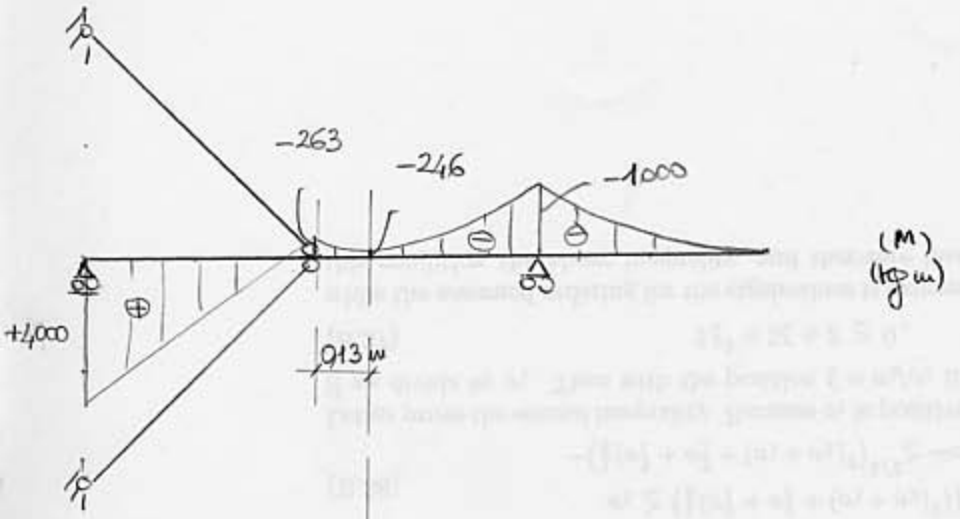
A3) Carico termico in AB

$$\eta_{1t} = \int_{AB} M_1 \chi_t = -\frac{\not{3}\alpha\Delta T}{H}\frac{\ell}{\not{2}}$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1T}}{\eta_{11}} = -\frac{7}{16}q\ell^2 + \frac{\alpha\Delta T\not{\ell}}{H}\frac{3EI_1}{2\not{\ell}} = -875 + \frac{\not{10}^{-5}\cdot\not{20}\cdot 3\cdot 2{,}1\cdot\not{10}^{\not{5}}}{2\cdot\not{20}}\frac{1943}{\not{10}^{\not{5}}}\ \text{kg m}$$

$$= -875 + 612 = -263\ \text{kg m}$$

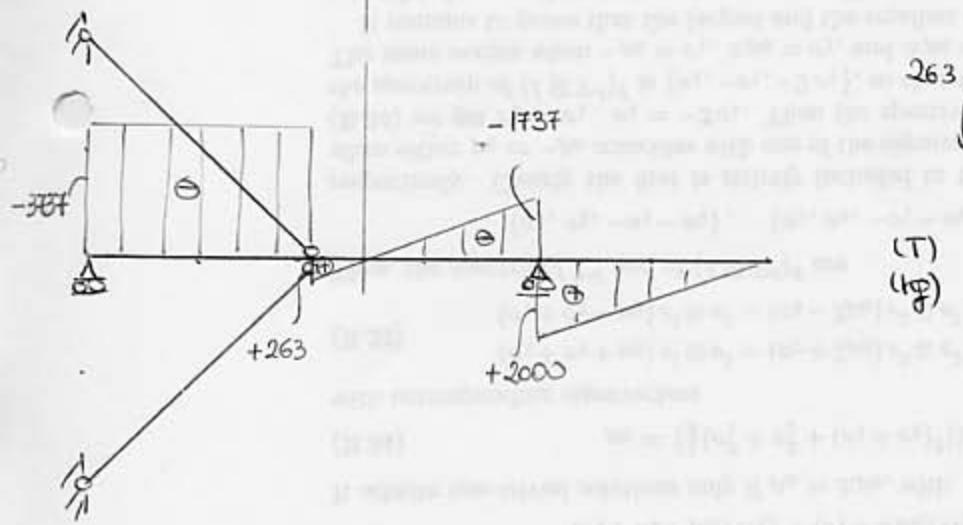
Diagrammi complessivi di q, T e di ΔT:

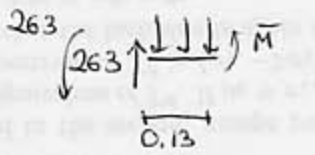


$T_A = -4000 + 263 = -3737$ kg

$T_B^+ = +263$ kg

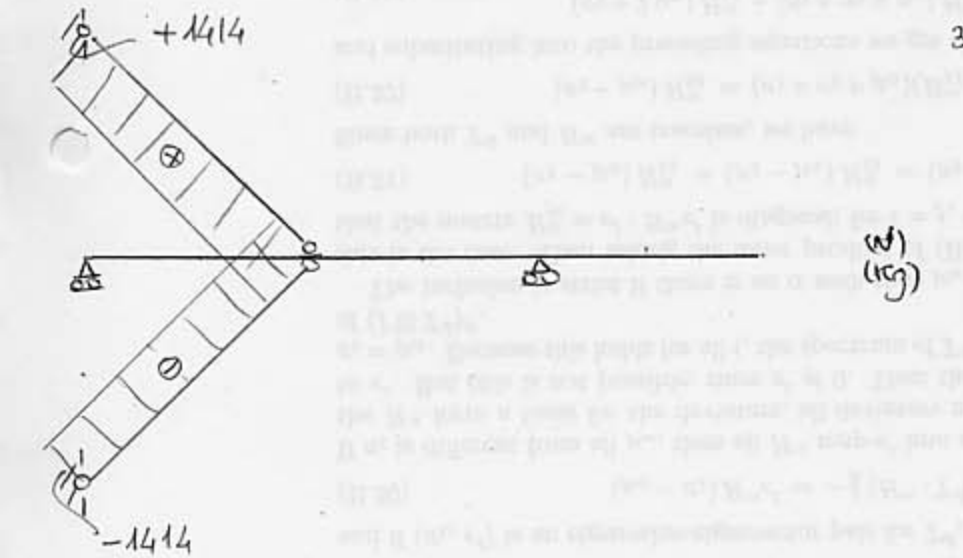
$T_C^- = -2000 + 263 = -1737$ kg

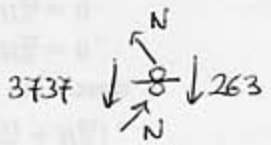




$\bar{M} = -263 + 263 \cdot 0{,}13 - 2000 \cdot \frac{0{,}13^2}{2}$

$= -246$ kgm





$2N\frac{\sqrt{2}}{2} = 3737 + 263$

$\hookrightarrow N = \frac{2000\sqrt{2}}{2} = 1414$ kg