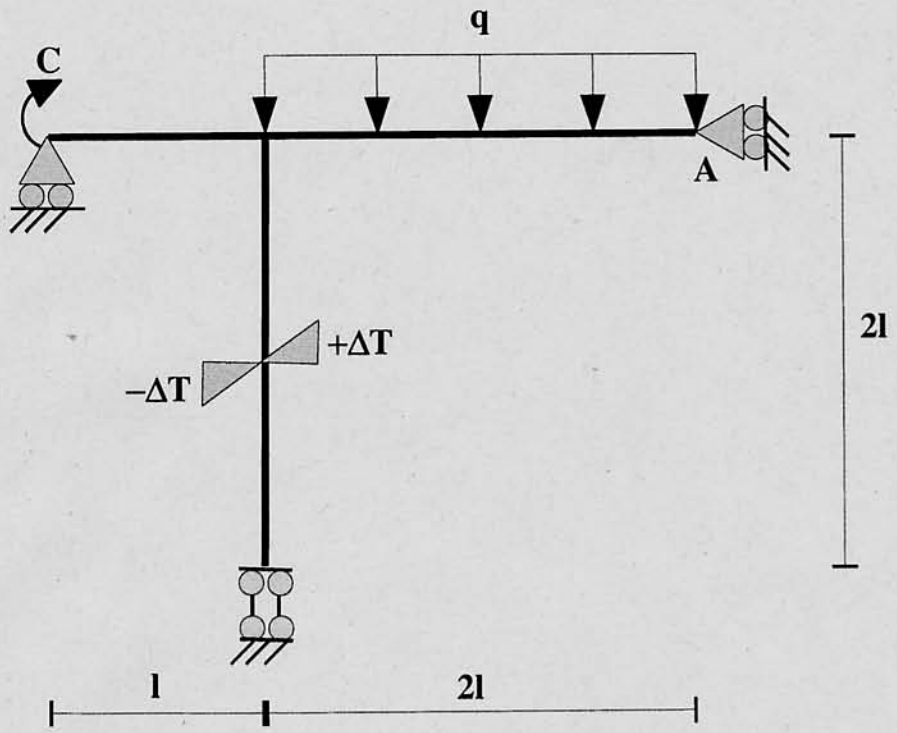
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
20/01/2011

A

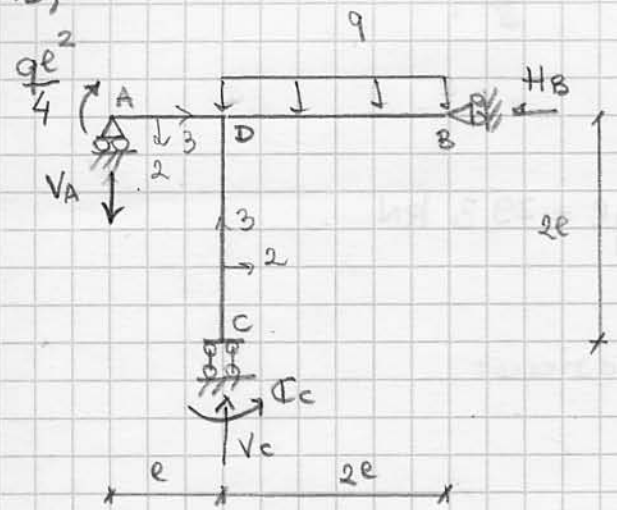


l = 2 m, q = 20 kN/m, C = 20 kNm
E = 210 GPa, σ_{amm} = 240 MPa, $\alpha = 10^{-5}\,°C^{-1}$
$\Delta T = 20\,°C$

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in presenza dei soli carichi q e C.
- Progettare la travatura.
- Calcolare lo spostamento verticale del punto A.
- Disegnare nuovamente i diagrammi quotati considerando, in aggiunta ai carichi q e C, anche un carico termico a farfalla, di intensità massima ΔT, agente solamente sul tratto verticale.

A1)



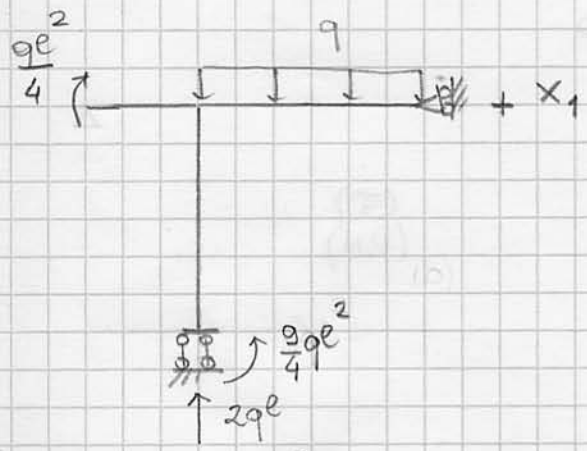
$H_B = 0$

$-V_A + V_C = 2ql$

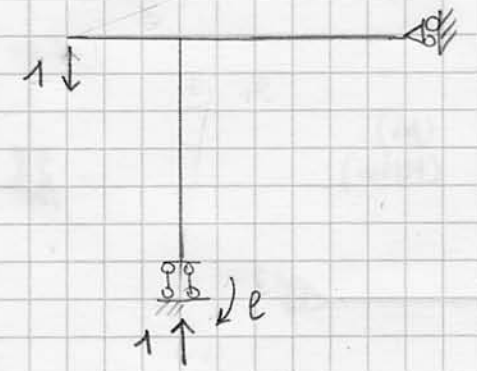
$(\curvearrowright)\quad V_C l + \mathcal{C}_C = 2ql\,2l + \frac{qe^2}{4} = \frac{17}{4}ql^2$

Trovata una volta iperstatica.

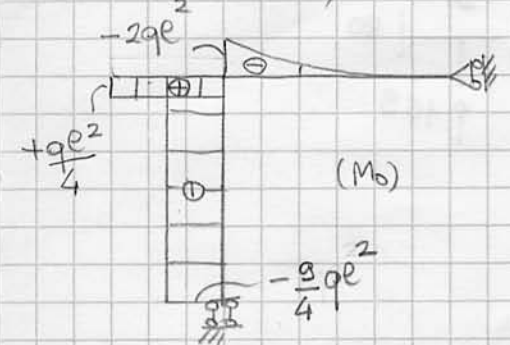
Incognita iperstatica $X_1 = V_A$.

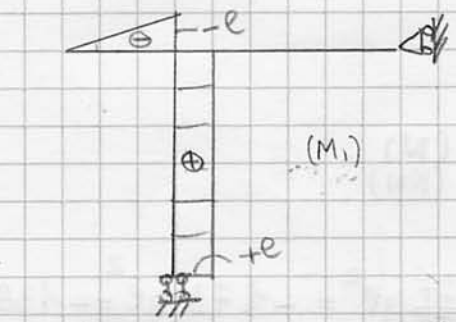


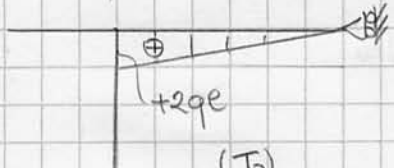
(0)

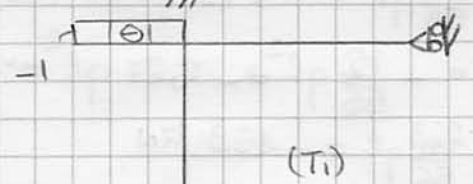


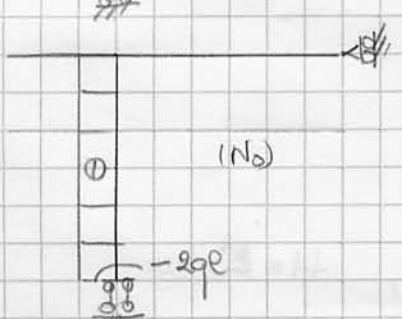
(1)

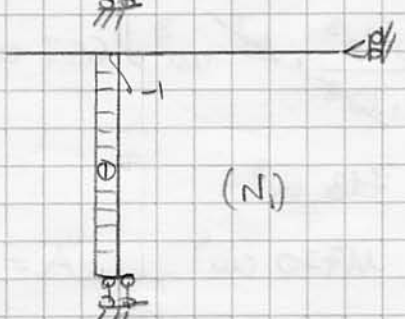










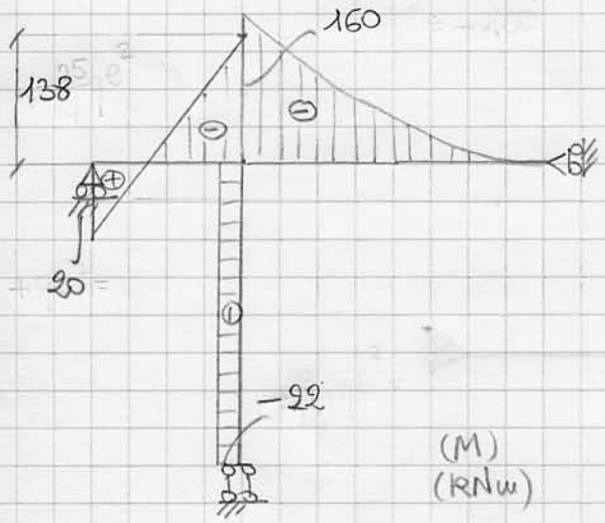


$$EI_1 \gamma_{10} = 2\ell\left(-\frac{9}{4}q\ell^3\right)\ell + q\ell^2\,\frac{\ell}{2}(-\ell) = \left(-\frac{9}{2}-\frac{1}{8}\right)q\ell^4 = -\frac{37}{8}q\ell^4$$

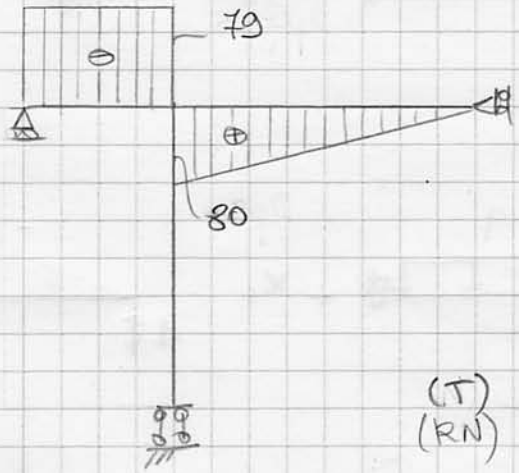
$$EI_1 \gamma_{11} = \frac{1}{3}\ell\,\ell^2 + 2\ell\,\ell^2 = \frac{7}{3}\ell^3$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} = \frac{37}{8}\,\frac{3}{7}\,q\ell = \frac{111}{56}q\ell = 1.98\,q\ell = 79{,}3\ \text{kN}$$

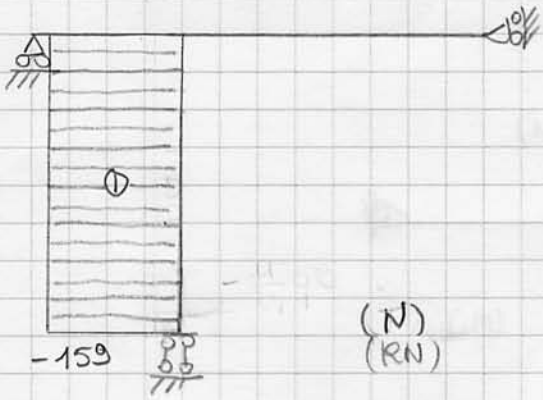
Diagrammi delle caratteristiche della sollecitazione:



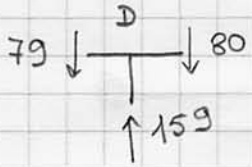
(M)
(kNm)



(T)
(kN)



(N)
(kN)



$$M_{DA} = \frac{q\ell^2}{4} - \frac{111}{56}q\ell^2 = -\frac{97}{56}q\ell^2 = -1{,}73\,q\ell^2 = -138{,}6\ \text{kNm}$$

$$M_{DC} = -\frac{9}{4}q\ell^2 + \frac{111}{56}q\ell^2 = -\frac{15}{56}q\ell^2 = -0{,}27\,q\ell^2 = -21{,}4\ \text{kNm}$$

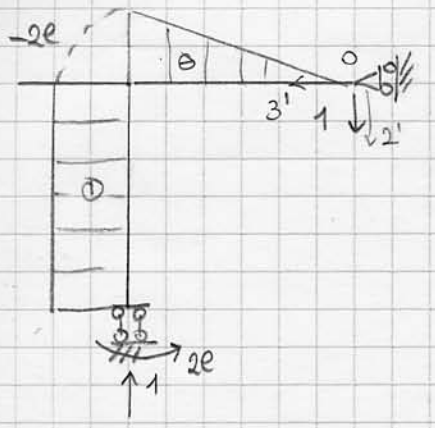
$$N_{DC} = -2q\ell - \frac{111}{56}q\ell = -\frac{223}{56}q\ell = -159\ \text{kN}$$

A2) Dimensionamento:

$$W_1 > \frac{M_{max}}{\sigma_{AMM}} = \frac{160\cdot 10^3}{240\cdot 10^6}\cdot 10^6\ \text{cm}^3 = 667\ \text{cm}^3$$

IPE 330 $\begin{cases} W_1 = 713\ \text{cm}^3 \\ I_1 = 11770\ \text{cm}^4, \quad A = 62{,}6\ \text{cm}^2, \quad H = 33\ \text{cm} \end{cases}$

A3) Spostamento verticale in A



$$1\cdot v_A = \frac{1}{EI_1}\,2e\left(-\frac{15}{56}qe^2\right)(-1e) + \frac{1}{EI_1}\int_0^{2e}(-x_3')\left(-9\frac{x_3'^2}{2}\right)dx_3'$$

$$= \frac{qe^4}{EI_1}\left[\frac{15}{14} + \frac{1}{2}\,\frac{1}{4}\,16^2\right] = \frac{43}{14}\,\frac{qe^4}{EI_1}$$

$$= \frac{43}{14}\,\frac{20\cdot 10^3\cdot 16}{210\cdot 10^{\cancel{3}}\,11770\cdot 10^{\cancel{-8}}} = 3{,}9\ \text{cm}$$

A4) Carico termico

$$\gamma_{1t} + \gamma_{10} + \gamma_{11}X_1 = 0$$

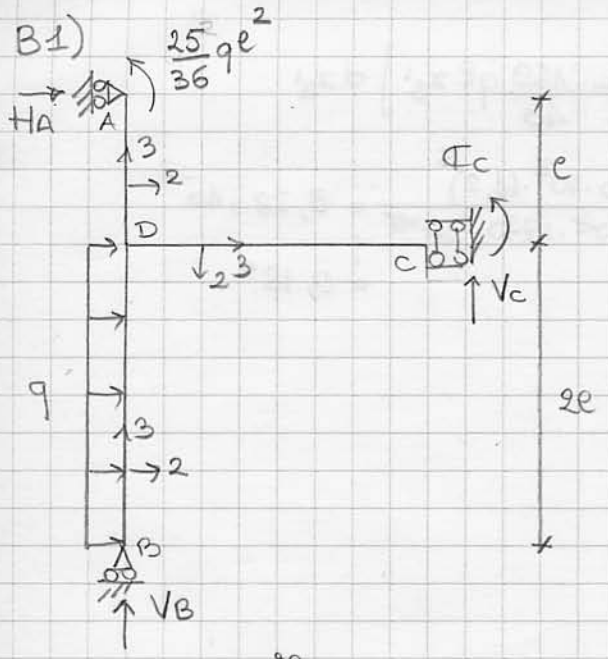
$$\gamma_{1t} = \int_0^{2e} \ell\,\chi_t\,dx_3 = 2e^2\chi_t = 2e^2\,\frac{2\alpha\Delta T}{H} = \frac{4\alpha\Delta T e^2}{H}$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} - \frac{\gamma_{1t}}{\gamma_{11}} = \frac{111}{56}qe - \frac{4\alpha\Delta T\cancel{e^2}}{H}\,\frac{3EI}{7\cancel{e^3}}$$

$$= \frac{111}{56}qe - \frac{12\alpha\Delta T EI}{7He} = 79{,}3 - \frac{12\cdot 10^{-5}\cdot 20\cdot 210\cdot 10^{\cancel{3}}\cdot 11770\cdot 10^{\cancel{-8}}}{7\cdot 0{,}33\cdot 2}$$

$$= 79{,}3 - 12{,}8\ \text{kN} = 66{,}5\ \text{kN}$$

B1)



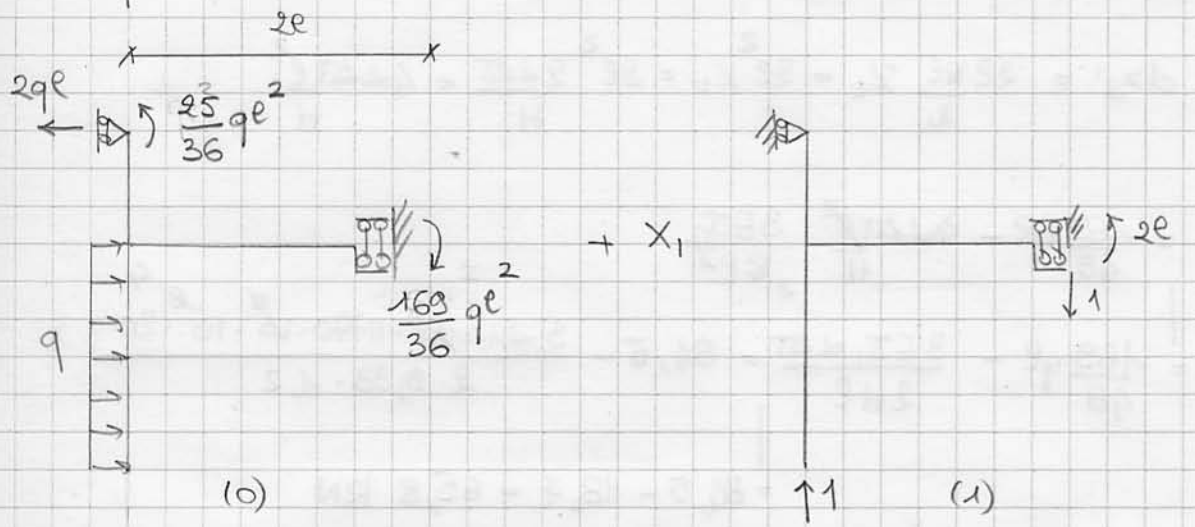
$H_A = -2qe$

$V_B + V_C = 0$

(A↑) $I_C + V_C 2e + \frac{25}{36}qe^2 + 4qe^2 = 0$

Individuata una volta iperstatica.

Incognita iperstatica: $X_1 = V_B$



(Diagramma della pagina seguente).

$EI_1 M_{10} = \left(-\frac{169}{36}qe^2\right)\frac{1}{2}\,2e\,2e = -\frac{169}{18}qe^4$

$EI_1 M_{11} = \frac{1}{3}\,2e\,4e^2 = \frac{8}{3}e^3$

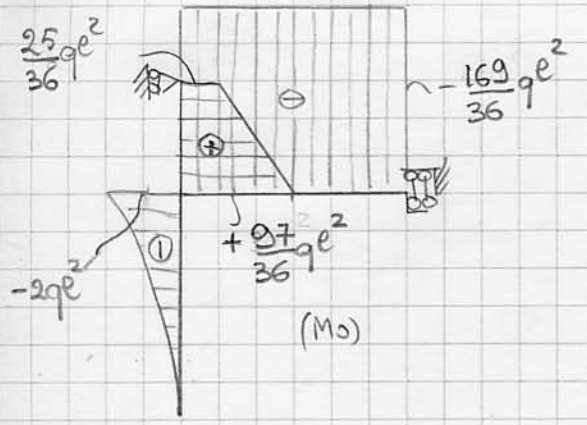
$X_1 = -\frac{M_{10}}{M_{11}} = \frac{169}{18_6}\,\frac{3}{8}\,qe = \frac{169}{48}qe = 84,5\ kN$

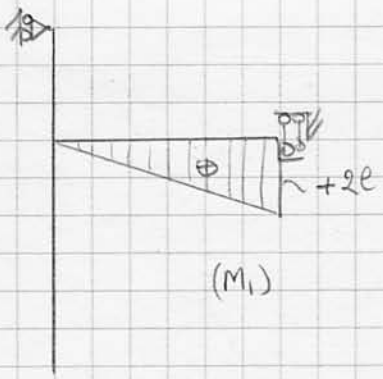
$M_C = -\frac{169}{36}qe^2 + \frac{169}{24}qe^2 = \frac{169}{72}qe^2 = 67,6\ kNm$

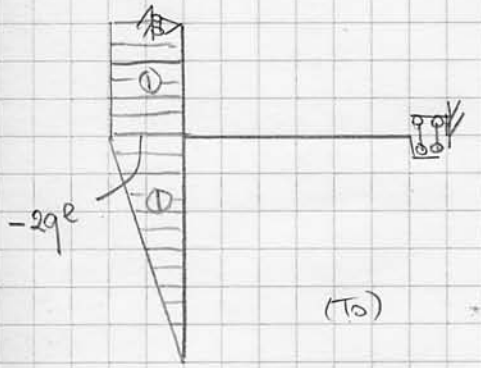
B2) Dimensionamento:

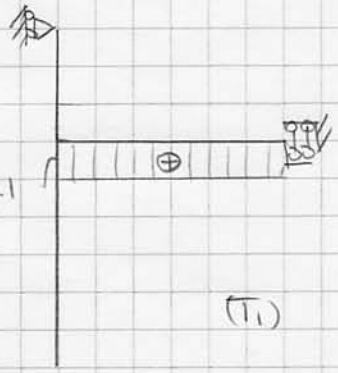
$W_1 \geq \frac{M_{max}}{\sigma_{amm}} = \frac{135,2\ 10^3}{240 \cdot 10^6} \cdot 10^6 = 563\ cm^3$

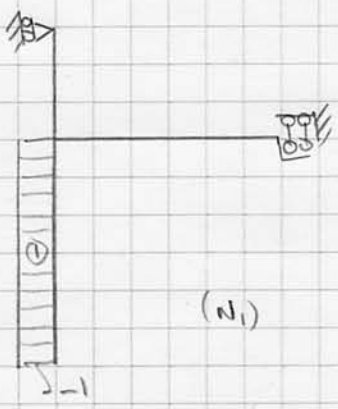
IPE 330 $\begin{cases} W_1 = 713\ cm^3 \\ I_1 = 11770\ cm^4, \quad A = 62,6\ cm^2, \quad H = 33\ cm \end{cases}$



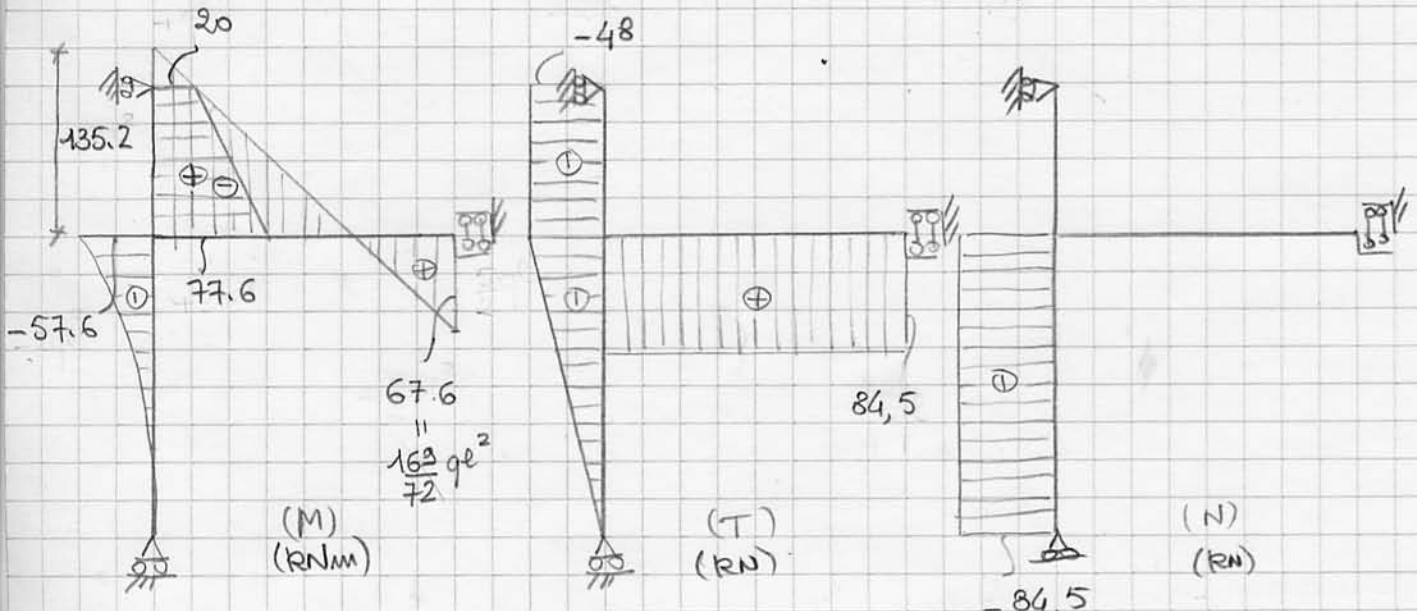




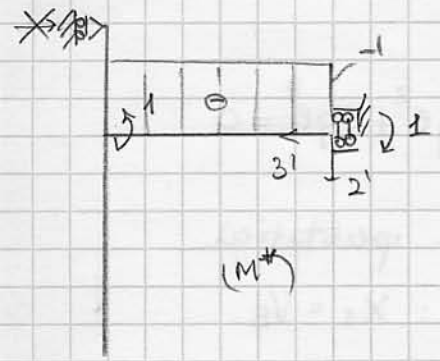




Diagrammi quotati:



B3) Rotazione di A.



$$1 \cdot \varphi_A = \frac{1}{EI_1}(-1)\int_0^{2\ell}\left[\frac{169}{72}q\ell^2 - \frac{169}{48}q\ell x_3'\right]dx_3'$$

$$= \frac{169}{72}\frac{q\ell^3}{EI_1} = \frac{169 \cdot 20 \cdot 10^3 \cdot (1,2)^3}{72 \cdot 210 \cdot 10^8 \cdot 1770 \cdot 10^{-8}} = 3,28 \cdot 10^{-3}$$

$$= 0,18°$$

B4) Carico termico

$$\gamma_{1t} + \gamma_{11} X_1 + \gamma_{10} = 0$$

$$\gamma_{1t} = \int_0^{2\ell} M_1 \chi_t \, dx_3' = 2\ell \frac{2\ell}{2} \chi_t = 2\ell^2 \chi_t = 2\ell^2 \frac{2\alpha\Delta T}{H} = 4\alpha\Delta T \frac{\ell^2}{H}$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} - \frac{\gamma_{1t}}{\gamma_{11}} = \frac{169}{48}q\ell - 4\alpha\Delta T\frac{\ell^2}{H}\frac{3EI_1}{8\ell^2}$$

$$= \frac{169}{48}q\ell - \frac{3EI_1\alpha\Delta T}{2H\ell} = 84,5 - \frac{3 \cdot 210 \cdot 10^3 \cdot 1770 \cdot 10^{-8} \cdot 20}{2 \cdot 0,33 \cdot 1,2}$$

$$= 84,5 - 18,7 = 65,8 \text{ kN}$$