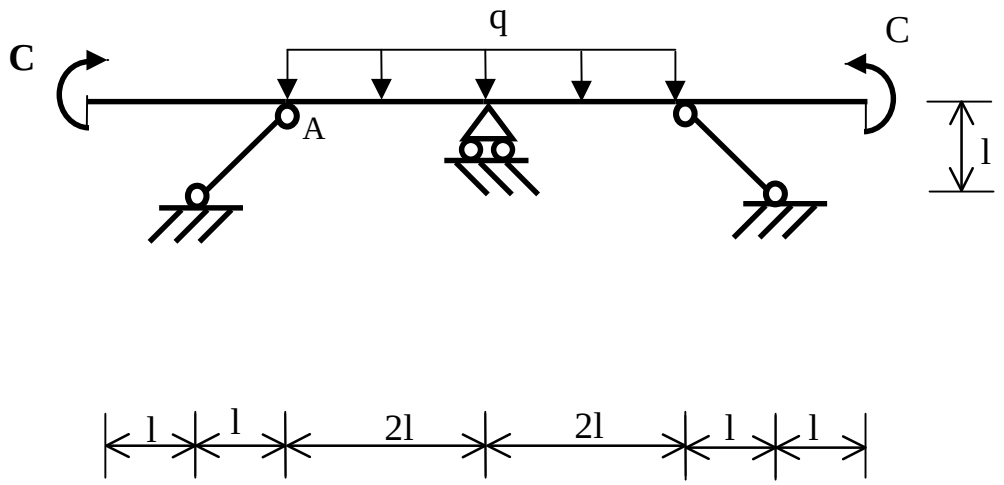
**CORSO DI LAUREA IN INGEGNERIA MECCANICA**
**UNIVERSITÀ DEGLI STUDI DI FERRARA**
**SECONDA PROVA SCRITTA IN ITINERE DI STATICA**
**FERRARA, 17/12/20109**

A




l = 1 m, q = 25 kN/m, C = 25 kNm
E = 210 GPa, $\sigma_{AMM}$= 240 MPa

La travatura iperstatica di figura è realizzata con profilati IPE 200.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo A.
3. Risolvere nuovamente la travatura considerando anche un abbassamento verticale del carrello pari a 1 cm e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q e di C che del cedimento.

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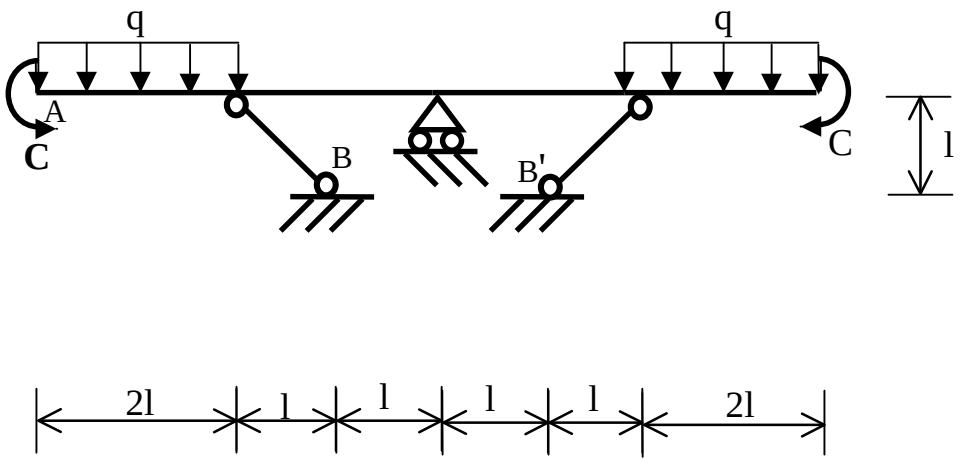
B




l = 1 m, q = 20 kN/m, C = 20 kNm
E = 210 GPa, $\sigma_{AMM}$= 240 MPa

La travatura iperstatica di figura è realizzata con profilati IPE 240.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è possibile trascurare le deformazioni assiali.
2. Calcolare lo spostamento verticale del nodo A.
3. Risolvere nuovamente la travatura considerando anche un abbassamento verticale dei punti B e B' pari a 1 cm e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q e di C che del cedimento.

**CORSO DI LAUREA IN INGEGNERIA MECCANICA**
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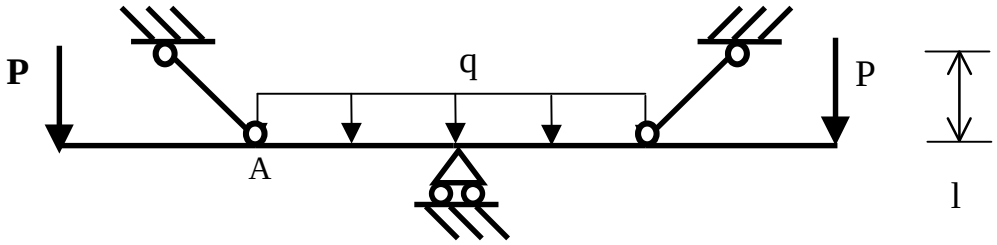
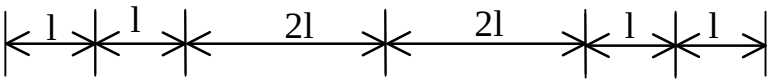
**FERRARA, 17/12/20109**

**C**



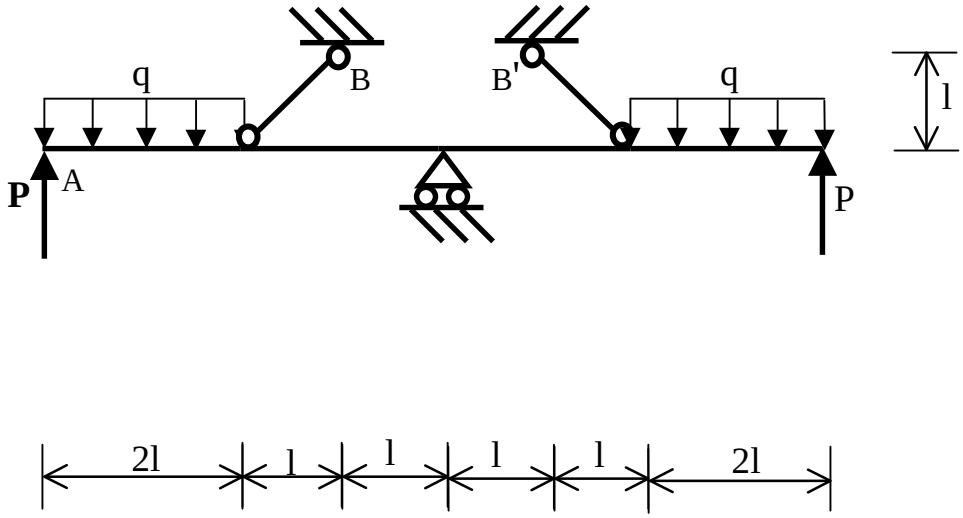

l = 1 m, q = 25 kN/m, P = 25 kN
E = 210 GPa, $\sigma_{AMM}$= 240 MPa

La travatura iperstatica di figura è realizzata con profilati IPE 220.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo A.
3. Risolvere nuovamente la travatura considerando anche un innalzamento verticale del carrello pari a 1 cm e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q e di P che del cedimento.

**CORSO DI LAUREA IN INGEGNERIA MECCANICA**
**UNIVERSITÀ DEGLI STUDI DI FERRARA**
**SECONDA PROVA SCRITTA IN ITINERE DI STATICA**
**FERRARA, 17/12/20109**

**D**

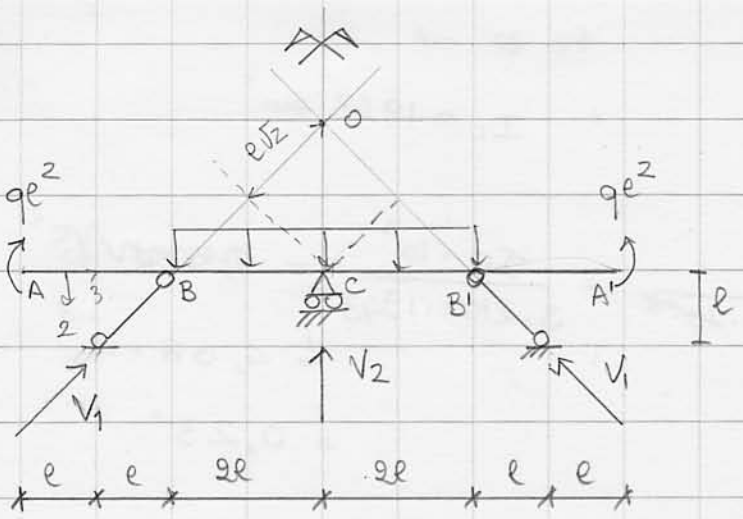



l = 1 m, q = 20 kN/m, P = 20 kN
E = 210 GPa, $\sigma_{AMM}$= 240 MPa

La travatura iperstatica di figura è realizzata con profilati IPE 180.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è possibile trascurare le deformazioni assiali.
2. Calcolare lo spostamento verticale del nodo A.
3. Risolvere nuovamente la travatura considerando anche un innalzamento verticale dei punti B e B' pari a 1 cm e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q e di P che del cedimento.

A1)

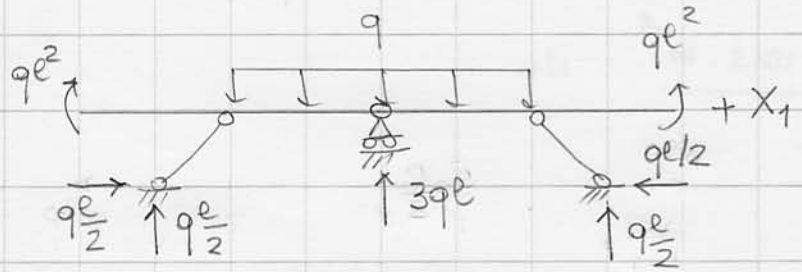


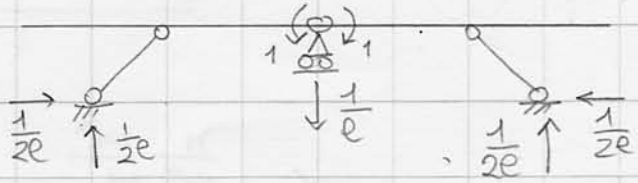

$$(\rightarrow)\ V_1\frac{\sqrt{2}}{2} = V_1\frac{\sqrt{2}}{2}$$

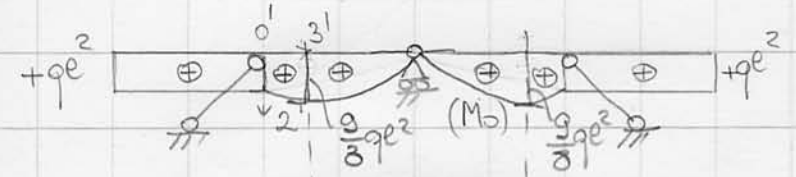
$$(\uparrow)\ 2V_1\frac{\sqrt{2}}{2} + V_2 = 4qe$$

$$(\circlearrowleft)\ qe^2 - qe^2 = 0$$

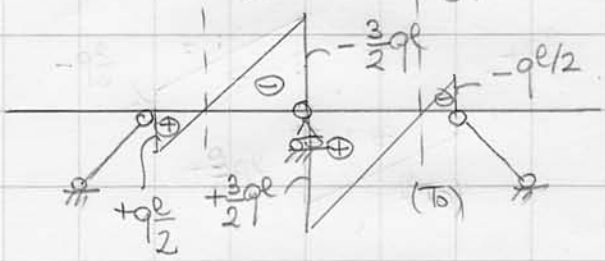
$$X_1 = M_C = V_1 e\sqrt{2} + qe^2 - 2qe^2$$
$$\overset{!}{=} V_1 e\sqrt{2} - qe^2$$

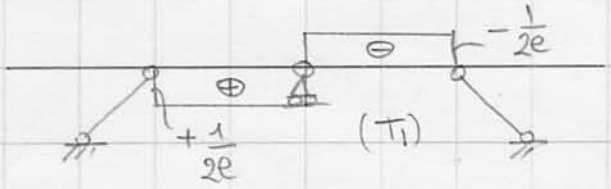


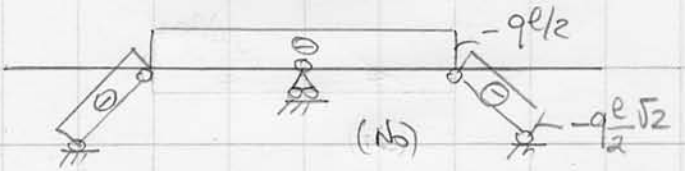



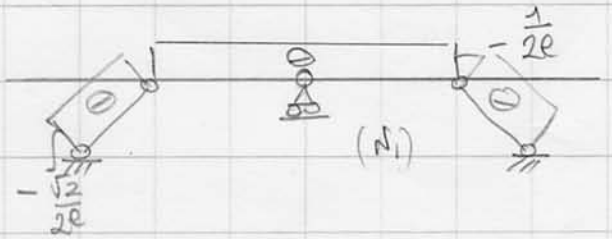








$$0 = M_C = V_1 e\sqrt{2} - qe^2$$
$$\hookrightarrow V_1 = \frac{qe\sqrt{2}}{2}$$
$$V_1\frac{\sqrt{2}}{2} = qe\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}$$

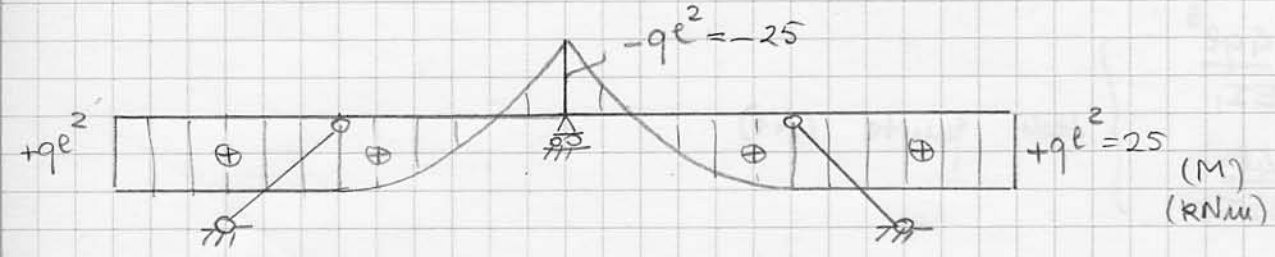
$$1 = M_C = V_1 e\sqrt{2}$$
$$\hookrightarrow V_1 = \frac{1}{e\sqrt{2}} = \frac{\sqrt{2}}{2e}$$
$$V_1\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2e}\frac{\sqrt{2}}{2} = \frac{1}{2e}$$

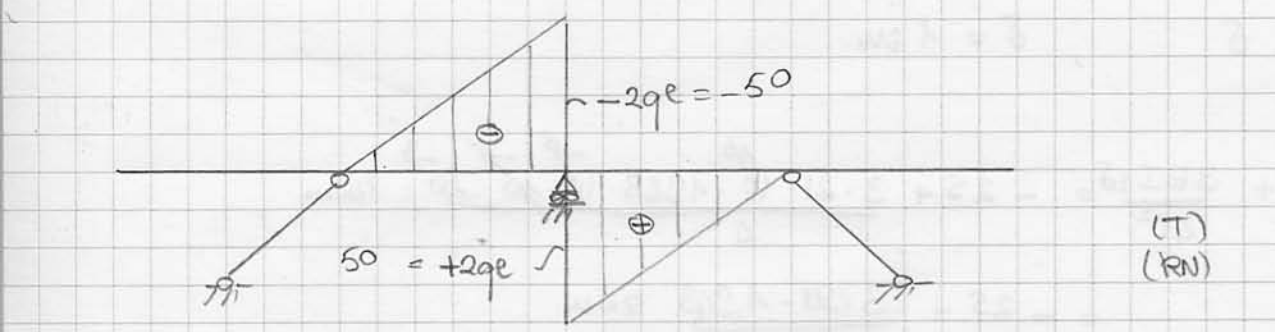
$$EI_1 M_{10} = 2\int_0^{2e}\left(\frac{x_3'}{2e}\right)\left(qe^2 + q\frac{e}{2}x_3' - q\frac{x_3'^2}{2}\right)dx_3' = \frac{4}{3}qe^3$$

$$EI_1 M_{11} = 2\cdot\frac{1}{3}\cdot(1)^2 2e = \frac{4}{3}e$$

$$X_1 = -\frac{4}{3} q\ell^3 \frac{3}{4\ell} = -q\ell^2 = -25 \text{ kNm}$$

Diagrammi delle caratteristiche di sollecitazione:



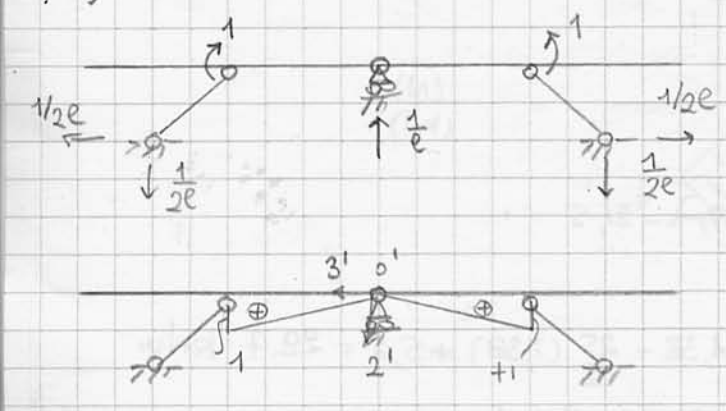


$$T_B^+ = q\frac{\ell}{2} - q\frac{\ell^2}{2\ell} = 0$$

$$T_C^- = -\frac{3}{2}q\ell - \frac{q\ell^2}{2\ell} = -2q\ell$$

$$N_{B^+} = -q\frac{\ell}{2} + q\frac{\ell^2}{2\ell} = 0 \quad ; \quad N_{biella} = -q\frac{\ell}{2}\sqrt{2} - q\ell^2\left(-\frac{\sqrt{2}}{2\ell}\right) = 0$$

A2)




$$2\cdot\varphi_A = \frac{1}{EI_1}\int_0^{2\ell}\left(\frac{x_3'}{2\ell}\right)\left(-q\ell^2 + 2q\ell x_3' - q\frac{x_3'^2}{2}\right)dx_3'$$

$$= \frac{q}{2\ell EI_1}\int_0^{2\ell}\left(-\ell^2 x_3' + 2\ell x_3'^2 - \frac{x_3'^3}{2}\right)dx_3'$$

$$= \frac{q}{2\ell EI_1}\left[-\frac{\ell^2}{2}4\ell^2 + \frac{2\ell}{3}8\ell^3 - \frac{1}{8}16\ell^4\right]$$

$$= \frac{q\ell^3}{EI_1}\left[-2 + \frac{16}{3} - 2\right] = \frac{4}{3}\frac{q\ell^3}{EI_1}$$

$$\varphi_A = 4{,}03\cdot 10^{-3} = 0{,}23°$$

$$\varphi_A = \frac{2}{3}\,\frac{qe^3}{EI_1} =$$

A3) $\quad \gamma_{10} + \gamma_{11} X_1 = \gamma_1$

$$\gamma_{10} = \frac{49e^3}{EI_1}$$

$$\gamma_{11} = \frac{4e}{3EI_1}$$

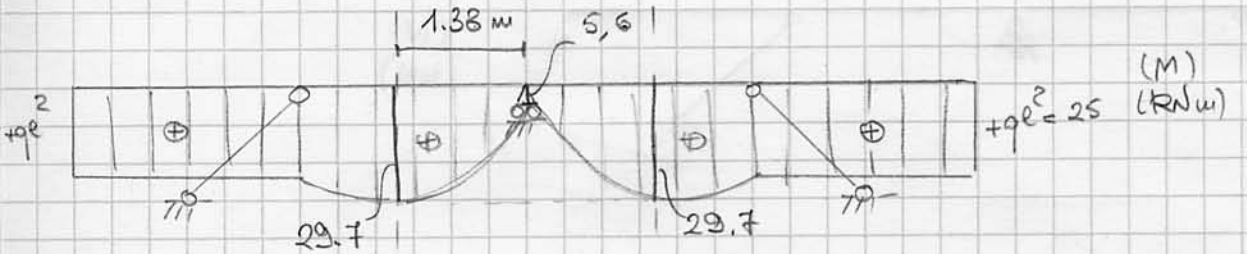
} vedi punto (A1)

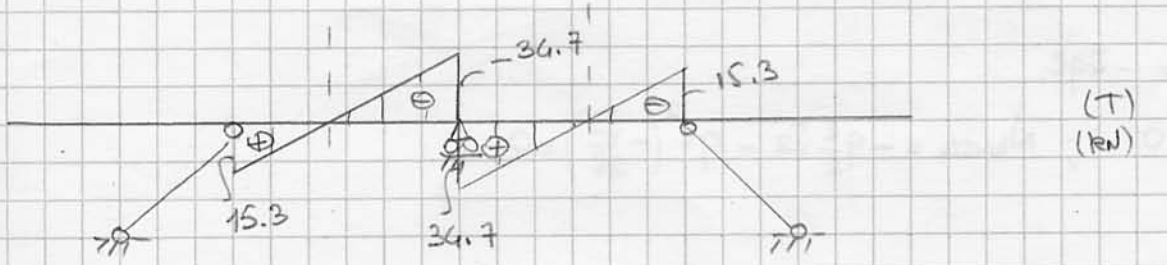
$$\gamma_1 = \frac{1}{e}\,\delta \quad , \quad \delta = 1\ cm$$

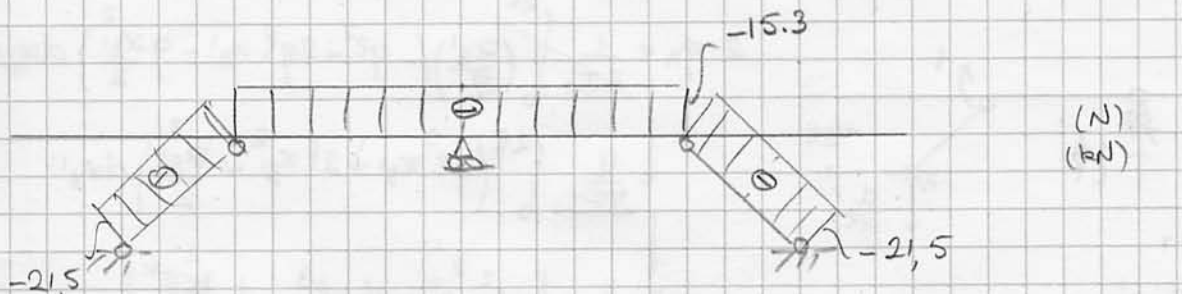
$$X_1 = -qe^2 + \frac{3EI_1\delta}{4e^2} = -25 + \frac{3 \cdot 21 \cdot 10^{10} \cdot 1943 \cdot 10^{-8} \cdot 10^{-2} \cdot 10^{-3}}{4}\ kNm$$

$$= -25 + \frac{3 \cdot 21 \cdot 1{,}943}{4}\ kNm$$

$$= -25 + 30{,}6\ kNm = 5{,}6\ kNm$$




$T_{B^+} = 12.5 + \frac{5.6}{2} = 15.3\ kN$

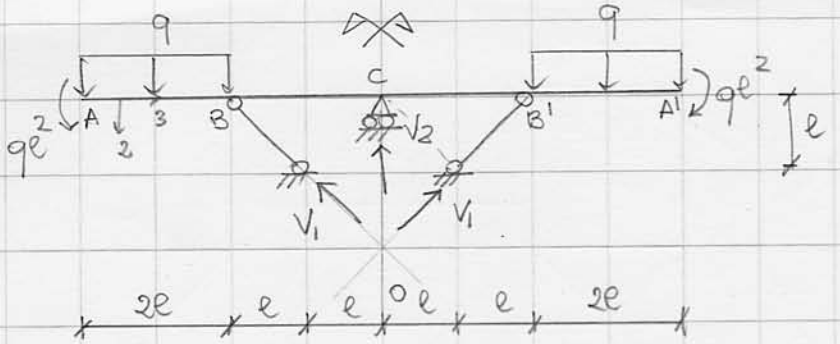
$T_{C^-} = -37.5 + \frac{5.6}{2} = -34.7\ kN$

$M_{max} = 34{,}7 \cdot 1{,}38 - \frac{25}{2}(1{,}38)^2 + 5{,}6 = 29.7\ kNm$

$N_{beta} = -17{,}6 - 3{,}9 = -21{,}5\ kN$

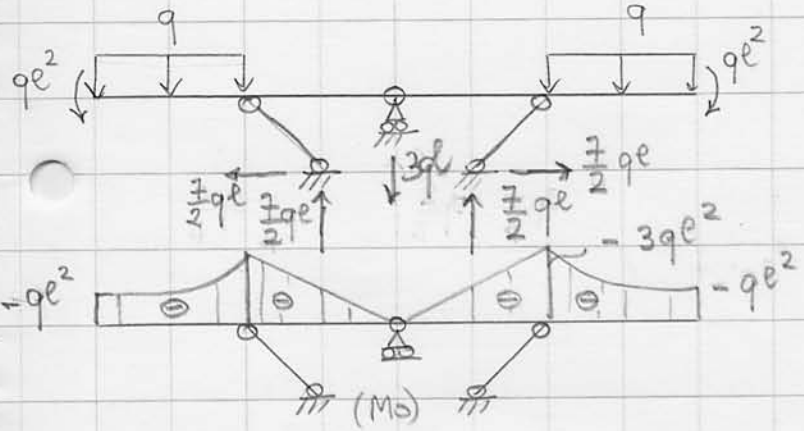
$N_{B^+} = -12.5 - \frac{5.6}{2} = -15.3$

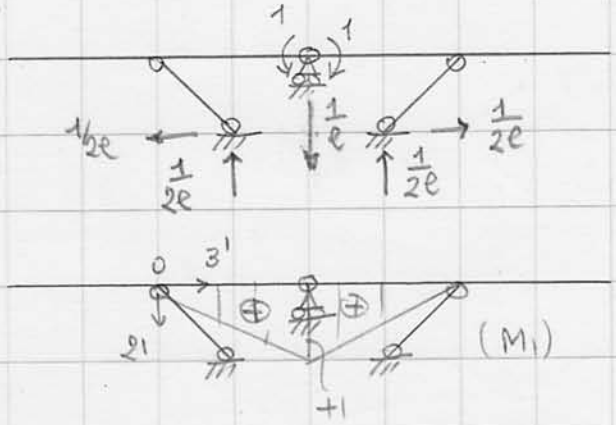
B1)

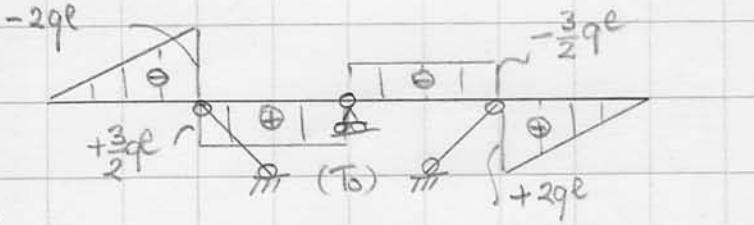


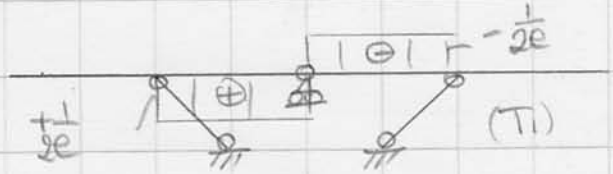

$$\begin{cases} (\rightarrow) & V_1\frac{\sqrt{2}}{2} = V_1\frac{\sqrt{2}}{2} \\ (\uparrow) & 2V_1\frac{\sqrt{2}}{2} + V_2 = 4q\ell \\ (\circlearrowleft) & q\ell^2 - q\ell^2 + 2q\ell\,3\ell - 2q\ell\,3\ell = 0 \end{cases}$$

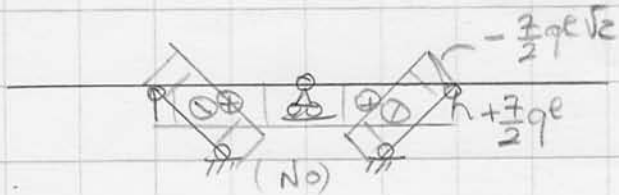
$$X_1 = M_C = V_1\,\ell\sqrt{2} - q\ell^2 - 2q\ell\,3\ell$$
$$\overset{!}{=} V_1\,\ell\sqrt{2} - 7q\ell^2$$

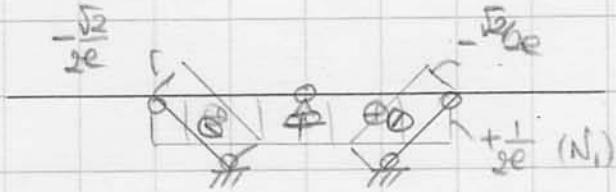


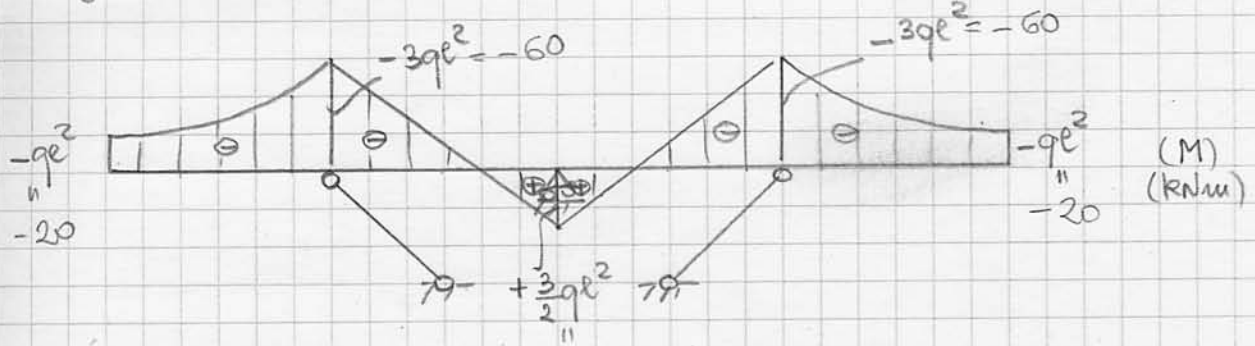

$$EI_1 M_{10} = 2\int_0^{2\ell}\left(\frac{x_3'}{2\ell}\right)\left(-3q\ell^2 + \frac{3}{2}q\ell x_3'\right)dx_3' = 2\cdot\frac{3q}{2\ell}\int_0^{2\ell}\left(-x_3'\ell^2 + \frac{1}{2}\ell x_3'^2\right)dx_3'$$

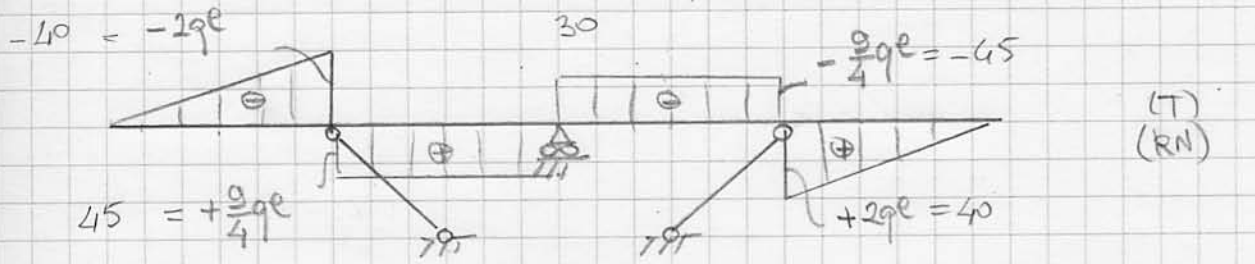
$$= 2\cdot\frac{3q}{2\ell}\left[-\ell^2\frac{1}{2}\cancel{4}\ell^2 + \frac{\ell}{2}\frac{1}{3}\overset{\#^2}{8\ell^3}\right| = 2\cdot 3q\ell^3\left[-1+\frac{2}{3}\right] = -2q\ell^3$$

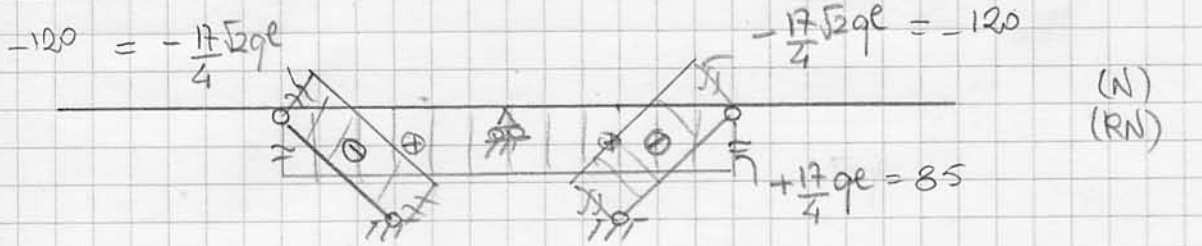
$$EI_1 M_{11} = 2\cdot\frac{1}{3}\,2\ell\cdot(1)^2 = \frac{4\ell}{3}$$

$$X_1 = 3q\ell^3\,\frac{3}{4\cancel{8}_2} = \frac{3}{2}q\ell^2 = 30\ \text{kNm}$$

Diagrammi delle caratteristiche della sollecitazione:

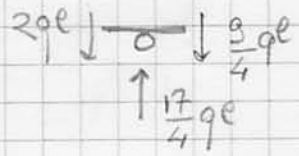


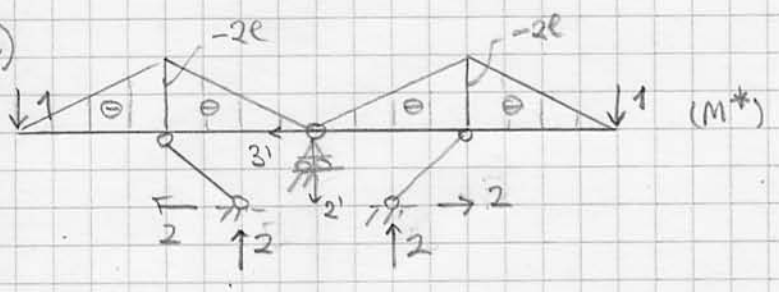




$$T_B^+ = \frac{3}{2}ql + \frac{3}{2}ql^2 \cdot \frac{1}{2l} = \frac{9}{4}ql$$

$$N_{biella} = -\frac{7}{2}ql\sqrt{2} - \frac{3}{2}ql^2\frac{\sqrt{2}}{2l} = -ql\frac{\sqrt{2}}{2}\left(7+\frac{3}{2}\right) = -\frac{17}{4}ql\sqrt{2}$$




B2)




$$2\cdot\delta_A = \frac{1}{EI_1}\int_0^{2l}\left(-x_3\right)\left(-ql^2 - q\frac{x_3^2}{2}\right)dx_3 + \frac{1}{EI_1}\int_0^{2l}\left(-x_3'\right)\left(\frac{3}{2}ql^2 - \frac{9}{4}ql\,x_3'\right)dx_3'$$

$$= +\frac{q}{EI_1}\int_0^{2l}\left(l^2x_3 + \frac{x_3^3}{2}\right)dx_3 - \frac{39}{2EI_1}\int_0^{2l}\left(x_3'l^2 - \frac{3}{2}l\,x_3'^2\right)dx_3'$$

$$= \frac{q}{EI_1}\left[\frac{l^2}{2}4l^2 + \frac{1}{8}16l^4\right] - \frac{3}{2}\frac{q}{EI_1}\left[\frac{l^2}{2}4l^2 - \frac{1}{8}\frac{l}{8}8l^3\right]$$

$$= \frac{ql^4}{EI_1}\left[4 + \frac{3}{2}\cdot 2\right] = \frac{7ql^4}{EI_1} = \frac{7\cdot 90\cdot 10^{3^2}}{210\cdot 10^{3}\cdot 3892\cdot 10^{-8}} = 1{,}70\ cm$$

↳ $\delta_A = 0{,}85\ cm$

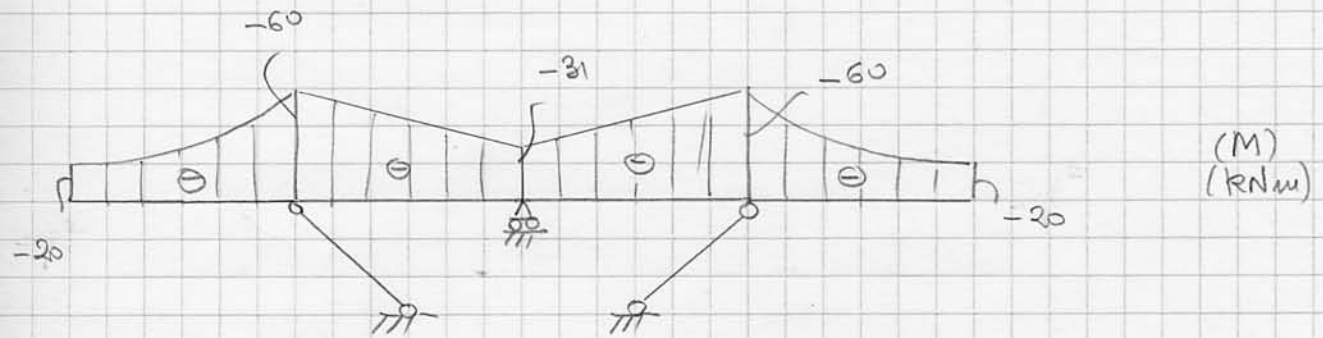
B3)

$$\eta_{10} + \eta_{11} X_1 = \eta_1$$

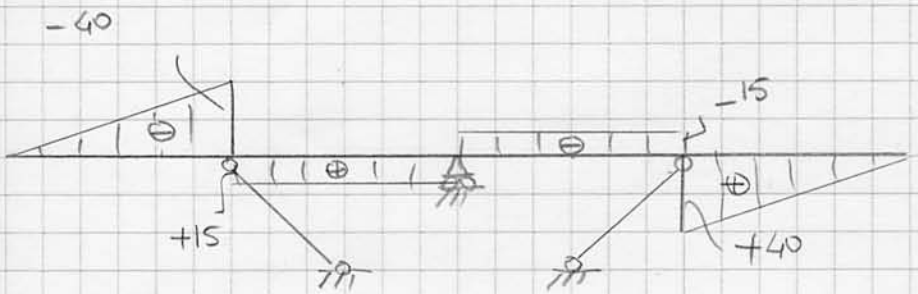
$$\eta_1 = -2\delta \frac{1}{2e} = -\frac{\delta}{e}$$

$$X_1 = \frac{3}{2} q e^2 - \frac{3EI_1\delta}{4e^2} = 30 - \frac{3 \cdot 210 \cdot 10^{8} \cdot 3892 \cdot 10^{-8} \cdot 10^{2} \cdot 10^{-3}}{4} \text{ kNm}$$

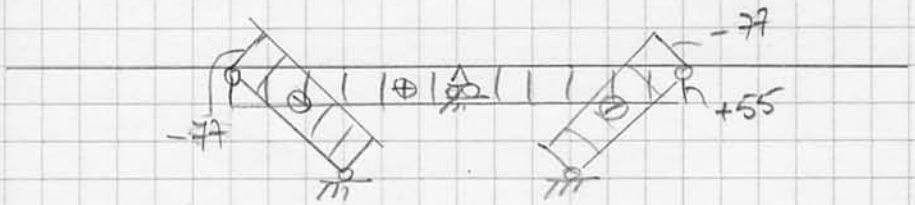
$$= 30 - 61 = -31 \text{ kNm}$$



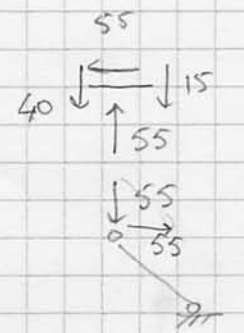

(M)
(kNm)



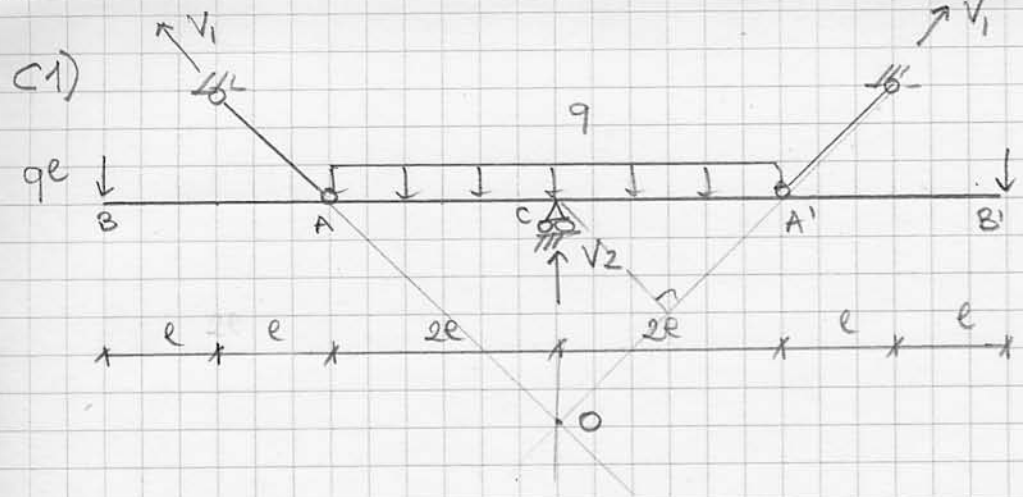

(T)
(kN)




(N)
(kN)



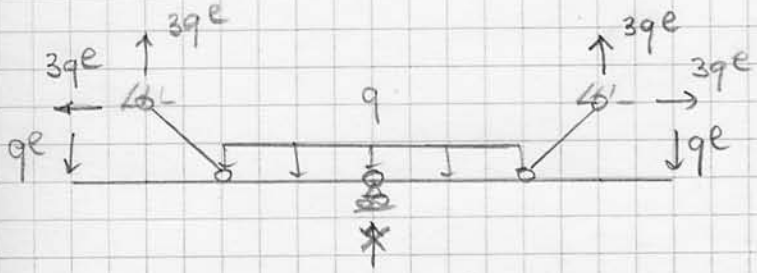
C1)

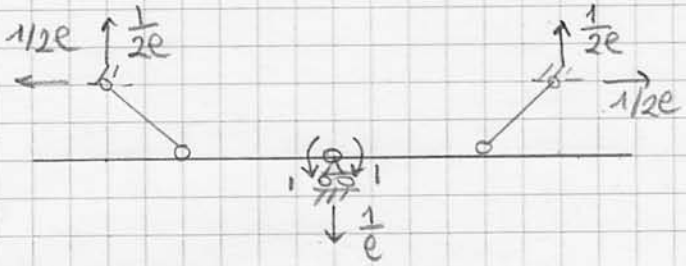


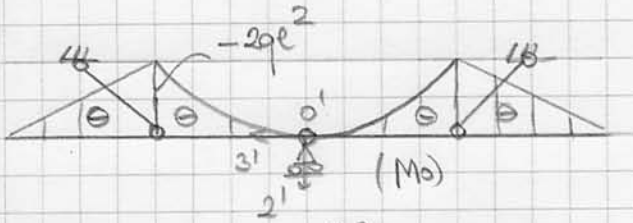

$$\begin{cases} 2\,V_1\frac{\sqrt{2}}{2} + V_2 = 6qe \\ (O') \quad qe\,4e - qe\,4e = 0 \end{cases}$$

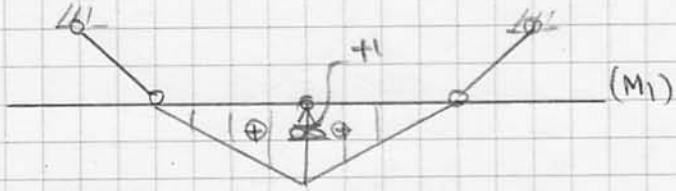
Trave una volta staticamente indeterminata.

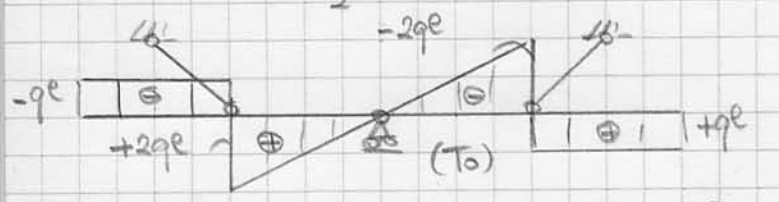
$$X_1 = M_C = V_1\,2e\frac{\sqrt{2}}{2} - 4qe^2 - 2qe^2$$
$$= V_1 e\sqrt{2} - 6qe^2$$



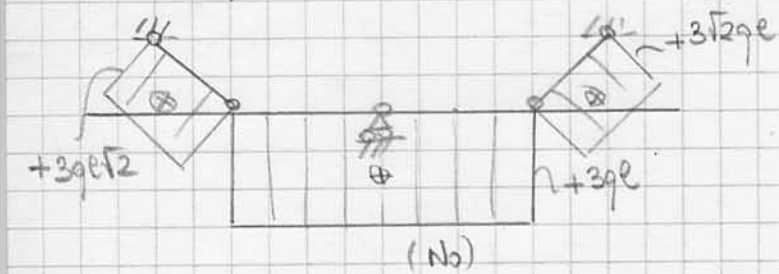



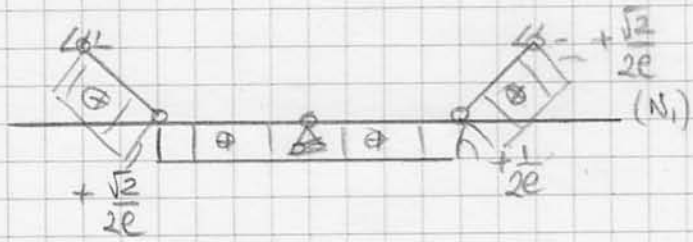







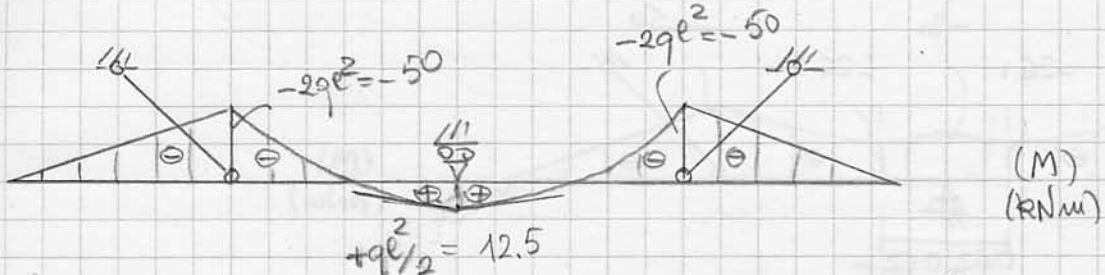



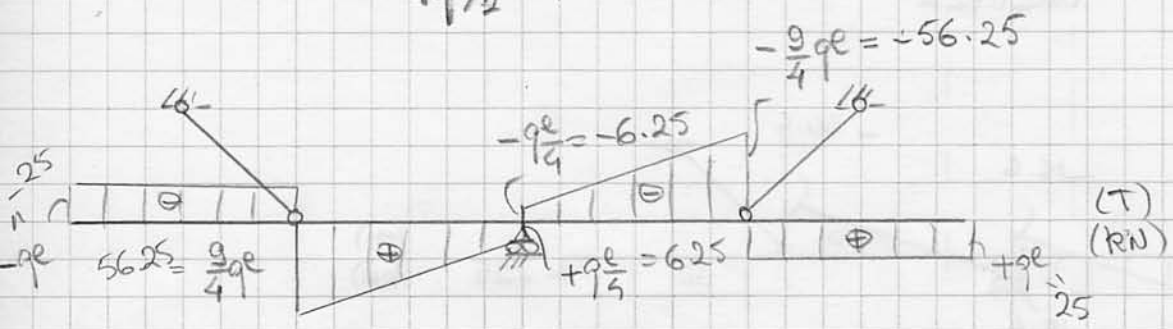

$$EI_1\,\delta_{10} = 2\int_0^{2e}\left(1-\frac{x_3'}{2e}\right)\left(-q\frac{x_3'^2}{2}\right)dx_3' = 2\left(-\frac{q}{2}\right)\int_0^{2e}\left(x_3'^2 - \frac{x_3'^3}{2e}\right)dx_3' = -\frac{2q}{2}\left[\frac{1}{3}8e^3 - \frac{1}{8e}16e^4\right]$$
$$= -2q\frac{e^3}{e}\left[\frac{8}{3} - 2\right]$$
$$= -\frac{2}{3}qe^3$$

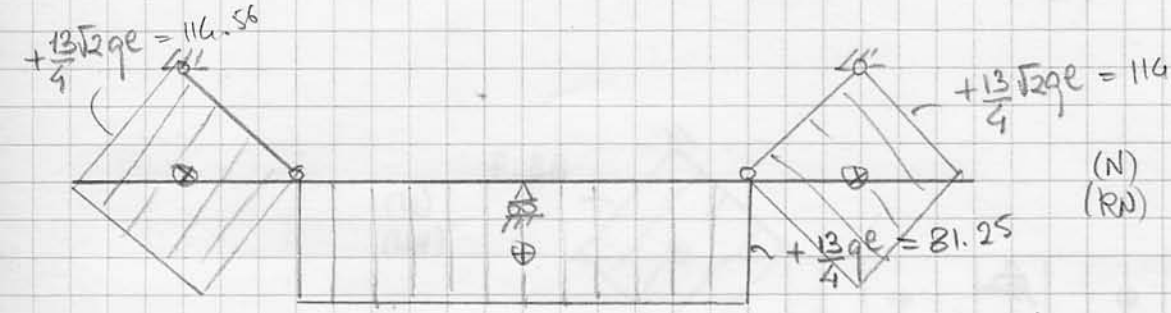
$$EI_1\,\delta_{11} = 2\cdot\frac{1}{3}\,2e = \frac{4}{3}e$$

$$X_1 = +\frac{qe^2}{2} = 12.5\ \text{kNm}$$

Diagrammi delle c.s.:



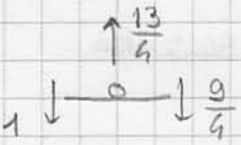




$T_A^+ = 2qe + \frac{qe}{4} = \frac{9}{4}qe$

$T_C^- = \frac{qe}{4}$

$N_{biella} = 3\sqrt{2}qe + qe\frac{\sqrt{2}}{2} = qe\sqrt{2}\left(3+\frac{1}{4}\right) = \frac{13}{4}\sqrt{2}qe$




C2)


$$2\cdot\varphi_A = \frac{2}{EI_1}\int_0^{2e}\left(\frac{x_3'}{2e}\right)\left(\frac{qe^2}{2} - \frac{qe}{4}x_3' - q\frac{x_3'^2}{2}\right)dx_3'$$

$$\hookrightarrow \varphi_A = \frac{5}{6}\frac{qe^3}{EI_1} = \frac{5}{6}\frac{25\cdot 10^{3\cdot 2}}{210\cdot 10^{8}\cdot 2772\cdot 10^{-8}} = 0,00357 = 0,2°$$

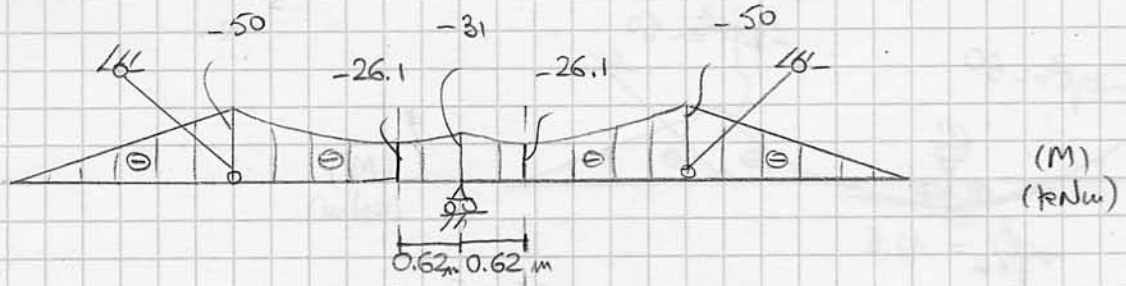
C3) $M_{10} + M_{11}X_1 = M_1$

$M_1 = -\frac{\delta}{e}$

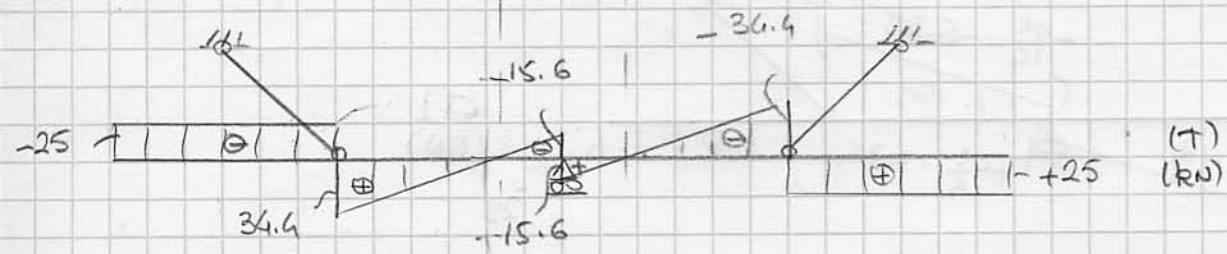
$$X_1 = \frac{qe^2}{2} - \frac{3EI_1\delta}{4e^2} = 12.5 - \frac{3\cdot 210\cdot 10^{8}\cdot 2772\cdot 10^{-8}\cdot 10^{-7}}{4}\cdot 10^{-3} \quad kNm$$

$$= 12.5 - 43,6\ kNm = -31,1\ kNm$$

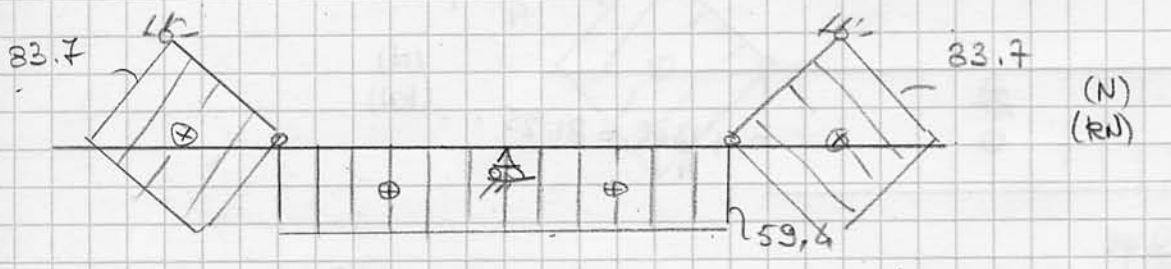
Diagrammi delle c. s.




(M)
(kNm)




(T)
(kN)

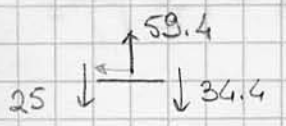



(N)
(kN)

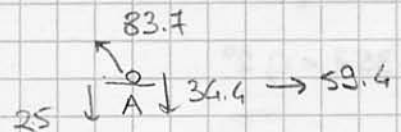
$T_{A^+} = 50 - \frac{31.1}{2} = 34.4$ kN

$T_{C^-} = -15.6$ kN

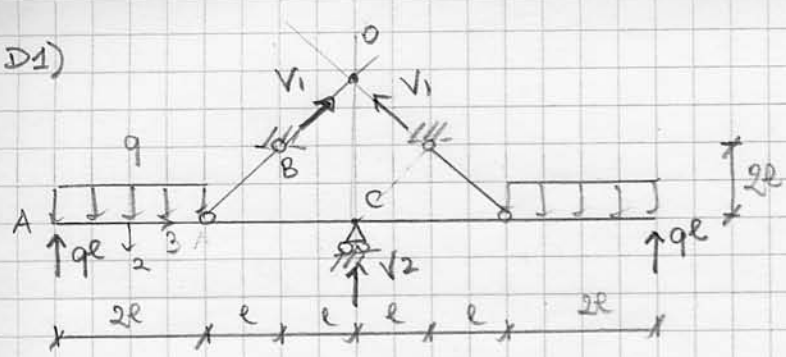
$M_{max} = -31 + 15,6 \cdot 0,62 - 25 \cdot \frac{(0,62)^2}{2} = -26,1$




$N_{bielle} = 59,4 \cdot \sqrt{2} = 83,7$ kN

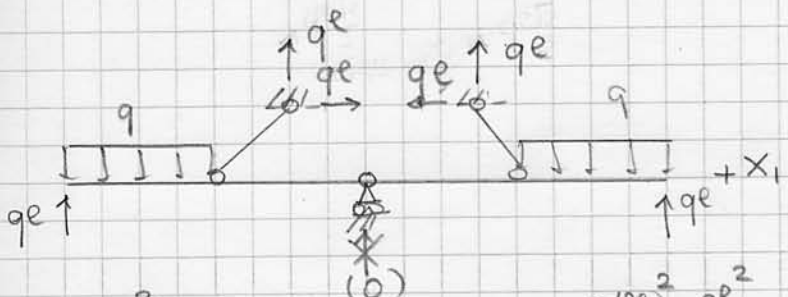


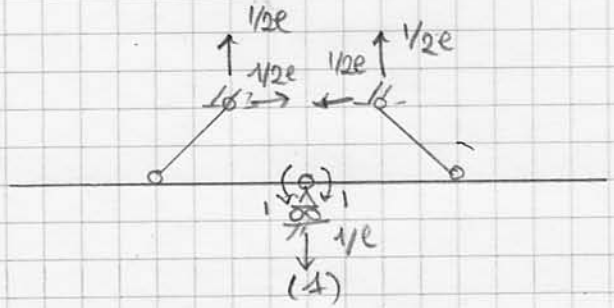
D1)

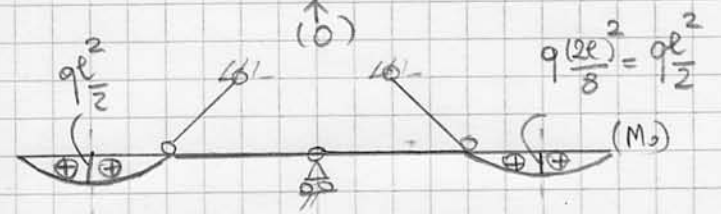


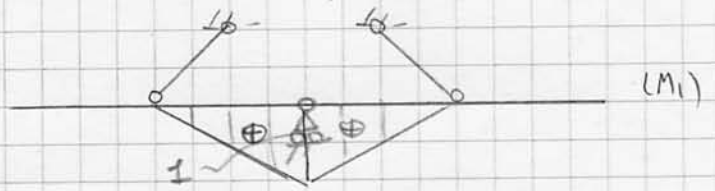

$$\begin{cases} 2V_1\frac{\sqrt{2}}{2} + V_2 = 4q\ell - 2q\ell = 2q\ell \\ (O\uparrow)\quad q\ell\, 4\ell - q\ell\, 4\ell + 2q\ell\, 3\ell - 2q\ell\, 3\ell = 0 \end{cases}$$

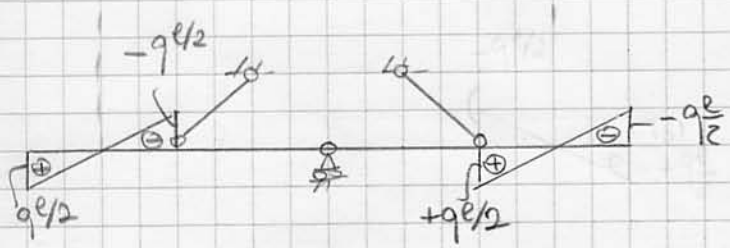
$$X_1 = M_C = V_1\, 2\ell\sqrt{2} - 2q\ell\, 3\ell + q\ell\, 4\ell$$
$$\overset{!}{=} V_1\, 2\ell\sqrt{2} - 2q\ell^2$$

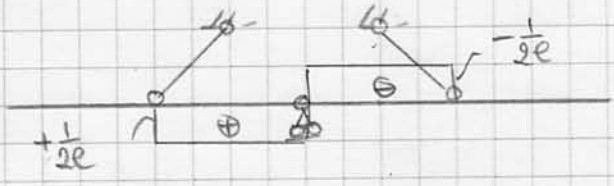


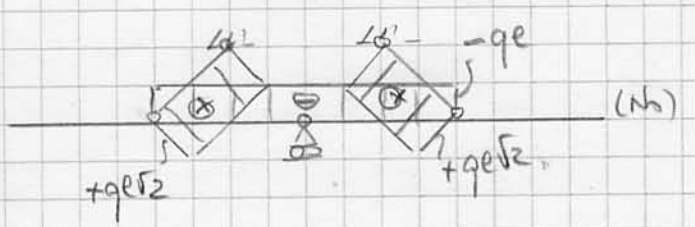



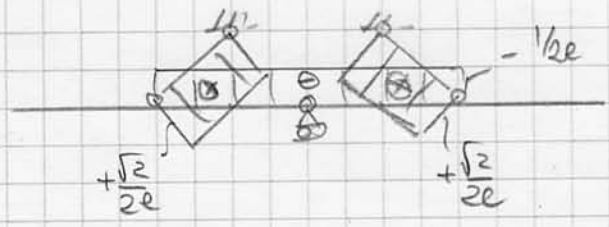



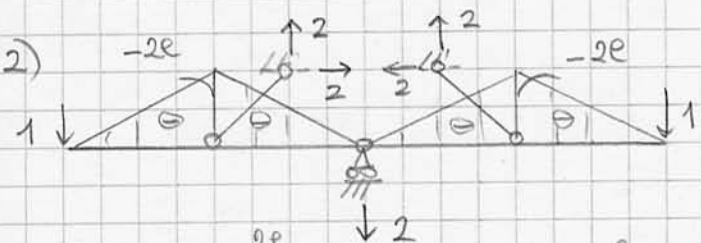

$$EI_1 M_{10} = 0$$

$$EI_1 M_{11} = 2 \cdot \frac{1}{3}\, 2\ell = \frac{4\ell}{3}$$

$$X_1 = 0$$

Diagramma delle caratteristiche della sollecitazione: vedi sistema (0)

D2)




$$2\cdot\delta_A = \frac{2}{EI_1}\int_0^{2\ell}(-x_3)\left(q\ell x_3 - q\frac{x_3^2}{2}\right)dx_3 = -\frac{2q}{2EI_1}\int_0^{2\ell}\left(2\ell x_3^2 - x_3^3\right)dx_3$$
$$= -\frac{2q}{2EI_1}\left[\frac{2\ell}{3}8\ell^3 - \frac{1}{4}16\ell^4\right] = \frac{4q\ell^4}{3EI_1}$$

D.1

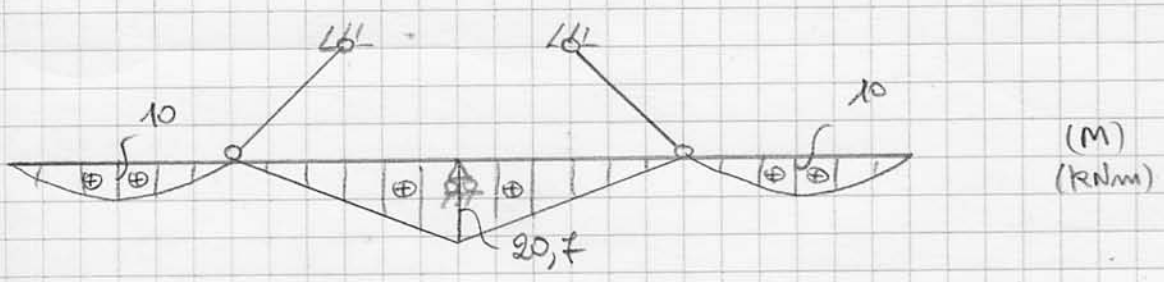
$$\delta_A = \frac{2qe^4}{3EI_1} = \frac{2 \cdot 20 \cdot 10^{3^2}}{3 \cdot 210 \cdot 10^{3} \cdot 1317 \cdot 10^{-8}} = 0{,}48 \text{ cm}$$

D3) $\mu_{10} + \mu_{11} X_1 = \mu_1$

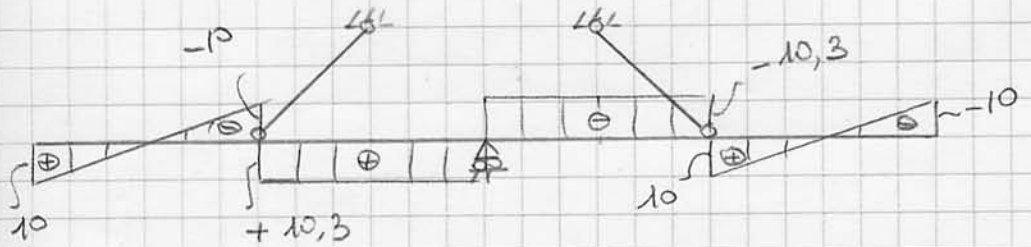
$\mu_1 = \frac{2\delta}{2e}$ , $\delta = 1$ cm

$$X_1 = \frac{3EI_1\delta}{4e^2} = \frac{3 \cdot 210 \cdot 1317 \cdot 10^{3} \cdot 10^{-8} \cdot 10^{-2} \cdot 10^{-3}}{4} = 20{,}7 \text{ kN}$$

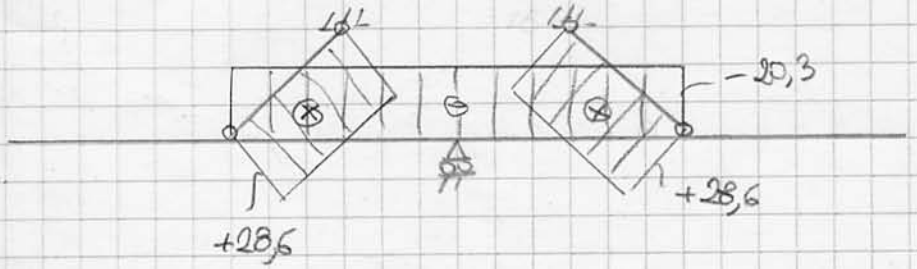
Diagramma delle caratteristiche della sollecitazione:



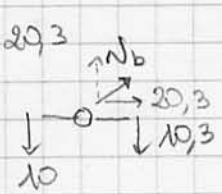

(M)
(kNm)




(T)
(kN)




(N)
(kN)




$N_b = +20{,}3\sqrt{2} = +28{,}6$ kN